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One, two, three: elementary school teachers' knowledge of the decimal numeral system

Um, dois, três: o conhecimento de professores dos anos iniciais do ensino fundamental sobre o sistema de numeração decimal

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Abstract
Over the last few years, there have been many discussions about teacher training, especially in regards to further education. Thus, this article presents partial data from the project "Creation of children's literature stories for the teaching of Mathematics". It also presents an analysis of continuing educations for teachers from the early years of Elementary School. Our goal was to identify the knowledge of teachers from the early years of Elementary School in the process of creating stories for teaching the decimal numeral system. We used the Design Experiment methodology, and the theoretical framework used for the analysis was the Mathematics Teacher's Specialised Knowledge - MTSK. In general, we identified that the training process for creating stories promotes teacher reflection, especially concerning the domain of Pedagogical Content Knowledge - PCK.

Keywords: Mathematics Education; Teacher Training; Elementary School.

Resumo

Nos últimos anos, muitas têm sido as discussões a respeito da formação de professores, principalmente no que tange sobre a formação continuada. Assim, este artigo apresenta dados parciais do projeto "Criação de histórias de Literatura infantil para o ensino de Matemática" e traz uma análise da formação continuada com as professoras dos anos iniciais do Ensino Fundamental. Nosso objetivo foi identificar o conhecimento de professores dos anos iniciais do ensino fundamental no processo de criação de uma história para o ensino do sistema de numeração decimal. Utilizamos como metodologia o Design Experiment e o referencial teórico utilizado para a análise foi o Mathematics Teacher's Specialised Knowledge - MTSK. De modo geral, identificamos que o processo formativo para a criação de histórias promove a reflexão dos docentes, principalmente em relação ao domínio do Conhecimento Pedagógico do Conteúdo - PCK.

Palavras-chave: Educação matemática; Formação de professores; Ensino Fundamental.

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Introduction

> A young man who lived near the river bank was
> crossing the river by the stone path and
> came across a monster.
> Let's bet who can jump more stones at once?
> The young man, also afraid to disagree,
> nodded with his head affirmatively.
> “I jumped them all”, celebrated the monster.
> The monster jumped ten stones at once.The young man jumped two stones, with great difficulty.
> (Kim & Jung, 2009, p. 5)

Our reflections in this article started with an excerpt from a storybook for children called “Bet with a Monster”, written by Kyoung Hwa Kim and Yoon Chul Jung. It should be pointed out that this story was selected because it is part of one of the stories used for the development of teacher education in the early years of elementary school, and it presents a possible task related to the decimal numeral system. In a fun way, the plot tells the story of a monster that likes to bet and always wins. He makes his opponents pay something or do something for him. During the betting stage, various situations of numerical comparison and counting take place. We therefore believe that it is a book that provides teachers with a wide range of planning possibilities for teaching mathematics.

This article presents part of the training process of teachers from the early stages of elementary education developed in the research project “Creation of children's literature stories for the teaching of Mathematics” and aims to “identify the knowledge of early years elementary education teachers”, as regards to their development of creating a story for teaching the decimal numeral system.

In the following sections we present the research project, how the content of the numeral system is presented in the curriculum, as well as the theoretical framework used for the analysis - Mathematics Teacher’s Specialized Knowledge - MTSK and part of the training carried out and its analysis.

The research project

The “Creation of children's literature stories for the teaching of Mathematics” research project was developed in 2017, and it was financed by Instituto Serrapilheira for one year. It should be noted that the Instituto Serrapilheira is a private development institution that values projects that develop scientific knowledge and visibility, its headquarters is in Rio de Janeiro, Brazil.

This research project was developed in a city in the interior of the state of Mato Grosso do Sul, Brazil. The Ethics Committee had a favorable opinion of the research project under the number CAEE 90142518.0.0000.5160 under ruling 2.756.607.

This investigation was founded on studies carried out in the research group TeiaMat -

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Web Research in Mathematics Education on international theorists: Hong (1996); Zazkis and Liljedahl (2009); Van den Heuvel-Panhuizen and Elia (2012); Flevares and Schiff (2014); Toor and Mgombelo (2015); Nurnberger-Haag, Alexander and Powell (2020); and national theorists: Galperin (2013); Zacarias and Moro (2005); and Alencar and Silva (2017). In addition, researchers in the group are authors of stories to teach mathematics. It has enhanced their development and the reflections of the researchers involved on the possible use of children's literature as a methodological resource.

Hong (1996) in his investigations considers that the use of children's literature in activities for the teaching of Mathematics allows them to study more about mathematical concepts. Likewise, it is stated by Zazkis and Liljedahl (2009), that children's literature is essential for the teaching of Mathematics and demonstrates the main benefits of its use. We therefore consider it relevant to support this investigation in more recent studies to identify which new aspects are being revealed.

Van den Heuvel-Panhuizen and Elia (2012) demonstrate in their studies that the use of children's literature aimed at reflecting on mathematical content enhances students' learning. The authors also demonstrate characteristics of children's books, with relevance, connection with the contents of mathematics, anticipation of concepts and offering levels and promoting work with emotions.

Complementing the previous studies, Flevares and Schiff (2014) mention that since the 90s, publications on the use of children's literature for the teaching of Mathematics have intensified, specifically the relevance of the use of children's literature is corroborated for the teaching of Mathematics, this investigation enhances its use for training actions.

As for formative aspects, Toor and Mgombelo (2015) mention the main aspects about a survey with teachers and how they are good mediators and need to be alert to the relationships that can occur with the use of children's stories.

In a more recent study, Nurnberger-Haag and Alexander and Powell (2020) still reveals the need for further studies on the use of children's literature for the teaching of Mathematics in order to advance and deepen the discussions about the real benefits and their use for teaching.

In national research, besides bringing important contributions in relation to children's stories as a resource for teaching, Galperin (2013) also presents its interdisciplinary potential. In addition to Galperin (2013), the authors Zacarias and Moro (2005) and Alencar and Silva (2017) reveal possibilities for how Children's Literature can be used in activities which teach mathematics. Alencar and Silva (2017) further expand the discussion on how children's books can be used for inclusive education.

Thus, the objective of the research project is to identify how creating children's literature stories (animated e-books and printed books), for the development of mathematical concepts, influence practices and/or professional knowledge of a group of teachers of Early Childhood Education and of Elementary School. The project hypothesizes that the creation of

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accessible children's stories can promote greater understanding and development of mathematical concepts.

In order to carry out this research, weekly meetings of 4 hours were held, using “Design Experiments”, referenced by Cobb, Confrey, Di Sessa, Lehrer and Schauble (2003), as a methodological ideology. The steps developed in the research project were: 1) Administering a questionnaire, 2) Children's literature, Mathematical studies and its curriculum and presentation of a didactic sequence to teachers, 3) Creation of children's stories collectively for the development of mathematical concepts, 4) Discussion and analysis of collective creations for rewriting and adjustments, 5) Making illustrations and their analysis, 6) Layout for animated e-book and printed books.

As a theoretical reference for the teacher’s knowledge, the Mathematics Teacher's Specialized Knowledge - MTSK, by Carrillo-Yañez, Climent, Miguel Montes, Contreras, Flores-Medrano, Escudero-Ávila, Vasco, Rojas, Flores, Aguilar-González, Ribeiro and Muñoz-Catalán (2018) was utilized.

Presented in the following sections, are how the decimal numeral system content is addressed in the Brazilian curriculum, the strengthening of our theoretical reference on teacher’s knowledge as well as the training development process.

The decimal numeral system content in the Brazilian Education Curriculum

This section will analyze the Principles and Standards for School Mathematics of the early years of Elementary School and the Brazilian Common Core Standards. We justify this section because knowledge of the Brazilian curriculum documents is necessary for there to be a better understanding of local conditions and the public selected for this research.

Despite the fact that the principles and standards for school Mathematics was written more than twenty years ago, we have chosen it because it is still widely used by schools.

The principles and standards for school Mathematics was written in 1997 and its focus is on providing support for the work developed in schools. This document contains blocks of content, which are: number & operations, geometry, measurement & data. The importance of the mathematical content of the decimal numeral system for the formation of citizens and citizenship is covered throughout the document.

In keeping with this notion, the decimal numeral system content is covered in the number and operations block, in which it appears in “number representation and operations in problem solving” (Brasil, 1997, p. 48).

This document is organized by conceptual, procedural and behavioural contents. The following details have been identified regarding the conceptual and behavioural content for the second and third grade of Elementary School:

> Recognize numbers in an everyday context.

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> Use different strategies to quantify elements of a collection: counting, matching, estimating and clustering.
> Use different strategies to identify numbers in problems which involve counting and measuring.
> Compare and order collections by the number of elements and size ordering according to their size.
> Formulate hypotheses about magnitude, by identifying the number of figures and their place in written form.
> Read, write, compare and sort familiar or frequent numbers.
> Identify the criteria which defines the classification of numbers (greater than, less than, equal) and rules used (plus 1, plus 2, double, half).
> Count forwards and backwards in ones, twos, fives, tens, etc., from any given number.
> Identify patterns in sets of numerals to name, read and write less frequent numbers.
> Use a calculator to produce and compare written numerals.
> Organize into clusters to facilitate counting and comparing large collections.
> Read, write, compare and order number notation by understanding the characteristics of the decimal numeral system (base, place value). (Brasil, 1997, p. 50).

Regarding the behavioural contents: “Curiosity to question, explore and interpret the different uses of numbers, recognizing their usefulness in everyday life.” (Brasil, 1997, p. 52)

On the fourth and fifth grade, the decimal numeral system is covered in the conceptual and procedural content:

> Identify natural numbers and rational numbers in daily routines.
> Understand and use the decimal numeral system rules to read, write, compare and order natural numbers of any relative magnitude.
> Formulate hypotheses about relative magnitude, by identifying the algorithm place in a decimal representation of a rational number.
> Extend the decimal numeral system rules to understand, read and represent the rational numbers in decimal form.
> Compare and order rational numbers in decimal form.
> Find rational numbers in decimal form on a number line. Read, write, compare and order fractional representations in frequent use.
> Recognize that rational numbers allow different (infinite) representations in fractional form.
> Identify and create equivalent fractions, by observing representations as graphs and regularities in written numbers.
> Explore different meanings of fractions in problem situations: whole-part, quotient and ratio.
> Observe that natural numbers can be expressed in fractional form.
> Relationships between fractional and decimal representations of the same rational number.
> Recognize the use of the percentage in daily routines. (Brasil, 1997, pp. 58-59)

There are several teaching guidelines that enable teacher reflection on different planning possibilities.

The brazilian common core standards was developed in 2017 and it promoted gatherings of different teaching communities across the country for development and improvement. Regarding the Mathematical content covered by the brazilian common core standards, it is composed of four strands: number, algebraic thinking, geometry, magnitude

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and measurement, statistics and probability. Since this article observes the approach of the decimal numeral system, we will focus on the number strand.

Each strand has it's own knowledge and skills' objectives organized by grade. In this article we will focus on the first elements, present in the curriculum documents Brasil (2017) as presented briefly below:

- For the 1st year, the main objects of knowledge are centered on counts of different types such as routine, ascending and descending. Identification of numbers in daily contexts, as well as the recognition of numbers as an identification code. Composition and decomposition of numbers. In addition to quantification of elements and estimates. Reading, writing and comparing natural numbers up to 100. Use of the number line. Construction of basic facts of addition and use for problem solving. Solve and elaborate problems involving different meanings of addition and subtraction.

- In the 2nd year, the objects of knowledge are reading, writing, comparing and ordering, composing and decomposing numbers up to 1000. Understanding the characteristics of the decimal numeral system (place value and the zero role). Make estimates and record the results. Build basic facts of addition and subtraction and use them in mental or written calculations. Solve and elaborate problems. Uses of the number line.

- For the 3rd year the main objects of knowledge are Reading, writing, comparing, ordering, composing and decomposing natural numbers up to 4 orders. Construction of fundamental facts of addition, subtraction and multiplication. Use the number line. Solve and elaborate problems of addition, subtraction, multiplication and division.

- In the 4th year the main objects of knowledge are reading, writing, comparing and ordering, composition and decomposition of natural numbers up to 5 orders. Solve and elaborate problems of addition, subtraction, multiplication and division with different solving strategies. Recognize the most common unit fractions (1/2, 1/3, 1/4, 1/5, 1/10 and 1/100). Use the number line.

- For the 5th year, the main objects of knowledge are reading, writing and ordering, composition and decomposition of natural numbers (up to six orders) Fractional representation of rational numbers: recognition, meanings, reading and representation on the number line. Identify equivalent fractions. Compare and order positive rational numbers (fractional and decimal representations), relating them to points on the number line. Calculation of percentages and fractional representation. Solve and elaborate problems of addition, subtraction, multiplication and division with natural numbers and rational numbers.

We note that both documents have similarities in the presentation of the contents to the respective years. We identified that the main difference between the documents is that the

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principles and standards for school mathematics aspects bring possibilities for reflections on teaching actions, not being restricted only to the presentation of the contents to be developed, which is what the brazilian common core standards presents. One of the examples that we can cite is that the principles and standards for school mathematics vestments proposes the use of problem solving as a teaching strategy for the decimal numbering system and intends to promote reflections on this content. The brazilian common core standards, on the other hand, brings the objects of knowledge and skills, without further reflections on the theme. Following on from this information, the MTSK model will be covered in the next section. Some of the aspects dealt with will be thoroughly investigated in the analysis and have been presented in the subdomains of the MTSK model, which will be discussed therein.

Mathematics Teacher's Specialised Knowledge –MTSK

The theoretical framework utilized for analysis of the training actions developed is to be based on the studies developed by Carrillo-Yañezet al. (2018) and Ribeiro, Mamore and Alencar (2019). These papers explain the Mathematics Teacher's Specialized Knowledge — MTSK. The model contemplates domains and subdomains, presents some beliefs and the effective domains that compose the mathematics teacher's work.

Two main domains that have been identified in the investigations and were consequently observed: 1) Mathematical Knowledge (MK) and 2) Pedagogical Content Knowledge (PCK). Mathematical Knowledge comprises teachers and their knowledge, the connections, rules and features that they establish in the mathematical content. The Pedagogical Content Knowledge is composed of methodologies, students' strategies, curriculum knowledge, means and resources for teaching and learning mathematics. Each domain comprises three subdomains that provide depth to these features. We also highlight that the subdomains are interrelated to foster Mathematics teaching and learning as shown in Figure 1.

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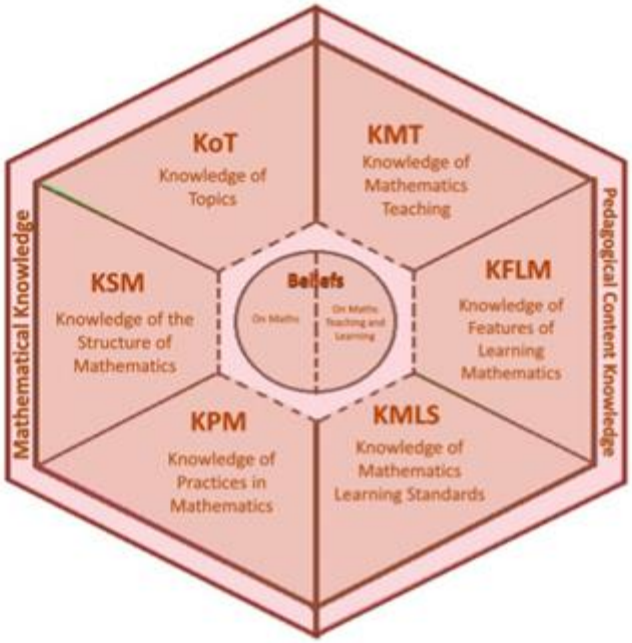

Figure 1 – Domains of the Mathematics Teacher's Specialised Knowledge
Source: Carrillo-Yañez et al. (2018)

In this model, the Mathematical Knowledge (MK) is composed of the following subdomains: i) Knowledge of Topics (KOT); ii) Knowledge of the Structure of Mathematics (KSM); iii) Knowledge of Practices in Mathematics (KPM).

The Knowledge of Topics (KOT) establishes the reason why such solutions and procedures are taken in different mathematical registers. This knowledge allows the teacher to identify what is done, why it is done and how the different types of resolution of a given result are made. It also allows foreseeing the existing connections between mathematical knowledge. An example is the teacher's knowledge when knowing that the number can take on different meanings such as: Location; Ordering; Identification; Measurement; Estimation. (Turkel and Newman, 1988).

The Knowledge of the Structure of Mathematics (KSM) involves understanding the knowledge thoroughly and perceiving the content transition in the following years. It involves connections for simplification, those of more complexity, auxiliary and transversal. An example is that the teacher knows about the context of the numbering system, he should know about the property of numeric symbols as a positional number system and non-positional (Ribeiro, Mamoré and Alencar, 2019).

The Knowledge of Practices in Mathematics (KPM) involves the synthetic capacity, of both demonstrations and valid solutions. This knowledge is also composed of demonstrations, justifications, definitions, deductions and inductions, example and counter—examples, presenting an understanding of each of the stages. This knowledge is also related to the understanding of mathematical content and its social function. An example of knowledge are some questions for reflection by the teacher: “how to present knowledge

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through children's stories and make students understand? What characteristics are important for students' understanding?" (Ribeiro, Mamoré and Alencar, 2019, p.54)

The second main, Pedagogical Content Knowledge (PCK) comprises: i) Knowledge of the characteristics of Learning Mathematics (KFLM); ii) Knowledge of the teaching of Mathematics (KMT); iii) Knowledge of mathematics learning patterns (KMLS)

The Knowledge of Features of Learning Mathematics (KFLM) as related to learning knowledge and its cognitive learning theories, and the students' learning difficulties. This knowledge covers the theories of mathematical learning, the different aspects of mathematical learning, the way in which the student understands the mathematical content and the emotional aspects of learning. An example is "knowing that one of the students' difficulties concerns the lack of differentiation regarding the cardinal and ordinal character of numbers" (Ribeiro, Mamore and Alencar, 2019, p.59)

The Knowledge of Mathematics Teaching (KMT) is related to the use of resources and different teaching strategies. It also involves the different ways of presenting content, the most suitable examples for understanding and knowing how to identify the sequence of approach to topics that makes sense for learning. We can present as an example, that the teacher knows that children's literature is an element that enhances students' reflection and learning (Carey, 1992; Kliman, 1993; Welchman-Tischler, 1992; Silva, 2012; Alencar, 2019)

The Knowledge of Mathematics Learning Standards (KMLS) and curriculum and deepening the Mathematical content in teaching. It covers the expected learning results at each level, the expected level of procedural and conceptual development and the sequencing of topics to be taught in mathematics. As an example, we can mention the analysis and previous knowledge carried out by the teachers to the curriculum documents for their planning actions.

We also emphasize that all these subdomains are related. We will see in the following section the training activities developed.

The process of creating stories: training activities

We emphasize that this article identifies the knowledge of early years elementary education teachers, as regards to their development of creating a story for teaching the decimal numeral system. In order to carry out this research, weekly meetings of 4 hours were held, using "Design Experiments", referenced by Cobb, et. al. (2003), as a methodological ideology. The group was composed of five individuals who work for public schools in Mato Grosso do Sul State. All formative stages were recorded and filmed using audio recorders and camcorders. Thus, all data were transcribed in a file and in this article, we use excerpts from the training carried out.

The realization of our investigation was carried out in six stages as specified in the sequence:

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1) Administering a questionnaire,

In this initial stage, we administered a questionnaire with open questions to become acquainted with their professional experiences and memories regarding their teachers when they were students, and their relationship with children's literature for teaching mathematics. The questions asked were as follows: 1) Have you ever seen, experienced or used children's literature to teach Mathematics? If so, provide some details about your experience. 2) Which factors do you consider relevant/irrelevant when using children's literature to teach Mathematics? 3) What mathematical content in Early Childhood Education do you consider important to be covered through children's literature? Explain. 4) What mathematical knowledge of the early years of Elementary School do you consider important enough to be covered through children's literature? Explain. 5) Have you ever created or adapted any stories for children to teach Mathematics or other subjects?

It should be noted that this was an initial calendar to get to know the profile of teachers who were in continuing education and there was no questionnaire at the end of the training.

2) Children's literature, Mathematical studies, and its curriculum and presentation of a didactic sequence to teachers

With the analysis of the answers to the questionnaire, we intend to use them for the composition and elaboration of didactic sequences for the group of teachers. To carry out the didactic sequences, we made preliminary studies on the use of children's literature for the teaching of Mathematics. These studies supported the organization of didactic sequences.

In addition, at this stage studies were carried out on the curriculum documents in order to assist teachers' learning about the content of the decimal numbering system.

3) Creation of children's stories collectively for the development of mathematical concepts.

In this stage, moments were held for teachers to create stories collectively, for that we made dynamics and made the stories in stages and following items of reflection such as: what specificity of the content to address, how to mention, which characters and which illustrations.

4) Discussion and analysis of collective creations for rewriting and adjustments

With the completion of the initial writing of the stories, we present the records of the teachers' writings, exposing them for the collective interpretation and analysis of the group. These records served as a basis for analyzing the most efficient and appropriate interventions and approaches. In this stage, versions of the stories were rewritten for possible adjustments. It should be noted that the university's teacher was present in all the training courses carried out.

5) Making illustrations and their analysis.

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With the final writing of children's stories for the teaching of Mathematics, these were taken to the professional who made the illustrations. With the first layouts, collective analyzes were carried out by the group of teachers who will approve or disapprove the illustrations and indicate possible corrections.

6) Layout for animated e-book and printed books

With the final elaboration of the illustrations, the conventional book was designed and began the process of elaborating the animated e-book. And the book was in the final process of publication.

In the next section, we will present the analysis of the data revealed in the continuing education.

Analysis: the knowledge revealed

Through the analysis of the teachers' answers of the questionnaire, we identified, regarding the first question, that the majority (four of them, which is 80%) had already experienced it as students and/or teachers and some of the teachers had already developed activities related to children's literature in other subjects. However, two of them said that they had difficulties in relating Literature to Mathematics. Some of the group presented adaptations of stories, such as: Snow White, Little Red Riding Hood, and Goldilocks to develop situations which contained mathematical content. Usually, they were connected to situational problems which involved the four operations. We identified that despite the use of children's literature being limited to only four operations, this concern directed us towards subdomain knowledge, the Knowledge of Mathematics Teaching (KMT), which shows the importance of the teacher's knowledge with regards to the usage of different resources to teach Mathematics.

We identified through the analysis of the second question that all of the teachers considered the use of children's literature important for teaching Mathematics and that they believed it can lead to a more pleasant and comprehensible learning experience for students. We perceived that this particular data indicates the Knowledge of Features of Learning Mathematics (KFLM). The use of children's literature suggests an interdisciplinary purpose; however, it also stirs and awakens feelings involved in moments of learning that we infer to be closely linked to theories of learning psychology.

Therein, we identified a positive acceptance on behalf of the group regarding the training proposal. In this article, we will not present the data for question number three since this covers the particulars of the Early Childhood Education teachers' group.

The decimal numeral system arose as a result of the teachers' answers to question number four, in which we identified that the greatest subject of interest was found in multiplication, addition and the numeral system. The numeral system was chosen due to the importance of being familiar with this content in order to teach addition and multiplication. The following graph presents these statements:

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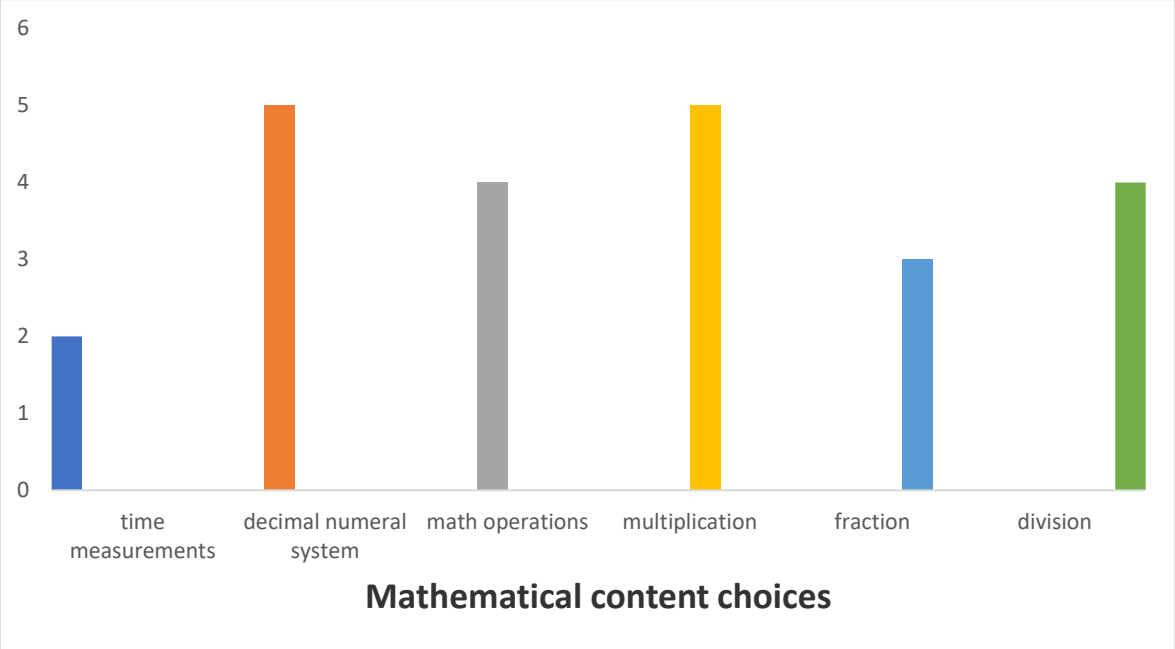

Figure 2 – Areas of Interest
Source: author.

It should be noted that each teacher was able to suggest more than one type of content, which explains the numbers in this graph.

Through the analysis of the fifth question, we identified the use of children's literature on behalf of the teachers as a methodological resource for teaching. It should be noted that 60% (three teachers) use this resource within their pedagogical practices and 40% (two teachers) have difficulties in developing mathematical activities using children's literature. We noticed that the diversity of resources and methodologies to be used in class was a concern for the teachers. This fact leads us towards the Knowledge of Mathematics Teaching (KMT). However, we realize the difficulties and challenges they face to propose new resources and teaching strategies. We characterized this difficulty within the subdomains Knowledge of Topics (KOT) and Knowledge of Practices in Mathematics (KPM). We can infer that the teachers' difficulties in the analysis of different mathematical registers of representation and different mathematical procedures might promote difficulties in pedagogical activities.

The second stage of the training proposed studying the use of children's literature to teach Mathematics, the Brazilian core curriculum, as well as those of other nations (Mexico, Italy, Norway, Spain, Portugal, etc.) and the definition of aspects of decimal numeral system content which could be addressed when developing a story for children, and how the plot could make for a better understanding.

Throughout the following training, we provided lessons on how to use children's literature to teach Mathematics. It started with the study and reflection of some literary works, such as the book "Bet with a Monster", written by Kyoung Hwa Kim and Yoon Chul

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Jung. It is a short story which leads the reader to think about the different bets the monster had made. The illustration fosters the idea of counting and reasoning among quantities, which enables the development of a study about the decimal numeral system in the Elementary School.

This narrative was presented at the introduction of this article and a further excerpt which follows:

> One day, an old man who lived on top of a hill went to the mountains to cut wood and met the monster.
> - Old man, let's bet who can eat more watermelons?
> - Yes, let's go!
> Disagreeing with the monster could be dangerous! So, the old man agreed to the bet.
> The monster devoured NINE watermelons at once.
> The old man ate only ONE PIECE of watermelon, with great difficulty.
> - I won! - the monster cheered. (Kim & Jung, 2009, pp. 3-4, emphasis added by the authors)

Among the several excerpts of the book that provide this comparison among quantities, we have selected this one to analyse due to the questions which were raised by the teachers during the training. They were concerned about which aspects of the decimal numeral system content were involved, since numbers, counting and quantity are usually used in school activities. One of them shared the following:

> "Teaching the decimal numeral system comprises only numbers and counting, one, two, three, doesn't it? Do I need to teach comparison? What exactly makes up the decimal numeral system? (Teacher D)

We identified, in this comment, the teacher's difficulty regarding the Knowledge of Topics (KOT). One of the difficulties found by the group was the question of how to compare the number nine to a piece.

Therefore, the trainer-researcher had to mediate and review what the topics of the decimal numeral system content were and suggest some ways to teach them. It was necessary to study the Brazilian core curriculum at first. Then, some core curricula from other countries were analysed to make the group see the differences and similarities between the documents.

The analysis of both Brazilian documents Brasil (1997) and Brasil (2017) brought some interesting elements to the study and reflection and enabled us to resolve the teachers' doubts. This also encouraged them to become more informed about the organizational content involved in their teaching. The same process occurred with foreign curricula, which expanded the elements for study and discussion. Therefore, we can deduce that studying the curricular aspects promoted to the teachers strengthened their knowledge and understanding of Knowledge of Mathematics Learning Standards (KMLS).

It should be noted that foreign curricula analysis was necessary in this training process; the stories which were created are available as animated e-books translated in different languages (English, Guarani, Spanish and Portuguese).

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Through the teachers' answers, it was possible to identify the challenges they deal with when teaching Mathematics for students in the early years of Elementary School. Some studies and reflections had to be studied and remembered during the training process: What is the decimal numeral system? What are its features? What are the elements of this content? All these questions served as a basis for creating a story.

During the creating process of "The magical tree of numbers", the teachers were concerned about which elements of the content knowledge should be used and how to do so. In this instance, they had to reflect on the importance of the decimal numeral system for society: Why should we know the decimal numeral system? These questions led us to the Knowledge of Topics (KOT) and it was one of the moments for learning reflection.

It was noted in one of their answers the concern over the knowledge of the decimal numeral system to be used in later content:

> We know the decimal numeral system is important since it will be used in different Mathematical content. For example, when solving problems and operations (Teacher B).

We identified from this comment, training activities and their practice made this teacher become aware of the Knowledge of the Structure of Mathematics (KSM). It was also noted that the teacher established a relationship between the decimal numeral system and different content, which we can identify as the Knowledge of Mathematics Learning Standards (KMLS).

The process of creating stories made the teachers start to reflect on the importance of knowing the decimal numeral system for our daily activities and how they could present this, using stories as a resource. It refers to the Knowledge of Practices in Mathematics (KPM), which promotes explanations and definitions of mathematical knowledge socially.

The process of choosing the illustrations and plot also fostered the Knowledge of Mathematics Teaching (KMT). The teachers were concerned about choosing words which were appropriate for each age and grade. The evidence of a good use of mathematical language and the use of illustrations were elaborated so that the students would have a better understanding.

Conclusion

This article presented partial data of a teachers' training group from the early years of Elementary School. The goal was to identify their knowledge during the creation process of a story to teach the decimal numeral system.

In general, we presented the research project characteristics and the *Design experiment* methodology used during the training process. It should be noted that our objective was to know more about the mathematics teacher's specialized knowledge in the early years of Elementary School. The Mathematics Teacher's Specialised Knowledge

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(MTSK) was used as the theoretical analysis. We affirm that the use of this theoretical framework was important to understand the training practice performed and the professional knowledge it revealed.

With this in mind, through each training stage, we tried to show examples of the MTSK subdomains, which allowed us to have greater knowledge regarding this training process.

The results of the analysis show the PCK as a positive factor in the teachers' understanding. We identified its subdomains (KFLM, KMT & KMLS), which provided enrichment of the actions carried out during the training.

We identified that teachers had difficulties with the MK, especially the Knowledge of Topics (KOT) and Knowledge of Practices in Mathematics (KPM). With regards to the former, we inferred that one of the probable causes is the initial training of these professionals, and the latter, we believe, is a consequence of the former, which makes it difficult for teachers to identify the possible relationships between mathematical content, their demonstrations and validate them socially.

This research project, whose main action was continuous training, enabled teachers to create stories and with them the possibility of reflections and learning, the limitation was the fear of the teacher to participate actively in their learning. We infer that this situation may have occurred due to the formative model that has been used in the last decades for teachers, in which they are only recipients of knowledge. Therefore, this research project promoted a training that included a differentiated formative model. In this sense, creating stories can be a new resource to be used when developing training activities that might benefit reflections upon specialized knowledge.

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