



Design principles of a TPACK-based experience in Mathematics Pre-service Teacher Education

Princípios de design de uma experiência baseada no TPACK na formação inicial de professores de Matemática

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Abstract

TPACK (*Technological Pedagogical Content Knowledge*) conceptualizes the teacher's professional knowledge for an effective integration of technology in education, which is often a challenge in mathematics teaching. Since it is essential to guarantee its development in pre-service mathematics teacher education, this study aims, following a Design-Based Research, to understand how the design principles adopted in a teacher education experience centred on developing the TPACK of mathematics prospective teachers (PTs) of 7th to 12th school years contribute to promote this knowledge. The results, based on the analysis of PTs' written solutions of the tasks proposed in the teacher training and responses to an interview and questionnaire, show that the teacher education experience promoted PTs' technological skills, evidencing the mobilization of TK (*Technological Knowledge*), TCK (*Technological Content Knowledge*) and TPK (*Technological Pedagogical Knowledge*) in articulation with the PCK (*Pedagogical Content Knowledge*) and, consequently, the TPACK.

Keywords: TPACK, Pre-service mathematics teacher education, *Design-Based Research*

Resumo

O TPACK (*Technological Pedagogical Content Knowledge*) conceptualiza o conhecimento profissional do professor para uma efetiva integração da tecnologia na educação, que é um desafio frequente no ensino da Matemática. Sendo essencial garantir o seu desenvolvimento na formação inicial de professores de Matemática, este estudo, seguindo uma Investigação Baseada em *Design*, visa compreender como os princípios de design adotados numa experiência de formação centrada no desenvolvimento do TPACK de futuros professores (FP) de Matemática do 7.º ao 12.º anos de escolaridade contribuem para promover esse conhecimento. Os resultados, com base na análise de resoluções escritas dos FP das tarefas propostas na formação e respostas a entrevista e questionário, mostram que a experiência de formação promoveu competências tecnológicas dos FP, evidenciando a mobilização do TK (*Technological Knowledge*), TCK (*Technological Content Knowledge*) e TPK (*Technological Pedagogical Knowledge*) em articulação com o PCK (*Pedagogical Content Knowledge*) e, consequentemente, com o TPACK.

Palavras-chave: TPACK, Formação inicial de professores de Matemática, Investigação Baseada em *Design*.

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Introduction

The evolution of digital technologies in the 21st century and its wide accessibility and potential for educational purposes raises several questions about the professional knowledge of the teacher to answer the challenges and demands of an education significantly transformed by its use (Albuquerque et al., 2006; Mishra & Koehler, 2006; Niess, 2012a). The TPACK (Technological Pedagogical and Content Knowledge) model, proposed by Mishra and Koehler (2006), has been consolidated in the conceptualization of the knowledge that teachers need for an effective integration of digital technology in teaching, influencing the recent research on the development of that knowledge (Gutiérrez-Fallas & Henriques, 2020; Niess & Gillow-Wiles, 2017).

In Mathematics Education, in particular, curriculum guidelines recommend the use of technology as an essential resource in students' learning, supporting them to assign meaning to mathematical ideas, to reason and communicate their thinking (NCTM, 2014). In order to respond to these demands, which have proved to be a challenge for teachers, it is essential that pre-service teacher education programs “integrate diverse educational technologies that are accessible to prospective teachers” (Gutiérrez-Fallas & Henriques, 2020. p. 200), to involve them in their exploration, providing opportunities for mathematics prospective teachers (PTs) develop their TPACK to create learning contexts, integrating technologies, that support students' mathematics learning (AMTE, 2017; Niess, 2012a). However, despite the effort that has been made to integrate technology in pre-service teacher education, several studies (Niess, 2012a; Niess & Gillow-Wiles, 2017) reveal difficulties in this integration, which is still little considered, so the PTs' preparation for teaching mathematics with technology is a theme that requires more research (Niess, 2012b), namely focused on design principles to consider in specific contexts of pre-service teacher education to promote TPACK.

In this context, it was pertinent to conduct a study based on an innovative training experience with mathematics PTs, following a design-based research methodology (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003), which aims to develop a local theory on how develop their TPACK in the context of pre-service teacher education.

To contribute with an innovative and specific way of developing TPACK in the context of pre-service teacher training, which can be useful for the scientific and educational community, in this paper we present a study aiming to understand how the design principles adopted in a training experience, focus on developing the TPACK of mathematics prospective teachers of 7th to 12th school years, contribute to promote this knowledge.

TPACK on mathematics pre-service teacher education

Mathematics teachers have been encouraged to use technology in their teaching practices, particularly educational technology, as it is a resource that influences the

mathematics taught and the way students learn, and its potential to improve these processes is recognized (NCTM, 2014; AMTE, 2017). However, as Earle (2002) argues, the integration of technology in teaching should not focus on the technology itself, but on the implementation of practices in which students are actively involved, to make their learning more meaningful.

For Krumsvik (2014) "there is a need to develop both theoretical foundations and models for a more in-depth understanding of digital competence in teacher education" (p. 272). In this sense, some international organizations have proposed models that aim to define principles or standards to teachers' use of technology and guide its integration in pre-service teacher education programs. For example, UNESCO (2008) presents a set of guidelines for the use of technology in pre-service teacher education in a framework (*UNESCO ICT Competency Standards for Teachers, ICT-CFT*), in which is defined the following three levels of teachers' skills in the use and integration of technology:

1. **Technology literacy.** At this first level, teachers' skills are expected to include basic digital literacy and digital citizenship skills, along with the ability to select and use tutorials, games, software and the web. They must also use technologies to manage classroom data and support their own professional learning.
2. **Knowledge deepening.** In the second level, the teacher's competencies include the ability to manage information, structure problematic tasks and integrate tools, such as software, in teaching and learning specific content through student-centred teaching strategies. They must also support collaborative projects, for which they must use networked and web-based resources, to create and monitor students' project plans, as well as to collaborate with other teachers and to support their own professional learning.
3. **Knowledge creation.** In the third level, competent teachers are expected to design learning environments based on technological tools, and to use them to support the development of students' knowledge and critical thinking skills, their continuous and reflective learning, and to create knowledge communities for students and colleagues. They will also be able to play a leading role with colleagues in creating and implementing a vision of the school as a community based on innovation and continuous learning, enriched by technology.

This emphasis on the integration of technology in education has raised the need to develop models to represent the knowledge of the teacher necessary to make this integration successfully. Mishra and Koehler (2006) propose a theoretical model, TPACK, characterized by the simultaneous and relational integration of three domains of the teacher's professional knowledge: content, pedagogy and technology (Figure 1). This integration results in the emergence of seven types of knowledge that compose the TPACK framework (Mishra & Koehler, 2006): content knowledge (CK), pedagogical knowledge (PK), technological knowledge (TK), pedagogical content knowledge (PCK), technological and pedagogical knowledge (TPK), technological content knowledge (TCK), and technological, pedagogical and content knowledge (TPACK).

In particular, we consider CK, PK and PCK knowledge as classic knowledge that has been discussed by different authors since 1980 (Grossman, 1989; Shulman, 1986), and we consider TPK, TCK and TPACK as emerging knowledge, as they emerge by integrating TK with CK and PK.

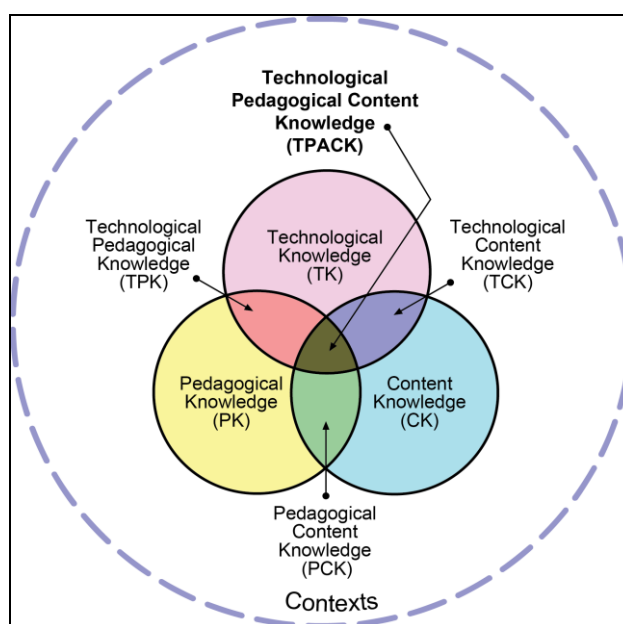


Figure 1. *Technological Pedagogical Content Knowledge Model*

Source: Koehler et al. (2014, p. 103). Reproduced with permission of the publisher, © 2012 available on tpack.org

The TPACK model suggests that teachers need to have a deep understanding of each of the domains of this knowledge to plan and develop curriculum activities that aim to guide and promote students' learning with technology. However, TPACK is more than content knowledge, pedagogy and technology, considered individually, but it involves a dynamic relationship between these domains of knowledge and the teacher's skills to teach specific content at specific school levels (Koehler, Mishra, Kereluik, Shin & Graham, 2014; Niess, 2012b).

According to Ponte (2012), planning, developing the curriculum, promoting learning and teaching specific content, are teacher's actions associated with his didactic knowledge. For this author:

The professional knowledge of mathematics teacher includes several aspects, of which we are mainly interested in what refers to the teaching practice, the one where the specificity of the Mathematics discipline is most strongly felt, and that we designate as didactic knowledge (Ponte, 2012, pp. 86-87).

In this perspective, TPACK is assumed as specialized professional knowledge, and in this study in particular, as the didactic knowledge of the teacher necessary to effectively integrate technology in the teaching and learning of Mathematics.

The pre-service training of mathematics teachers is a complex process and influenced

by several elements that interact with each other (Ponte & Chapman, 2008). A mathematics teacher should have knowledge of: the nature of mathematics; mathematical content; curriculum objectives; the way students understand and learn mathematical content; how to present ideas to students so that they are learned; and classroom management (Albuquerque et al., 2006). This complexity of the teacher's professional knowledge raises questions about the PTs' training process, namely how to prepare them to face the challenges they will face in future professional practice.

Part of these challenges emerge from the constant technological development, especially the educational software, and the curriculum recommendations for its use (NCTM, 2014). Thus, it is necessary that the pre-service teacher training would promote the development of a deepen knowledge regarding the integration of technology in the teaching and learning of mathematics, fulfilling the recommendations of AMTE (2017) about the mathematics PT's profile that characterize him as proficient in technological tools, both for their own in mathematics and to support students' mathematical learning.

Regarding to this, Niess (2012a) suggests that training programs be planned to prepare the PTs for teaching mathematics with technology, with this learning being a process of acquiring technological knowledge and of articulation with didactic knowledge, considering how technologies can impact teaching strategies, in the school curriculum and the way students learn contents.

Several approaches have emerged in the literature to develop the PTs' TPACK that, according to Koehler et al. (2014), can be classified in three possible ways (Figure 2): (i) from PCK to TPACK, (ii) from TPK to TPACK, and (iii) PCK and TPACK simultaneously. The authors also emphasize that these paths are not necessarily disjoint, on the contrary, there is some overlap between these different approaches.

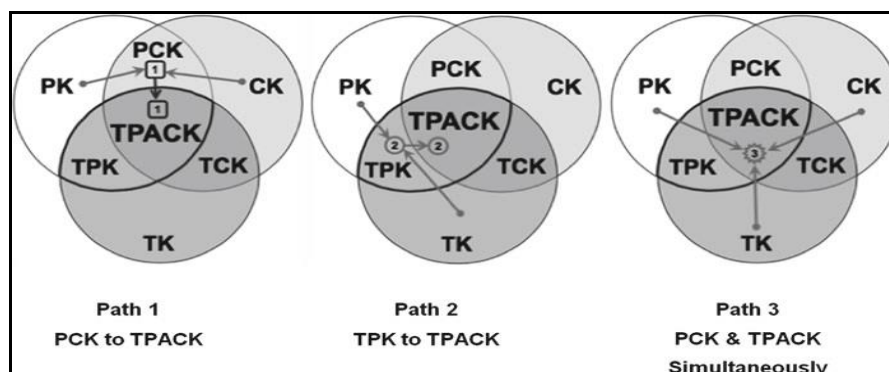


Figure 2. Paths to develop TPACK

Source: Koehler et al. (2014, p. 106)

From PCK to TPACK. In this approach, technology is introduced as a way to support and deepen the teaching strategies already established and used by the teacher. That is, the teacher first develops the PCK through experiments that do not involve the use of technology and later, the teacher learns how technology can be used to improve the strategies

with which he is already familiar. The authors point out that one of the limitations of this path is shaped by the set of conceptions and beliefs that the future teacher has internalized about the teaching and learning of certain content, which limits his vision and willingness to try new strategies supported by technology.

From TPK to TPACK. This approach is used when a prospective teacher participates in a technology training course before developing the PCK, which aims to broadly address technology in different areas, such as the use of Web 2.0, software and other digital tools, but does not yet know specific pedagogical strategies for teaching content. Thus, the first step on this path is to develop TK and TPK in initial experiences, and then going through the pedagogical disciplines the future teacher develops the PCK and expands the TPK in TPACK. Finally, the authors point out that this “is the “default approach” in most institutions of higher learning. Technology is relegated to a few courses, and teachers are left to take those lessons and apply them to their own content areas” (p. 107, quotation marks in the original).

PCK and TPACK simultaneously. In a context of pre-service teacher training, this path consists of systematically integrating technology in the disciplines of methodology, pedagogy or didactics in the specific content area. That is, a teacher training program that follows this approach, may not have a specific technology discipline, but promote the use of technology to teach content in the different disciplines of methodology and didactics in their respective professional area. Thus, the PTs will develop their PCK and TPACK simultaneously. According to the authors, one of the challenges that arises in this approach is the cognitive load to which the PTs are exposed when trying to simultaneously develop knowledge associated with the pedagogical, content and technological domains.

In the study presented in this paper, the training experience follows the third path, PCK and TPACK simultaneously, assuming that this approach promotes opportunities for the PTs to integrate technology in conjunction with mathematics and its didactics in tasks and activities proposed in the course of didactics of mathematics. And it is also intended that “Teachers must experience for themselves, as learners, the potentials and pitfalls of digital tool in the learning of mathematics, thus gain knowledge about how students can learn mathematics in various digital environments” (Leung, 2017, p. 6).

Methodology of the study

Methodologic options, context and participants

This study, based on a pre-service training experience (Gutiérrez-Fallas, 2019), follows a Design Based Research (DBR) methodology, covering two complete design cycles (preparation, experimentation and retrospective analysis) (Cobb et al., 2003; Ponte, Carvalho, Mata-Pereira & Quaresma, 2016), to develop a local theory on how to promote the TPACK of mathematics PTs.

The **preparation phase** included formulating the training objectives of the experience to develop the TPACK, planning tasks and formulating the design principles of the

intervention to be carried out. The **experimentation phase** contemplated the implementation of the tasks in 11 training sessions (2 hours each), following an exploratory teaching approach (Canavarro, 2011) that contemplates three moments: (i) presentation and introduction of the task, giving instructions for solving the task; (ii) PTs' autonomous work in solving the task, individually or in pairs; and (iii) collective discussion, consisting of a space for reflection and sharing the task solutions. Finally, the **retrospective analysis phase** had the purpose of redefining the design principles based on the reflection about the obtained results in the experimentation of the proposal.

In both design cycles, the training experience focused on the TPACK development of mathematics PTs of school levels from 7th to 12th year, was carried out in a Didactics course from the 2nd semester of the 1st year of a master's degree in mathematics teaching from a Portuguese university that provides professional qualification for teaching. This course aims to provide PTs with fundamental didactic instruments for the teaching of Mathematics, addressing themes associated with the learning of the theme, curriculum management, learning assessment and didactic resources, including technology. This paper authors had the role of teacher educators, the second author being the full professor of the course and the first author a collaborating professor.

The PTs who are attending the Didactics course in each design cycle in which the training experience took place, voluntarily participated in this study through an informed consent given to them in the first week of the academic semester. In the 1st cycle, participated 6 female PTs (Patrícia, Cristina, Sara, Vitória, Marta and Paula) and, in the 2nd cycle, 4 female and 2 male PTs (Ana, Sofia, Glória, Isabel, Samuel and Tiago), whose names are fictitious. These PTs, mathematics graduates, acquired some pedagogical knowledge in a previous course of pre-service teacher education, but without involving technology. So, only one PT indicated that had been trained in technology in extracurricular courses, the rest of PTS indicated that they had not previously additional training in the area of technology in general nor in education. However, they assume that they have some technological knowledge acquired autonomously through their experience with daily lives technologies.

The descriptive and interpretative (Coutinho, 2011) analysis of data, collected in the experimentation phase of the training experience, in both DBR cycles, was focus on the design of the experience based on the TPACK and respective theoretical framework, described at follow. In this study, in particular, were analysed: PTs' written work on task solving (RT #) in the training sessions; audio and video records of the semi-structured interview (I) and the answers to the open questionnaire (Q), both carried out with the PTs at the end of each experimentation phase and focus on the acquired knowledge in the training experience, being asked to report: their attitudes, use and knowledge about the technology; and conceptions about the contribution of the work carried out to their professional knowledge and the integration of technology in future practice.

These data were selected because they are illustrative of knowledge related to PTs' TPACK acquired, and whose analysis is intended to be representative of the training

experience carried out and how it can contribute to develop the TPACK as a professional knowledge of the PTs necessary to effectively integrate the technology in teaching and learning mathematics.

The training experience based on TPACK: characteristics and design principles

The training experience is part of a study by Gutiérrez-Fallas (2019), realized in the Didactics course program and supported on the TPACK framework (Mishra & Koehler, 2006), considering that its main objective is to develop the mathematics PTs' TPACK. In particular, the objectives aimed to develop in initial training are:

- Improve the PTs' conceptions about the integration of technology in the teaching and learning of Mathematics.
- Articulate PTs' technological knowledge with didactic knowledge for an efficient use of technological tools in the teaching and learning of Mathematics.

To achieve these objectives, the experience based on TPACK contemplated two main dimensions. The dimension called **pedagogical practice** consisted in defining an approach that guides the experimentation of the training experience in the classroom. This approach follows an educational and learning trajectory (Figure 3) that involved twelve open tasks that stimulate the PTs' autonomous inquiry and protagonism in their realization and moments of sharing and collective discussion of processes and responses (Ponte & Chapman, 2008), and included three phases: (i) initial experiences; (ii) training and learning experiences; and (iii) production experiences.



Figure 3. Educational and learning trajectory of the experience based on TPACK

In phase 1 it is intended to explore the PTs' conceptions about the integration of technology in mathematics education and to promote the articulation of these conceptions with their didactic knowledge. Phase 2 aims to develop the PTs' technological knowledge by exploration different technological tools for the teaching of Mathematics. Finally, in phase 3, we seek to promote the mobilization and operationalization of the PTs' TPACK to elaborate didactic proposals involving tasks and lesson plans, and moments of reflection and knowledge sharing. Table 1 presents a general description of the tasks performed by the PTs

in each of these phases.

Table 1 – General description of the tasks of the experience based on TPACK (Gutiérrez-Fallas, 2019)

Phase	Tasks	Description
Phase 1. Initial experiences	T1. Participation in a forum of moodle platform	Discussion about the technology integration in mathematics education, based on text reading
	T2. Exploring technologic resources	Recognize several technological tools available and free on the internet for teaching and learning mathematics
	T3. Analysis of mathematics task	Reflective analysis on the integration of technology in a mathematical task
	T4. Analysis of a lesson plan	Reflective analysis on a technology integration proposal in an available lesson plan
Phase 2. Training and learning experiences	T5. Exploring the <i>GeoGebra</i>	Use of <i>GeoGebra</i> to solve a plane geometry task by PTs
	T6. Exploring the calculator	Analysis of students' use of the graphic calculator in solving a task involving functions
	T7. Exploring <i>applets</i>	Research, exploration and reflective analysis of an applet available on a website for teaching and learning a mathematical content
	T8. Exploring <i>TinkerPlots</i> TM	Use of <i>TinkerPlots</i> in solving a statistical task by PTs
Phase 3. Production experiences	T9. Elaboration of a mathematics task	Selection and adaptation of a mathematical task that integrates the use of technology in solving it
	T10. Elaboration of a lesson plan	Didactic proposal for a mathematics class, based on a task that integrates the use of a technological tool explored in the discipline
	T11. Final reflection	Reflective written report on the integration of technology in mathematics education
	T12. Elaboration of a digital portfolio	Elaboration of a digital portfolio on the WIX.com platform as a resource for assessing the PTs' learning

Source: Own elaboration.

The second dimension referred as **theoretical-investigative**, involved the elaboration of a set of design principles of the experiment (Figure 4), which were refined in the retrospective analysis carried out after the experimentation phase in each of the DBR cycles. This improvement process is guided by the formulation of a conjecture, susceptible to be tested, which structures the experience design (Ponte et al., 2016).

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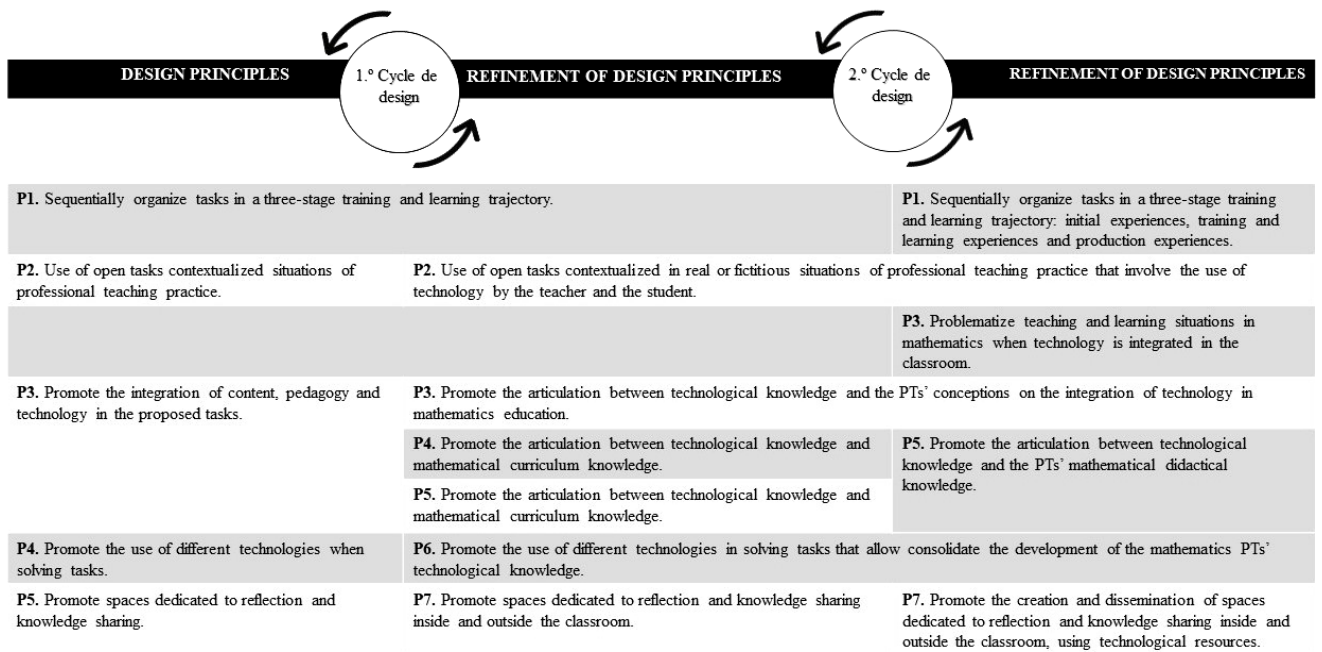


Figure 4. Refining the design principles of the TPACK-based experience

Therefore, after the two cycles of RBD developed in this study, the following seven design principles are defined (P1, ..., P7), which allow to support an experience in the pre-service teacher education of mathematics teachers to develop the TPACK:

- P1.** Sequentially organize tasks in a three-stage training and learning trajectory: initial experiences, training and learning experiences and production experiences.
- P2.** Use of open tasks contextualized in real or fictitious situations of professional teaching practice that involve the use of technology by the teacher and the student.
- P3.** Problematize teaching and learning situations in mathematics when technology is integrated in the classroom.
- P4.** Promote the articulation between technological knowledge and the PTs' conceptions on the integration of technology in mathematics education.
- P5.** Promote the articulation between technological knowledge and the PTs' mathematical didactical knowledge.
- P6.** Promote the use of different technologies in solving tasks that allow consolidate the development of the mathematics PTs' technological knowledge.
- P7.** Promote the creation and dissemination of spaces dedicated to reflection and knowledge sharing inside and outside the classroom, using technological resources.

Thus, the conjecture tested and verified during this RBD cycles, is that a training experience with the characteristics described and supported by these design principles contributes to promote the TPACK of mathematics PTs.

Analysis of the design of the experience based on TPACK

In this section we present the main results of the analysis, evidenced with excerpts from the PTs written work on solving tasks and their responses to interviews and questionnaire.

In relation to **what** the training experience aimed to develop, in this case the TPACK as PTs' professional knowledge necessary to effectively integrate technology in the teaching and learning of mathematics, the data show that the PTs acknowledged having acquired knowledge related to the integration of the three knowledge domains that constitute the TPACK: CK, PK and TK.

For example, Ana assures that this training experience “allowed me to become familiar with all these elements, (**mathematics, technology and didactics**), which strengthens the confidence of the future teacher” (RT11, bold author). Isabel also stated that “in my professional practices I intend to mobilize the **new knowledge acquired**, both in terms of content and technology” (RT5, bold author). Similarly, Sara makes reference to the knowledge acquired and their perspective of applying it in her future professional practice:

All the work done [in the course] allowed me to **acquire important knowledge**. In this way, **I became aware** that it will be important, in my future professional practice, to implement technologies in the best way, always aiming at learning and experiences that are quite enriching in **students** and in the **teaching** of Mathematics. (RT11, bold author)

Thus, the above statements reveal that, in addition to the acquisition of new and important knowledge, the PTs also recognized the importance and the intention to mobilize this knowledge in their future professional practice. Glória is also a case that highlighted the contact with educational technologies during this training experience as a promotor of her knowledge about technological tools (TK), and how to didactically integrate them in the mathematics classroom to benefit students' learning (TPK), stating:

It helped me a lot to know new technologies, to use the graphic calculator, which I had never used, software that I did not know. Everything will be of great value for my future, because already knowing these technologies I already have an idea of what I can use and how it contributes to the students' learning. I already have a first experience, because I was able in this discipline to avoid a possible problem, knowing a new didactic teaching tool that most probably I will use in the classroom (RT11).

Regarding **how** it was planned to develop the PTs' TPACK, the operationalization of the design principles analysis during the training experience, showed the diversity of the following results.

In the case of open tasks (P2), which sought to challenge the PTs in articulating the TK with the PCK, this aspect was recognized when, for example at T9, Isabel states that “this task involves research, choice of technology and adaptation of the learning situation, making it more challenging” (RT9).

The open tasks were also characterized by promoting collaboration between PTs, in

each of the cycles nine of the tasks were solved in pairs, always keeping them with the purpose of consolidating the affinity between the PTs, as well as also strengthening their performance and progress at work throughout the training experience.

The data show that PTs highlighted learning generated by collaborative work, as referred by Vitória, “due to the fact that it was a group work, it is also important to highlight, as one of my main learnings, the experience developed through collaborative work” (RT11), and similarly Paula says that “group work is one of the learning that in my opinion I take” (I).

The experience based on TPACK sought to problematize teaching and learning situations in mathematics when technology is integrated (P3), through tasks that appealed to contexts, hypothetical or real, that described classroom situations that involved the integration of technology. In this regard, Paula indicated that in general, during the course “was made an approach to technology and how it could help students in their teaching and learning process” (RT11). In turn, Glória specified that:

We had many tasks related to technology, we had many opinions, many discussions about the use of technology in teaching, technology in the classroom, the use of technology by teachers, by students, how does technology contribute to students' learning, which are the risks, potentialities, difficulties, etc. (RT11).

The aim was also to promote the articulation of technological knowledge with the conceptions of PTs on their integration (P4). The data show the PTs' recognition of the digital age that characterizes the 21st century and the implications that this has on mathematics education. For example, Ana argued that “in an era so technological and more and more digital as the one we live in, addressing the theme of using technology as a means to promote the teaching-learning process, is not only useful but essential for any future teacher” (RT2). In terms of how the conceptions about the importance of using technology in the teaching and learning of mathematics were consolidated, Isabel argues that “I have always been fan of technology, but with the experience in this discipline I have grasped even more arguments to continue using technology” (I).

The data also made it possible to identify changes in the PTs' conceptions. For example, Marta points out that her perspective and position about the use of technologies in educational contexts has changed after participating in the training experience, arguing:

First, with regard to the use and importance of technology in education, for a long time I took a position against the use of technology in the classroom, considering that it would be a very high distraction factor, taking students away from main focus of the class. However, after carrying out the work discussed here, I realized that technology can be quite useful, when well implemented, offering diverse tools for the exploration of some contexts, which, sometimes, becomes difficult to realize, in useful time, without the use of technological resources (RT11).

With regard to promoting the articulation between technological knowledge and mathematics didactic knowledge (P5), the data show that PTs mobilized TK in the didactic proposals they developed (PCK), such as the mathematical task or lesson plans. For example, Marta and Paula refer that “with the help of software, students will be able to develop some

transversal skills, such as inductive reasoning, mathematical language development, autonomous and cooperative work” (RT9). In the lesson plans, the PTs also highlighted the students' involvement in an active exploration of mathematical concepts and ideas (CK) with the technological tool (TK). Namely, in Sara and Vitória's lesson plan on the use of *TinkerPlots* in teaching statistical concepts, they present possible students' solutions of the task (Figure 5) and indicate that “during all task solving, the student remains sitting at the computer, work collaboratively, analyse and interpret graphics. And relates the statistical and physical concepts to support the possible answers to each question” (RT10).

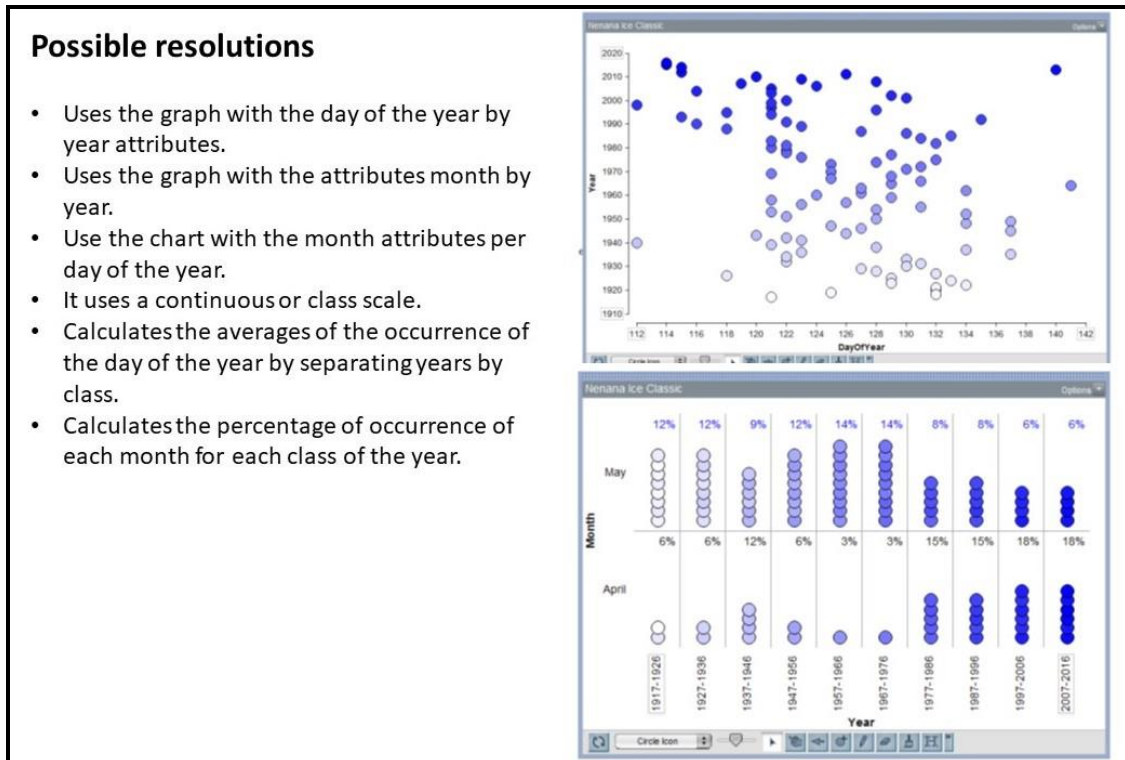


Figure 5. Foreseen solutions for the task elaborated for the lesson plan prepared by Sara and Vitória (RT10).

Still, about the articulation between technological and didactic knowledge, it was also evident the PTs' interest in continuing to explore technological tools with the purpose of integrating them in their professional practice. Glória exemplifies this by suggesting that “using the software we already know, such as Excel, GeoGebra and others, we can extract from them what is important for the objectives of our class, thus creating an applet, with only basic computer and software knowledge” (RT11). PTs also identified the potential of a technological tool and its applicability in the teaching and learning of mathematics, as in the case of Sofia, who recognized “the part of presenting the software and understand the potential of each one, what could we do with each one, was the most important part of our learning” (I).

In general, in both cycles, the data from the questionnaire (Figure 6) reveals that all PTs recognized that they had acquired knowledge and developed skills related to the

elaboration of a lesson plan that integrates technology.

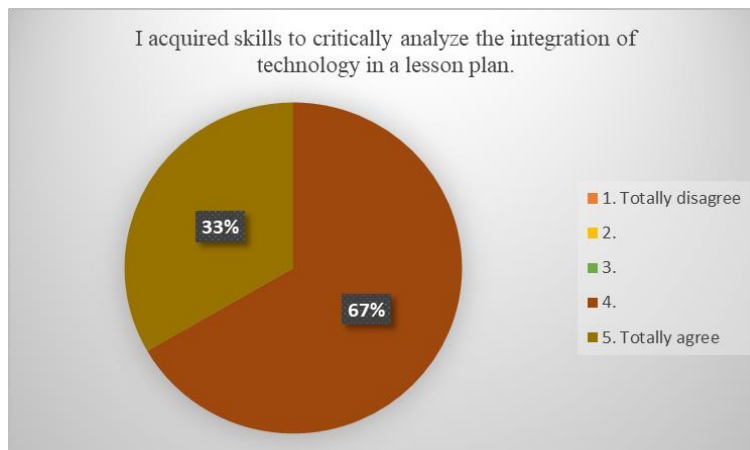


Figure 6. Responses to question 8 of questionnaire

With regard to the exploration of technological tools (**P6**), the PTs highlighted the opportunities that the training experience offered them to use and explore various technological resources available for teaching and learning mathematics. For example, Paula stated that “during the course we had opportunity to have contact with some of the technologies that can be used in the classroom, namely Geogebra and *TinkerPlots*” (RT11). Similarly, Sofia referred that “for us, as mathematics PTs, it has been important for our training the study of the various technologies currently available” (RT5), and in the interview highlighted some of these technologies: “when we thought on technologies in the classroom class was the projection of a PowerPoint and much less the calculator, so we have noticed that there are others, namely applets, that I had no idea that we could use, and that is a very useful resource ” (Sofia, E).

Another aspect that PTs recognized was the contributions that the exploration of these resources brought to their TK. Glória, for example, referred that the task of exploring applets (T7) allowed her to use “a resource with which I did not have much contact and helped me to research the unknown more deeply. So I made this task an opportunity to know one more tool” (RT7). The applets and the WIX platform were a novelty for PTs. According to the final questionnaire (Figure 7), more than half of the PTs agreed to have acquired knowledge regarding the use of these tools for teaching and learning mathematics.

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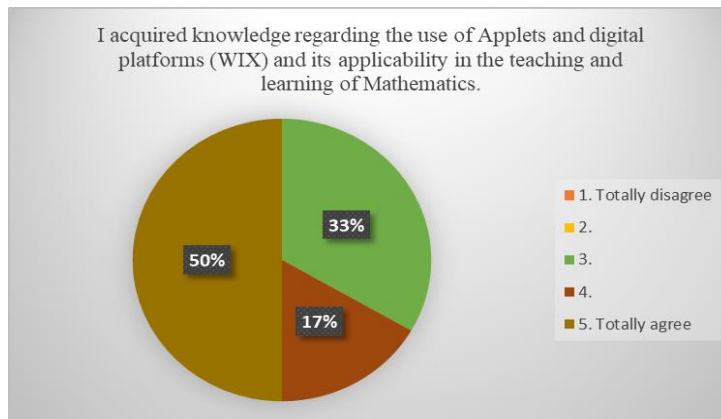


Figure 7. Responses to question 5 of questionnaire

In the case of the elaboration of a digital portfolio (T12), the PTs concluded that they acquired learning as a result of exploring and using the tools available on the WIX.com platform associated with the creation of a web page. As Sofia (Figure 8) stated, “learning a new platform like WIX and the video editor like POWTOON, were great consequences in elaborating the portfolio” (RT11).



[Hello dear colleagues and teachers. Digital Portfolio]

Figure 8. Image capture of the video made by Sofia (RT12)

However, the PTs also recognized that one of the limitations associated with the use of diverse technologies during this training experience, was the little time dedicated to deepen each one of them, as evidenced by Ana's argument: “the tasks included a great diversity of tools, but I wish I had go a little deeper, I wish I had time to go deeper, to familiarize myself with the tool, and I think it just did not happen”(I). This aspect can be considered a limitation of this training experience, which can be linked to the context in which the experience took place.

Regarding participation in spaces for reflection and knowledge sharing (P7), the results reveal that these spaces, like the digital portfolio, allowed the development of the PTs

self-reflection on the integration of technology in mathematics education. In the case of Vitória, this PT defends that “I think that I still have many ways to go, but the contributions of the discipline were essential for this path as it led me to be able to reflect on all aspects that I still need to improve” (RT11).

Sara, on the other hand, specified some of the aspects submitted for her reflection, indicating that “this was a very enriching work, which led me to reflect on fundamental aspects, of which I highlight the preparation of lesson plans and the use of tasks that integrate technology” (RT11). Associated with these results, the questionnaire data reveals that all PTs agreed to have acquired skills associated with self-reflection on the integration of technology in the teaching and learning of Mathematics (Figura 9).

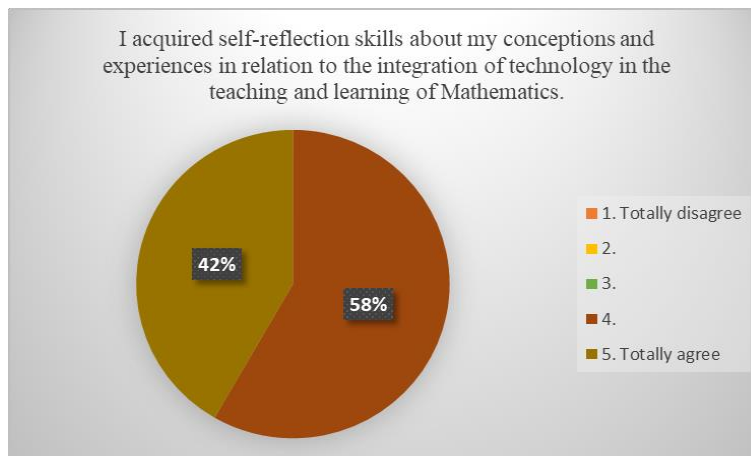


Figure 9. Responses to question 1 of questionnaire

In general, the questionnaire data (Figure 10) also shows that the PTs agree with the fact that the training experience contributed to the development of their professional knowledge to effectively integrate technology in mathematics education.

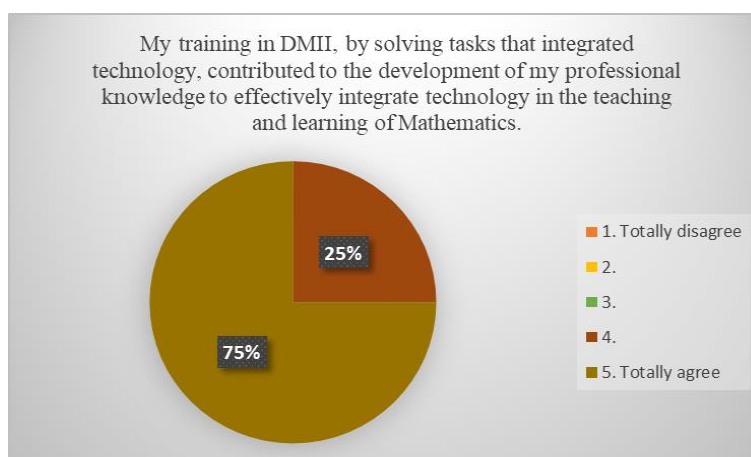


Figure 10. Responses to question 9 of questionnaire

These results support the options that guided the preparation of the tasks and verify the fulfilment of the objective that directed its implementation in the classroom - to develop the TPACK of the mathematics PTs.

Final reflection

This study reports the design principles of a pre-service teacher experience that aims to develop the PTs' TPACK, which is innovative in Portugal. The PTs' recognition of the knowledge they acquired and the analysis of the operationalization of the design principles during the experience, offered an opportunity to better understand and confirm that the design principles adopted, which were still little researched, can contribute to promote it.

The results obtained show that the training experience based on TPACK, carried out in this study, contributed to mathematics PTs develop the TPK and TCK knowledge in articulation with the PCK. In particular, the proposed tasks solving sequentially in the training and learning trajectory was the main resource to promote these knowledges, and the problematization of teaching and learning situations in mathematics through open tasks (Phase 2 and Phase 3) allowed PTs to explore different technologies in educational contexts, contributing to the development of TK, and recognize their potential by articulating this knowledge with TCK and TPK for the objectives of the tasks.

We can conclude, therefore, that this training experience allowed the PTs to develop knowledge related to PCK simultaneously with TK, TCK, TPK and, consequently, with TPACK (Koehler et al., 2014), corresponding to what Niess (2012a) defends as the focus of pre-service teacher education programs.

Another evident conclusion is that this training experience consolidated the PTs' conceptions, being more and more favourable to integrate technology in the teaching and learning of mathematics. As in other studies (Niess & Gillow-Wiles, 2017; Niess, 2012b), it is confirmed that the change in their conceptions resulted from the characteristics of the training program in which they participate. In this study, the characteristics of the experience that effectively contributed to these changes are: (i) offered an opportunity for reflection, discussion and analysis of teaching and learning situations in mathematics that integrate technology; (ii) allowed PTs to explore several technologies for teaching and learning mathematics; and (iii) promoted the articulated mobilization of the PCK, TPK and TCK of the PTs in the planning of teaching and learning situations in mathematics that integrate technology.

Associated with the knowledge of different technological tools, the results reveal that the PTs have significantly developed and consolidated their TK. This knowledge, according to Costa et al. (2008), although fundamental for understanding the potential of technologies for learning, it must be complemented with opportunities for the PTs to experience them "in concrete teaching and learning situations" (p. 42). Thus, we can conclude that this experience was not limited to contributing to develop the TK of the TPs, as it also created opportunities for them to try this TK in concrete situations such as mathematical tasks solving using

technology and in the planning of classes that integrate the use technological tools.

Finally, in the experience presented in this paper it was also found that, during the training process, the PTs reinforced competences that approach the second level of the UNESCO framework (2008): *Knowledge deepening*. For example, the results show that the PTs were able to: have a critical and reflective thought about the integration of technology in the school; increase their interest and curiosity about the use of technological tools for teaching and learning mathematics; identify the potential of specific technological tools and their applicability in mathematics education; and have an attitude of perseverance and confidence in the integration of technology in the teaching and learning of mathematics.

As a result of the reflection made on the results obtained in this study and monitoring as educators the experimentation phases, we identified that a limitation of the training experience was that it was not possible to consolidate the knowledge on: the curricular management of technological resources in the classroom, diversified teaching strategies to efficiently integrate technology, and the adaptation of assessment strategies when technology is integrated in the teaching and learning of mathematics. This limitation is mainly related to the context of the Didactics course where the experiment was carried out, in which it was not possible to propose a task that would ask the PTs to implement a class using technology with students from the 7th to the 12th years as it is a curricular unit of the 1st year of master's degree in mathematics teaching. Being "important that the PTs have more opportunities to get in touch with concrete experiences of using the technology with students and, hopefully, to take their plans into practice, reflecting on their implementation in the classroom" (Oliveira, Henriques & Gutiérrez-Fallas, 2018, p. 441), a proposal to overcome this limitation is, in the training experience, to offer opportunities for the PTs not only to plan teaching and learning situations but also to dynamize these situations in classes, such as micro-teaching practices (Niess & Gillow-Wiles, 2017), where they can teach a class with technology to their colleagues.

The results of this study, thus, highlight valuable design principles for pre-service teacher education, which are confirmed to promote the development of the TPACK of future teachers as the professional knowledge necessary to effectively integrate technology in the teaching and learning of mathematics, bringing them from the expected reality of their future practice.

References

- Albuquerque, C., Veloso, C., Rocha, I., Santos, L., Serrazina, L., & Nápoles, S. (2006). *A matemática na formação inicial de professores*. Lisboa: APM e Secção de Educação e Matemática da SPCE.
- Association of Mathematics Teacher Educators (AMTE) (2017). *AMTE Standards for Preparing Teachers of Mathematics*. Recuperado de <http://www.amte.net/publications>.
- Canavarro, A. P. (2011). Ensino exploratório da Matemática: Práticas e desafios. *Educação e Matemática*, 115, 11-17.

- Cobb, P., Confrey, J., diSessa, A., Lehere, R., & Schauble, L. (2003). Design experiments in education research. *Educational Researcher*, 32(1), 9–13.
- Costa, F. (Coord.), Peralta, H., Rodrigues, A., Dias, P., Osório, A. J., Gomes, M. J., Ramos, A., Ramos, J. L., Sebastião, L., Maio, V., & Valente, L. (2008). *Competências TIC. Estudo de Implementação* (vol. I). Lisboa: GEPE-Ministério da Educação.
- Coutinho, C. P. (2011). *Metodologia de Investigação em Ciências Sociais e Humanas, Teoria e Prática*. Coimbra: Edições Almedina.
- Earle, R. S. (2002). The integration of instructional technology into public education: Promises and challenges. *ET Magazine*, 42(1), 5-13.
- Grossman, P. L. (1989). A study in contrast: Sources of pedagogical content knowledge for secondary English. *Journal of Teacher Education*, 40(5), 24–31.
- Gutiérrez-Fallas, L. F. (2019). *O conhecimento tecnológico e pedagógico do conteúdo (TPACK) na formação inicial de professores de matemática do 3.º ciclo do ensino básico e do ensino secundário*. Tese de Doutoramento, Universidade de Lisboa, Portugal.
- Gutiérrez-Fallas, L. F., & Henriques, A. (2020). O TPACK de futuros professores de Matemática numa Experiência de Formação. *Revista Latinoamericana de Investigación en Matemática Educativa*, 23(2), 175-202. <https://doi.org/10.12802/relime.20.2322>
- Koehler, M. J., Mishra, P., Kereluik, K., Shin, T. S., & Graham, C. R. (2014). The technological pedagogical content knowledge framework. In J. Spector, M. Merrill, J. Elen, & M. Bishop (Eds.), *Handbook of research on educational communications and technology* (pp. 101-111). New York, NY: Springer.
- Krumsvik, R. J. (2014). Teacher educators' digital competence. *Scandinavian Journal of Educational Research*, 58(3), 269-280.
- Leung, A. (2017). Exploring techno-pedagogic task design in the mathematics classroom. In A. Leung & A. Baccaglioni-Frank (Eds.), *Digital technologies in designing mathematics education tasks: Potential and pitfalls* (pp. 3–16). Cham: Springer.
- Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for integrating technology in teachers' knowledge. *Teachers College Record*, 108(6), 1017–1054.
- National Council of Teachers of Mathematics (NCTM) (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.
- Niess, M., & Gillow-Wiles, H. (2017). Expanding teachers' technological pedagogical reasoning with a systems pedagogical approach. *Australasian Journal of Educational Technology*, 33(3), 77-95.
- Niess, M. L. (2012a). Rethinking pre-service mathematics teachers' preparation: technological, pedagogical and content knowledge (TPACK). In D. Polly, C. Mims and K. Persichitte (Eds.), *Developing technology-rich, teacher education programs: Key issues* (pp. 316–336). Hershey, PA: IGI Global.
- Niess, M. L. (2012b). Teacher Knowledge for Teaching with Technology: A TPACK lens. In R. Ronau, C. Rakes and M. Niess (Eds.), *Educational technology, teacher knowledge, and classroom impact: A research handbook on frameworks and approaches* (pp. 1-15). Hershey, PA: IGI Global.

- Oliveira, H., Henriques, A., & Gutiérrez-Fallas, L. F. (2018). A integração da tecnologia na planificação de aulas na perspectiva do ensino exploratório: um estudo com futuros professores de Matemática. *PERSPECTIVA*, 36(2), 421-446.
- Ponte, J. P., & Chapman, O. (2008). Preservice mathematics teachers' knowledge and development. In L. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 225–263). New York, NY: Routledge.
- Ponte, J. P. (2012). Estudiando el conocimiento y el desarrollo profesional del profesorado de matemáticas. *Teoría, crítica y práctica de la educación matemática*, 41, 83-98.
- Ponte, J. P., Carvalho, R., Mata-Pereira, J., & Quaresma, M. (2016). Investigação baseada em design para compreender e melhorar as práticas educativas. *Quadrante*, 25(2), 77-98.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.
- UNESCO (2008). *ICT Competency Standards for Teachers. Policy Framework*. Paris: UNESCO.