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An analysis of the state of the art of research on transition in mathematics education

Uma análise do estado da arte das pesquisas sobre transição na educação matemática

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Abstract

This study integrates research on the theme "transition" in mathematics education based on an analysis referenced in systematic reviews of the literature as a research practice. The objective is to identify the contexts and ways in which the topic has been approached, highlighting the diversity of treatments and common traits when communicating or conceptualizing and adding meaning to "transition" in the research field. The purpose is to understand and highlight the importance of the theme and of investigations on transitions in mathematics education – in particular, in teaching at universities. From the results, there is a predominance of a view on the transition in mathematics teaching in universities, conceived as a network of processes by which individuals "cross borders" or go through "rites of passage" from one culture to another, in the same community or social context. In doing so, individuals change their role in such environments.

Keywords: Transition in Mathematics teaching; Internal/External transition; Systematic Review; State of the Art.

Resumo

Este estudo integra pesquisas sobre o tema "transição" na educação matemática a partir de análise referenciada em revisões sistemáticas da literatura como prática de investigação. O objetivo é identificar os contextos e modos como o tópico vem sendo abordado, evidenciando a diversidade de tratamentos e traços em comum ao comunicar ou conceituar e significar "transição" no campo de pesquisa. Assume-se como propósito compreender e destacar a importância do tema e das investigações sobre transições na educação matemática – em particular, no ensino nas universidades. Dos resultados, destaca-se a predominância de uma visão sobre a transição em ensino da matemática nas universidades concebida como uma rede de processos pelos quais os indivíduos "atravessam fronteiras" ou percorrem "ritos de passagem" de uma cultura a outra, em uma mesma comunidade ou contexto social. Ao fazê-lo, indivíduos mudam o seu papel em tais ambientes.

Palavras-chave: Transição em ensino de matemática; transição interna/externa; revisão sistemática; Estado da Arte.

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Introduction

After all, what does transition mean?

In the analyzed studies, transitions are identified in moments of transmission of knowledge or of enculturation[3] in practices between different stages of teaching, in which adversities, resistance or deficiencies in the teaching-learning process stand out.

Here we seek to understand, from the examination of research works, the meanings and treatments given to the theme "transition in mathematics teaching" adopted in the area of education. We focus in particular on research on internal transitions within the same institution of higher mathematics education, referring to the content of calculus[4], fragmented into different disciplines. In these cases, transitions are identified between different mathematical practices, although within the same community or social context.

Such moments of transition are less studied and no less important to the questions that we elaborate when we perceive the diversity of modes in which research with this focus refers to the way that knowledge is legitimized within the scope of the practices addressed. These themes are often revealed by researchers and teachers when facing situations marked by students' difficulties, or performance in conflict with the teacher expectations during transitional stages in education. In particular, we investigated the open questions, in the related research literature, that reveal how the phenomenon recognized as a transition in mathematics teaching at the university is understood and treated.

To promote reflections on this matter, we survey the academic literature on this topic in the form of state of the art, adopting a systematic investigative model as a methodology to reveal the status quo of the specified object from the range of research produced on the subject in the area of mathematics education.

This study brings, in the following section, the state of the art as a research practice and the investigative procedure for the review of research produced on the subject, pointing out ways to make systematic the investigation on the same object. Then, we included the consulted material, organizing the information and possible inferences to analyze the subject.

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[3] The concept of "Mathematical Enculturation" in Bishop (1999) distinguishes between the processes of enculturation and acculturation, the former having the objective of preserving, or strengthening, the cultural values of a given community. The acculturation process is characterized by the induced transmission of elements from one culture to another, implying the acceptance and/or rejection of certain cultural elements. Thus, acculturation can promote the disintegration of one culture, superimposed by another. (Brandenberg, 2016, p.188)

[4] In our country, the content of calculus is worked on in higher education. This is not the shared practice in other countries, which address the content in schools corresponding to our secondary school. Cross-country comparisons must therefore consider such differences. The option was to maintain the centrality of the research in the country, situating it in an international scope for possible comparisons and dialogues.

A survey in the state of the art format

Which elements are common and which ones distinguish the notions of state-of-the-art, literature review and systematic review?

By adopting the state-of-the-art format, we remember they "are also known as state-of-the-art reports, or even as progress reports or progress reports in a given area" (Figueiredo, 1990, p.134). We are thus approaching a bibliographic review or literature review, which designates a representation of the state of research on a particular object of study. We add that the literature review

> ... is already a step towards the solution of theoretical problems insofar as it is not limited to making collages of the propositions contained in the various authors' reports. The review, in this way, seeks a theoretical statute that can be valid as a referential in the face of the various/same descriptive, methodological and technical postures towards the object, on the part of the texts that make up the literature. (Barone, 1990, p.138).

A systematic search of the literature, in a State of the Art format, contemplates our need as researchers when faced with an object of interest in the area, which is to situate ourselves in the face of questions already raised and to delve deeper, bringing up other considerations for a better understanding. of the observed situations.

By ascertaining what is already known about the subject, we investigate the obstacles and theoretical gaps for the understanding of the research object so far. In short, state-of-the-art research, developed according to a mapping of academic production, allows elaborations on discussions which are relevant to the theme. So:

> They are also recognized for carrying out an inventorying and descriptive methodology of the academic and scientific production on the subject they seek to investigate, in the light of categories and facets that are characterized as such in each work and as a whole, under which the phenomenon starts to be analyzed. (Ferreira, 2002, p. 258)

We resort to such an investigation format, focusing on research that addresses the transition in mathematics teaching. Prompted by the questions raised, we organized an inventory of academic production in the light of categories to map it, pointing out singularities of the identified object and theoretical-methodological perspectives used, or pointing out the different aspects used when approaching the theme.

Having stated this objective, our focus goes beyond the latent spaces of the term "transition", which seems to us to assume a status of a phenomenon in mathematics education. The results of previous research (articles in scientific journals, academic works or theses) are arranged and explored as sources of data for an investigative production in order to add information to our study, as in Creswell (2010, p.51).

With this understanding of the notion of state of art, we conducted our study using the resources of a digital database, gathering academic works related to the indicated topic and dialoguing with problems linked to objects of study of ongoing research.

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Methodological procedures

Adopting the procedures of a systematic investigation includes exposing the stages of the research, of a qualitative nature; the choice of terms to be used in the query and the justification for choosing the adopted database; the selection criteria of texts to be considered in the review and organization of topics; the processes for the categorization of subjects and approaches used in the description of contexts; the synthesis of the information contained in the consulted literature; and the discussion of the results that integrate the understanding of transition in the teaching of mathematics, answering the questions posed.

In order to survey and consult the literature that deals with the theme "transition in mathematics teaching", in particular the internal transition to calculus, we organized the texts as in Creswell (2010, p.49). We also consider the procedures recommended by the *PRISMA* (*Preferred Reporting Items for Systematic Reviews and Meta-Analyses*), discussed in Silva (2020).

Table 1: Methodological Procedures for the (Systematic) Review of Literature

Methodology	Procedures
Topic identification	● Define the research question. ● Identify keywords (or terms).
Data collection and organization	● Select the database for locating bibliographic items. ● Define inclusion/exclusion criteria for material selection. ● Raise jobs in databases of the selected base. ● Select texts applying the defined inclusion/exclusion criteria.
Structuring and analysis of information	● Summarize the literature, structure by similar themes to organize the topic and categorize the concepts covered. ● Elaborate the literature map, presenting an overview of the literature, after identifying the topic and subtopics correlated to the study. ● Develop a summary of research abstracts and discussions, connecting studies in progress with previous ones. ● Contribute with our impressions, analyzing the researched topic, pointing out gaps and verifying possibilities for expansion.

Source: Elaborated by the authors, 2020.

This option for the systematic review methodology is aligned with the need to avoid a partial, biased study, sharing the position that:

> All research is an extremely versatile process, making it possible to follow numerous paths that lead to desired or unexpected, casual or planned discoveries or conclusions, but which can also lead the researcher to mistakes (in good faith) or fraud (in bad faith). (Fiorentini & Lorenzato, 2012, p.126)

Another need is not to be limited to a restricted set of references, or just a review of previous studies that do not add new elements to the research. At the same time, we seek to "identify the existing convergences, relationships and approximations in research, presenting evidence and understanding of knowledge from the mapped studies" (Otero-Garcia, 2011, p.41).

The literature review is carried out in a methodical configuration, according to the steps described in Table 1, and presents characteristics of a sequence of procedures and objectives, following an eligibility criterion for the selection of texts.

The method can be reproduced or restructured to be adapted to other situations.

The research question, identification of keywords, text selection criteria and location of materials in the database

From the initial contact with the literature, we stated the research questions:

   i.   How is described the transition phenomenon in mathematics teaching research which was analyzed? What questions remain open?

  ii.   How are the transitions in university mathematics teaching understood?

Such questions emerge as initial reflections in a research on different institutional spaces and on the phenomenon of transition in higher mathematics education – in particular, on the transition from the calculus of one variable to that of more variables. From these reflections, the themes for localization in databases also emerge (mainly in Google Scholar: https://scholar.google.com/). The option for such a search tool concerns the possibility of accessing academic literature in a simple and broad way compared to other databases (Silva, 2020, p. 55) and the fact that we can access the publications in various formats.

Google Scholar effectively identifies often cited documents concerning the research in mathematics education published in South America.. Given its exclusive coverage (no restrictions on document type and origin), this makes it a valuable tool for bibliometric analysis (Martin-Martin, Orduña-Malea, Harzing & López-Cózar, 2017, p.16). Thus, we understand that the use of Google Scholar is sufficient for identifying the intended literature.

The selection of articles must be delimited while demarcating points related to the guiding question of the research. For this purpose, we adapted the text selection criteria in Silva (2020):

- Inclusion only of works on transition in mathematics teaching.
- Inclusion of works available for consultation online in full.
- Deleting duplicate jobs.
- Exclusion of works that are not available in English or Portuguese.
- Exclusion of works that are not published academic texts.

Our search was carried out in August 2020. Initially, we chose not to restrict the publication periods of the researched articles, allowing the queries to reveal the periods of interest in the topic.

In order to search for the research related to the central idea – the phenomenon of transition in the teaching of mathematics in higher education and the transition from the calculation of one variable to that of more variables –, we initially typed the keywords respecting the following logical syntax: "transition in mathematics teaching", "higher education" and "calculation of several variables".

To ensure greater diversity in the literature, we use the terms, in two languages:

(1)   "*transição no ensino*" and "*educação matemática*"[5];

(2)   "transition in teaching" and "mathematics education";

(3)   "transitions in mathematics education" and "Transition in mathematics teaching".

When synthesizing the stated issues, we identified in the literature a variety of investigations on the subject, bringing different approaches, meanings and contexts. A first case that has been extensively studied is the transition from high school to higher education (about 42% of the results obtained), while others focus on the other transitions from one level of education to another during school and university life of students.

Research whose theme is also "transition" are those that investigate the introduction of practices or methodologies that involve the transition from passive reception (as in the lectures) to active engagement in the construction of knowledge, using, for example, information technology tools (Kingundu, 2014); or even those that aim to study the transitions between forms of mathematical thinking during learning – for example, the case of the transition between arithmetic and algebraic thinking.

It is worth noting that the understanding of the term "transition" according to the perspective adopted in these various cases, is revealed by the ways in which transitions in mathematics teaching are perceived and investigated. In order to understand its scope as a research topic in mathematics education, including the particular case of internal transitions to the same institution of higher education, characterized between practices in the same community or social context, and from high school to higher education, we insert keywords specifying cases, including corresponding English terms. After the successive changes, we performed the searches according to the combinations of terms presented in Table 2. In the identified files, we performed a brief reading of the texts for the selection of the material.

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[5] Translator's note: "transition in teaching" and "mathematics education" in Brazilian Portuguese.

Table 2: Terms used in searches

Languages	Combinations of descriptor terms used	Contained results	Selected results
Portuguese	"Transição no ensino de matemática", "educação superior" e "cálculo de várias variáveis"	01	01
Portuguese	"Transição no ensino" e "educação matemática"	16	02
English	*"Transition in teaching" e "mathematics education"*	19	04
English	*"Transitions in mathematics education"*	79	04
English	*"Transition in mathematics teaching"*	04	01
Portuguese	"Transição interna do cálculo"	52	11
Portuguese	"Transição do cálculo para a análise"	40	08
Portuguese	"Transição do ensino médio para o superior"	148	20
Total	08	359	51[6]

Source: Elaborated by the authors, 2020.

We opted to consider research that recognized and meant the expression "transition in teaching"[7] as a field of investigation in mathematics education as a passage between stages in the formation of subjects during the teaching-learning process.

Literature summary, structuring and organization of topics

In order to structure similar themes and to organize the topics and concepts addressed in the studies on "transition", we reorganized the selected research by grouping them into similar subtopics, in more specific categories (or classes), to allow comparing or contrasting their results. Table 3 summarizes the subtopics corresponding to cases of transitions in teaching, in addition to correlated aspects or attributes; still, it brings identifications of continuities or discontinuities during the transition processes, identified in completed works (Gueudet, Bosch, Disessa, Nam Kwon & Verschaffel, 2016).

There are attributes that configured certain cases which are also present in the others, although each one has a prominent focus on a specific topic. The first attempt to summarize

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[6] The fifty-one results selected and shown in Table 2 were obtained from among three hundred and fifty-nine works found in eight searches performed. The reduction is due to: repeated work in different searches; publications containing discussions about transitions in another area of knowledge (biology, for example); similar works, by the same author, published on different websites; or even cases that departed from our theme.

[7] Transition cases have been included in table 3. For example, teacher education is included in the subtopic Transition from university to school.

and categorize the bibliographic results on transition is not enough to highlight or distinguish case by case the central topic each of them had addressed; on the other hand, they indicate directions for further development of empirical studies on the theme.

Table 3: Subtopics of the theme "transition" in mathematics teaching

	Subtopic	Marks/atributes
1	Transition from high school to college	Discussions focused on students' difficulties, alternatives to minimize such difficulties. Discontinuous process, with identified breaks or gaps.
2	Transition from one to more variables in calculus.	Identification of situations or contexts that present analogies and differences between the ways of treating mathematical concepts. Seemingly continuous process, including broken down steps.
3	Transition from calculus to analysis	Change of treatment of mathematical concepts, construction of formal mathematical knowledge. Seemingly continuous process, including broken down steps.
4	Transition between forms (levels) of mathematical thinking	Passage to formal mathematical reasoning, treatment of concepts and meanings. Continuous (or discrete, but composed of steps to be carried out) or discontinuous process, with identified breaks or gaps.
5	Transition from university to school (teacher training)	Changes in beliefs and practices from the university to the school. Second moment in the double discontinuity (Klein, 1908/1939).
6	Transition between levels (stages) of schooling	(For example, transition from preschool to elementary school.) Discontinuous, with identified breaks or gaps.
7	Transition to school	Analysis and stimuli to develop children's math skills. Discontinuous, with identified breaks or gaps.
8	Transition from university to professional life	Change in beliefs between the practices of university studies for professional practice. Discontinuous, with identified breaks or gaps.
9	Transition between teaching practices	Issues and challenges when developing new teaching practices.
10	General theoretical approaches (transitions in mathematics education)	Theories and methods of research in the field.

Source: Elaborated by the authors, 2020.

Such structuration into subtopics allows us to understand how the research field aggregates, expands or reproduces research that has already been completed (Creswell, 2014, p.84). A literature review relating different subtopics and cases or approaches can provide a view of the theme – in this case "transition" – with meaning, for example, for teaching; or even to propose new semantic fields for the notion of transition in mathematics education. Based on the organization in Table 3 and inspired by Creswell (2010), we elaborated a map of the literature, aligning the specific study on internal transitions to the discipline of Calculus with research conducted by other authors, with similar objectives.

The literature map

Creswell (2014) argues that "for a qualitative study, the literature review can explore aspects of the central phenomenon being treated and be divided into topics" (p.72). A

research summary would include: the problem that is being addressed; the central objective or focus of the study; brief information about the sample, population or individuals; the fundamental results. Here the general topic and the other related subtopics focus on "The phenomenon of transition of mathematics in higher education: the passage from the calculus of one variable to that of more variables".

A literature map provides an overview described in topics and subtopics (or themes and subthemes) correlated to the study whose development is proposed. This is not a hierarchical model, as in a flowchart; but rather a structure that identifies research topics, organized according to subtopics that allow a panoramic understanding of the topic in the field of investigation.

To the themes that emerged from the highlights in Table 3, we incorporated the organizers of the review that was presented at the XIII International Congress on Mathematical Education – ICME, referring to internationally developed research on "transitions in mathematics education" (Gueudet, Disessa, Kwon & Verschaffel, 2016, p.vii). Such new themes were:

• Conceptual change and learning processes as transition processes.

• Transition from university mathematics to high school mathematics teaching and double discontinuity.

• Institutional transitions.

• Transitions between out-of-school and in-school mathematics.

From the analysis of research presented at that congress, Gueudet et al. argue that transitory changes can (or should) occur at different times or places during the formation of individuals, that is, all "learning can be described as a transition process" (Gueudet, et al, 2016, p.ix) . They are perceived as distinguishable characteristics or attributes: "any process of change or transition can be (or appear) continuous (or discrete, but composed of successive steps to be carried out) or discontinuous, with ruptures or gaps identified" (Gueudet, et al, 2016, p.ix).

For those authors, such attributes are related to the two preliminary categories of transition types identified by them, which we rewrite as:
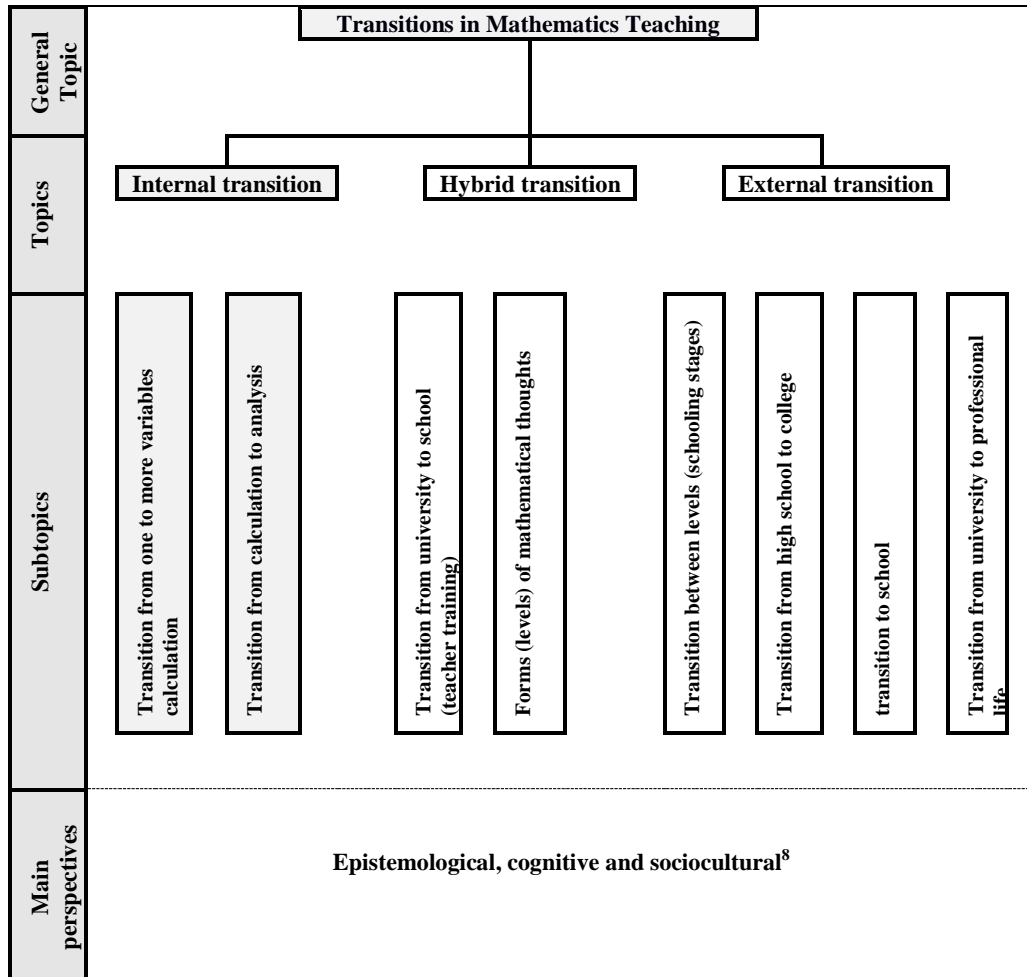
i. Conceptual changes in stages (or processes) of learning.

ii. Contextual variation in mathematical practices between social groups.

Table 1 (Map of the literature on transition) synthesizes the results from our systematic review and incorporates the two categories above, exposing the variety of transition situations (and possible connections) which were researched, and are related to the general (central) topic: transition in mathematics teaching.

The generated topics portray approaches that are attributed to the research themes. The subtopics discriminate the specific cases investigated in the area of mathematics

education. The approaches or perspectives that permeate the studies describing the transition in mathematics teaching are consistent with the results in Gueudet, Bosch, DiSessa, Nam Kwon & Verschaffel (2016, p.ix). Based on the synthesis shown in Table 1 below, we will describe some ways of understanding the phenomenon of transition.

Chart 1 –Map of the literature on transition

General Topic	Transitions in Mathematics Teaching							
Topics	Internal transition		Hybrid transition		External transition			
Subtopics	Transition from one to more variables calculation	Transition from calculation to analysis	Transition from university to school (teacher training)	Forms (levels) of mathematical thoughts	Transition between levels (schooling stages)	Transition from high school to college	transition to school	Transition from university to professional life
Main perspectives	Epistemological, cognitive and sociocultural[8]							

Source: Personal archive.

Given the scope of this journal, we present only two of the topics generated in the review carried out. In the first topic, we included the abstracts and discussions of research conducted by other authors, establishing a dialogue between these and our studies. This choice contemplates the multiple understandings of "transition in teaching" evidenced in the treatments and meanings of the phenomenon, in all three cases of transitions in mathematics teaching that we identified in the research literature. We therefore understand that there is no loss in the identification of the common traits when communicating and treating or conceptualizing and adding meaning to "transition" in the field of mathematics education; in

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[8] These are the main perspectives to describe the transition in mathematics teaching present in the literature review. (Gueudet, Bosch, DiSessa, Nam Kwon & Verschaffel, 2016, p.ix).

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particular, mathematics education in higher education.

Some ways of understanding the phenomenon of transition in universities

The theme "transition is a trend, and it has been debated by researchers and educators in the area for more than four years" (Gamoran, Porter, Smithson & White, 1997). In the case of inclusive research, although not the inclusion studies used for a review, which probably can be included in this topic, problems such as cases of learning difficulties related to changes in teaching internship and that student performance in a later course (Wisland, Freitas & Ishida, 2014), or studies that verify the factors that interfere in the student's adaptation in a new educational environment, from a psychosocial or sociocognitive approach (Seco, Casimiro, Pereira, Dias & Custódio, 2005; Azevedo & Faria, 2006).

Considering the area of interest – the transition from one to two variables calculus – we will discuss studies included in our bibliographic mapping in the topics "internal transition" and "external transition", in the following two subsections, in order to contrast and compare the ways in which the transitions in mathematics teaching are interpreted, addressing our specific interest, which falls under the first subtopic. We believe that the theme "hybrid transition" requires looking into new theoretical questions that will guide future studies.

In the following are the studies referring to an internal or external transition within educational institutions.

External transition

The topic includes about 20 research works related to passages (or pathways) between different stages of the complex teaching-learning system of mathematics, in which changes/alterations from one institutional culture to another occur. Common traits identified are problems, difficulties or poor student performance between transitional levels (e.g. Nasser, Sousa & Torraca, 2017; Gueudet, 2008; Lacaz, 2009; Palis, 2010). The perspective is the one of discontinuities, ruptures or gaps in the teaching-learning system (Meneghetti, 2017; Gueudet, 2008). The transition is interpreted, in many cases, as a discontinuous process during subjects' schooling or university pathway, with tensions and confrontation of the subjects through sociocultural, epistemological or cognitive changes, resulting from normative dissimilarities, knowledge, customs, habits or beliefs in a new cultural institution. The typical case of external transition in the literature on mathematics education is the transition from high school to higher education.

It is worth mentioning that Gueudet (2008) studies this case of the transition phenomenon analyzing results in a literature review, with a focus on three aspects linked to the existence of a transition problem: the difficulties presented by the students, the didactic action developed and the particular conception of transition observed in each perspective. For the author, investigation through the association and comparison of different perspectives,

which include dimensions related to the individual, social and institutional, allows building an adequate view of the transition phenomenon. Gueudet presents at first his reflections on research focused on students' ways of thinking and on the organization of knowledge during the transition processes from elementary mathematics to the field of advanced mathematical thinking (Tall, 1991) and on epistemological analysis according to the understandings of APOS theory (Edwards, Dubinsky & McDonald, 2005). Gueudet (2008) also focuses on issues related to proofs and mathematical communication (Weber & Alcock, 2004; Nardi & Iannone, 2005, for example); then, it approaches an institutional perspective, from the didactic transposition perspective (Chevallard, 1992) and from the notion of didactic contract (Brousseau, 1997). She argues that high school and university have different teaching contracts, and that the transition from one contract to another can generate ruptures. Furthermore, the discourse on what is expected at the university must be compared with what is actually evaluated. Finally, the author highlights the ruptures concerning the mathematical content, adding here the ways in which its teaching is didactically organized.

Internal transition

This topic includes the research in which we identify a perception of epistemological, cognitive or conceptual changes during the subjects' academic life, although immersed in (apparently) the same institutional culture. At the heart of the same university course, it seems natural to conjecture a continuous process at the institutional level; but there are traces of cracks, or borders caused during the formation of knowledge, identified between stages discriminated, internally, in the teaching-learning system. The most frequent subtopics found in the research literature in the area occur in works on transitions between the various calculus classes and from calculus to real analysis.

Transition from calculus to analysis

Garcia and Cammarota (2013) re-examine Otero-Garcia (2011), who had mapped Brazilian research on the teaching of analysis, considering the state of knowledge of the teaching of analysis from the perspective of inventive cognition. The authors explored the conditions for inventive learning in the teaching of analysis, and for the teacher's mathematical training as a problem linked to "cognitive politics".

Amorim's (2011) dissertation on the learning of limits of real functions as a conceptual reconstruction deals with the transition from calculus to analysis based on theoretical references from the advanced mathematical thinking group (Dreyfus, 1991; Gray, Pinto, Pitta & Tall, 1999) and referring to the notions of image and conceptual definition (Tall & Vinner, 1981). The same author makes use of such results and recommends a teaching proposal for the analysis course based on the students' conceptual images, believing to contribute to the teacher's work to situate the students' learning moment, to identify mistaken conceptual images and to explore situations that favor ideas which are consistent with formal definitions, directing consistent formal mathematical constructions.

Alves (2012) and Oliveira (2016), in their turn, use dynamic software to teach analysis. The first research explores topological notions that can help in the transition from calculus to analysis, believing that the exploration of graphics with the use of the computer and the identification of software limitations can provoke the intuitive perception of fundamental properties and provide the necessary meaning for understanding of some formal definitions. The second research focuses on the construction of the Riemann integral concept. It highlights possibilities for the discussion between teachers and students about the construction and resignification of the concept of Riemann's integral in the passage between calculus and analysis.

Reis (2001) aims to understand how the tensional relationship between rigor and intuition happens, and studied its manifestations in the teaching of calculus and analysis. The analysis of textbooks and semi-structured interviews with four teacher-authors showed that this relationship is almost always unequal and dichotomous in the approaches to textbooks. The positions defended by the deponents point to the need to break with the current formalist teaching, mainly in view of the formation of a mathematics teacher with multiplicity and flexibility of specific, pedagogical and curricular knowledge.

From another perspective, although dealing with the transition from calculus to analysis, we highlight the texts by Pinto (1998, 2001, 2002 e 2009). The author is based on empirical investigations into the teaching and learning of calculus and analysis. Pinto argues that, in the case of teaching calculus, the ideas initially proposed can be supported by a "naive" (or intuitive) approach with respect to definitions and other propositions; whereas in real analysis concepts are (intentionally) given formal treatment from the start. To her observations, we add that "while computational aspects and symbolic manipulation are emphasized in the calculation in order to obtain a final answer, the analysis is based on axiomatic and systematic principles, with formal definitions. This difference has a major impact on the transition from calculation to analysis" (Garcia & Cammarota, 2013, p.240). In her analysis, Pinto (1998, 2001, 2009) highlights the students' perception linked to the experiences that made them meaningfull. An example of this is the case of student Chris, who relates the contents of the discipline from multiple representations produced for mathematical objects when building and communicating mathematical knowledge in axiomatic-formal form.

(Internal) Transition of calculus

Alves (2011) focuses on the transition from one-variable calculus to multivariable calculus. It is included in the topic "internal transition". In his thesis, the elaboration and application of the Fedathi sequence aims to promote intuitive reasoning in multivariate calculus. The author identifies and describes categories for intuitive reasoning, referring to Fishbein (1987). Still in the theoretical framework, Alves (2011) makes use of the Theory of Registers of Semiotic Representation (Duval, 1995). In his literature review, Alves (2011) identifies that there are still few studies in Brazil and abroad on the teaching and learning of

differential and integral calculus for several variables. The main emphasis is on internal description in Calculus, that is, focusing on the barriers in the conduction between the calculus disciplines.

The author empirically verified the students' difficulties when faced with logical-formal arguments. He assumes the importance of perception and visualization in the teaching and learning processes of concepts in calculus. He argues that "the intuitive understanding brought about by visualization is necessary for understanding, but it is not sufficient for the evolution of further conceptual reasoning related to the same mathematical object" (Alves, 2011, p.51).

The didactic actions in Alves (2012, p. 5) include activities that involved a group of students selected and invited to participate of interviews. The researcher concludes that "we can discuss situations of the Calculus of Several Variables that admit an immediate interpretation in the context of the Calculus of One Variable, and possible conceptual connections evidenced by the CAS Maple". The expectation is that the support of the graphic representation can guide a didactic approach that promotes the understanding of the concept of functions treated in Calculus I and in Calculus II. From the results observed during proposed activities involving the formation of graphic records with the students, the author considers that the perception obtained through the visualization and geometric description of conceptual objects in Calculus is promising in an internal transition to the two different calculus.

In the same direction, Río (2016) takes up theoretical foundations to argue in favor of visualization using the Geogebra 3D graphic system as a tool for teaching and learning differential and integral calculus. Topics investigated included solids of revolution, functions of two variables, and limits calculus. In his didactic intervention, illustrated notes served to assist students in geometric constructions, as a way of collaborating with the learning process for the understanding of concepts which are considered obstacles, due to the great difficulty of representing them only using pencil and paper. The Theory of Registers of Semiotic Representation formulated by Raymond Duval (2006) is a reference for the author when arguing against an understanding of the content restricted to algebraic manipulation, noting that the possible constructions in a dynamic geometry environment can be included in the didactic proposals, as a way to improve understanding of concepts in differential and integral calculus – although one cannot conclude that the problems associated with teaching and learning these topics will be solved.

Discussion

From the analysis produced based on a systematic review of the research literature, a "framework for establishing the importance of the study and an indicator for comparing the results of a study with other results emerged" (Creswell, 2007, p.46). It allowed, in our study, to infer the transition in teaching as an educational phenomenon, that is, as an object and concept underlying the complex teaching-learning system in the area of education. Such a

system is constituted as a network of processes (cognitive, epistemological, social or cultural) identified in the revised texts, and by which, somehow, individuals (students or teachers) change their role when in distinct environments, scenarios and social spaces.

From the results of these works, we produced an analysis of the ways in which the phenomenon of transition in mathematics teaching in universities has been understood, based on the network of selected researches referring to the central topic.

First, the problem inferred from the dynamics of transition in mathematics teaching, which in the literature remains used as a common expression, refers to difficulties in understanding a concept in mathematics, whether arising from differences in approaches to previous subjects, or by the students or teachers' mathematical background. Topics related to teaching-learning processes in the individuals' academic preparation predominate in the studies analyzed. These are understood as discontinuous, marked by ruptures and perceived in the form of difficulties or deficiencies in the understanding of mathematical concepts during the passage of a stage of transmission of knowledge or practice of teaching to another – as "boundaries" that need to be crossed.

Stages are generally identified and theoretically referenced in cognitive, epistemological and sociocultural approaches. When adopting cognitive approaches, the expectation is the researcher's neutrality as a non-participant observer and the research focus on the subjects' learning stand out, with a focus on individual development, assuming the distance amongst the participants to guarantee impartiality in the analysis. Although the transition in teaching seeks to reveal phenomena of changes and conceptual development of individuals in the educational context, this context is sometimes disregarded in the unit of analysis of the research, with a short description and insuficient taken up in the analysis of the obstacles faced by subjects. Alves (2011), Garcia & Cammarota (2013), Amorim (2011) Río (2016) are some of the cases presented. Our interpretation is that, among other possible reasons related to and/or inherent to such approaches, researchers share a view that mathematics teaching is universal and homogeneous, especially in universities. This vision includes not only the universality of the content to be taught, but also the homogeneity in the relationships between students and teachers in the classroom. Research with an epistemological focus is sometimes articulated in didactic actions; and classroom intervention research, planned and developed by the researcher himself predominates (see, for instance, KOICHU & PINTO, 2019; GUEUDET, 2008; PALIS, 2010).

The sociocultural approaches, in smaller numbers and more recent research, suggest that understanding the educational formation of subjects is a process of continuous transition; or rather, learning can be understood as bringing together a diversity of transition processes (Corriveau & Bednarz, 2017; Azevedo & Faria, 2006; Sousa & Diogenes, 2016 are some of the studies that can be included here). For example, in a simple way with moments in transitions in early grades, we have that the transition to school corresponds to a period of adaptation of the child, who until then only recognized the home as a social environment, and now needs to live and interact with a new cultural universe. During the transition from high

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school to higher education, many young people start a new phase in their lives, with more independence, but which also requires autonomy, discipline and new commitments – a maturation cycle, which includes another praxis related to integration in the academic space, according to the stage that configures the student's incipient condition in relation to the course in higher education: as in an "initiation ritual" or "rite of passage"[9].

In a similar way, in the transition from calculus to analysis, we infer that there is an expectation that undergraduate students express themselves autonomously, thus modifying their role in the educational environment, and that they develop other perceptions and cognitive, epistemological or conceptual skills that modify their knowledge for mathematical training, as if at the dawn of scientific thinking and research activity.

In relation to teaching-learning problems, in the transition stages identified in the literature and organized into subtopics using corresponding traits and attributes in each case, we include issues of continuities or discontinuities in educational spaces according to the types of transitions that were found. In our analysis, we understand that the focus on discontinuities does not need to be associated with the idea of difficulty or insurmountable obstacles, as frequently evidenced in selected studies. We argue that they may be related to stages that correspond to moments of change in processes (cognitive, epistemological, social or cultural), in a way that modifies the understanding of the individuals regarding their role or conduct in environments, scenarios and social spaces. In other words, subjects are in the process of recognizing themselves in new paths (cognitive, epistemic, social or cultural) to go through, which are still unknown, and which, for this reason, demand other efforts. In other words, apparently continuous paths may have different stages, as if different paths to follow.

Since the same transitory effects are perceived at different moments in the formation of individuals, that is, they are propagated in our educational practices, between or internally to cultural institutions, our organization of the subtopics from the selected literature made it possible to identify the topics: internal, external and hybrid transition, containing distinguishable characteristics or attributes. Making it possible to perceive the traits of transitions in teaching as coming from constituent cause or effect and internal to the institution's own formation. When distinguishing the same transitions case by case, possibilities are generated to be more broadly studied in order to better understand the meaning of each of them in the field of education.

The subtopic of "transition from calculus from one to more variables", as well as that of "transition from calculus to analysis", was included in the topic of "internal transition", since the changes caused in each of these cases can be understood as immersed in the same institutional culture. However, research shows epistemological, cognitive and conceptual differences during the course of individual's schooling and academic life. The understanding of these possible conceptual changes in the learning processes – and the contextual variants

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[9] The "rites of passage" can be understood as "ceremonial sets that accompany, facilitate or condition the passage from one stage of life to another or from one social situation to another" (Van Gennep, 1981, p.155)

caused in these practices – demand further studies in the area.

Final Considerations

Putting into practice the design of an educational system that recognizes the phenomena of transitions in mathematics teaching seems to require an understanding of forms of dialogue in teaching that accommodate spaces for the production of knowledge in moments of educational transitions. For example, the study of knowledge in the teaching of Analysis in Otero-Garcia (2016), investigations whose proposal of the teacher's work is situated in the moment of student learning (Amorim, 2011), or discussing the importance of experience made significant to the understanding of students (Pinto, 1998), are studies in which the phenomenon of transition researched comprises a transition in teaching-learning in mathematics. In particular, which paths to weave in the internal transitions between the disciplines of Calculus? Answers to such questions can broaden our understanding of the transitional educational phenomenon as teachers and researchers in the field of mathematics education to develop situations that motivate student engagement.

Treating the theme transition as a meaning of its own in mathematics education, based on the processes identified from the literature research in the area, reveals/evidences moments or places where the conceptual or curricular elements which are present in the didactic organization of a certain institutional culture becomes eligible or preponderant in relation those which are present from other cultures that emerge in spaces, understood as borders. These are perceived by the students and teachers, and they seem to promote disintegration, or contempt for objects evoked or brought from other domains, instead of reestablishing, reconnecting or recontextualizing them. As perceived in our literature review (for example, in the research by Wisland, Freitas & Ishida, 2014; Meneghetti, 2017; Gueudet, 2008) the research treatments of the transition phenomenon provide evidence of a verticalization of knowledge as a common feature of the environments investigated, where the main objective of teaching would only be subordinated to a legitimate mathematical knowledge ideal, where what matters is the domain of formal theory. In this case, analysis is understood as a content to be retained in the cognitive structure of the subjects (Otero-Garcia & Cammarota, 2013, p. 241).

Revealing, in these cracks, the presence of a variety of other epistemologies, let's say educational ones. However, cracks turned conflicting and unimportant for knowledge construction and obstacles to surpass, in the face of a belief on a single and sovereign epistemology of mathematics.

In this sense, it seems that the teaching-learning system maintained institutionally does not use or allow other contexts, modes or treatments referring to the original forms or contexts or believed to pre-exist the mathematical content. Rather it assumes a position of rejection or rupture with them, bringing about an educational phenomenon that produces the transitory spaces revealed – the transition in mathematics teaching-learning.

However, reflecting on the notion of borders when conceptualizing transition presents

itself as a potentiality to rethink other ways of understanding transitions in teaching and learning, according to spaces of intercultural and decolonial dialogues amongst knowledge, from the formulation of questions that overcome statements of the didactic organization that supports a single constitution of Mathematics for teaching.

References

Alves, F. R. V. (2011). *Aplicações da sequência Fedathi na promoção do raciocínio intuitivo no Cálculo a Várias Variáveis*. Tese de Doutorado em Educação Brasileira. Ceará: Universidade Federal do Ceará. Retirado em 26 de setembro, 2020, de: http://repositorio.ufc.br/handle/riufc/3166

Alves, F. R. V. (2012). Exploração de noções topológicas na transição do Cálculo para a Análise Real com o GeoGebra. *Revista do Instituto GeoGebra Internacional de São Paulo*, 1(1), 1-14. Disponível em: https://revistas.pucsp.br/IGISP/article/view/8239

Amorim, L. I. F. (2011). *A (re)construção do conceito de limite do cálculo para a análise: um estudo com alunos do curso de licenciatura em matemática*. Dissertação de Mestrado Profissional em Educação Matemática. Ouro Preto: Universidade Federal de Ouro Preto. Retirado em 26 de setembro, 2020, de: https://repositorio.ufop.br/handle/123456789/2981

Azevedo, Â. S., & Faria, L. (2006). Motivação, sucesso e transição para o ensino superior. *Psicologia*, 20(2), 69-93.

Barone, F. (1990) A revisão de literatura em Biblioteconomia e Documentação: uma prática pedagógica. *Revista Brasileira de Biblioteconomia e Documentação;* 23 (1), 136-146.

Bishop, A. J. (1999). *Enculturación matemática: la educación matemática desde una perspectiva cultural*. Barcelona: Paidos Iberica Ediciones.

Brandemberg, J. C. (2016). Enculturação, Formação de Professores e Ensino de Matemática: Uma Discussão sobre Visão Ampliada dos Valores Culturais e Conhecimento Aprofundado do Conteúdo. *Revista Margens Interdisciplinar*, 9(12), 186-202.

Brousseau, G. (1997). *Theory of didactical situations in mathematics 1970–1990*. Dordrecht: Kluwer.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique: Perspectives apportées par une approche anthropologique (Fundamental concepts of didactics: Perspectives opened by an anthropological approach). *Recherches en Didactique des Mathématiques*, 12(1), 77–111.

Cooper, H. (1984). *The integrative research review: A systematic approach.* Newbury Park: Sage.

Corriveau, C. & Bednarz, N. (2017). The secondary-tertiary transition viewed as a change in mathematical cultures: an exploration concerning symbolism and its use. *Educational Studies in Mathematics*, 95(1), 1-19.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches*. Thousand Oakes: Sage.

Creswell, J. W. (2010). *Projeto de Pesquisa: Métodos Qualitativo, Quantitativo e Misto*. Porto Alegre: Artmed.

Creswell, J. W. (2014). *Research design: qualitative, quantitative, and mixed methods approaches*. Thousand Oakes: Sage.

Dreyfus, H. L., & Hubert, L. (1991). *Being-in-the-world: A commentary on Heidegger's Being and Time, Division I*. Cambridge: Mit Press.

Duval, R. (1995). *Sémiosis et pensée humaine: registres sémiotiques et apprentissages intellectuels*. Berne: Peter Lang.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1-2), 103-131.

Edwards, B. S., Dubinsky, E., & McDonald, M. A. (2005). Advanced mathematical thinking. *Mathematical Thinking and Learning*, *7*(1), 15-25.

Ferreira, N. S. D. A. (2002). As pesquisas denominadas "estado da arte". *Educação & Sociedade*, 23(79), 257-272.

Figueiredo, N. (1990). Da importância dos artigos de revisão da literatura. *Revista Brasileira de Biblioteconomia e Documentação*, 23(1), 131-135.

Fiorentini, D., & Lorenzato, S. (2012). *Investigações em Educação Matemática: percursos teóricos e metodológicos*. Campinas: Autores Associados.

Fischbein, H. (1987). *Intuition in science and mathematics: An educational approach*. Dordrecht: Kluwer.

Gamoran, A., Porter, A. C., Smithson, J., & White, P. A. (1997). Upgrading high school mathematics instruction: Improving learning opportunities for low-achieving, low-income youth. *Educational Evaluation and Policy Analysis*, 19(4), 325-338.

Garcia, S. C. O., & Cammarota, G. (2013). Releituras de um estado do conhecimento do ensino de análise a partir da noção de cognição inventiva. *Alexandria: Revista de Educação em Ciência e Tecnologia*, 6(1), 235-260.

Gray, E., Pinto, M., Pitta, D., & Tall, D. (1999). Knowledge construction and diverging thinking in elementary & advanced mathematics. *Educational Studies in Mathematics*, 38(1-3), 111-133.

Gueudet, G. (2008). Investigating the secondary–tertiary transition. *Educational Studies in Mathematics*, 67(3), 237-254.

Gueudet, G., Bosch, M., Kwon, O. N., & Verschaffel, L. (2016). *Transitions in mathematics education*. Hamburg: Springer Nature.

Kastrup, V. (2004). A aprendizagem da atenção na cognição inventiva. *Psicologia & Sociedade*, *16*(3), 7-16.

Kigundu, S. (2014). Engaging e-learning in higher education: issues and challenges. *International Journal of Educational Sciences*, 6(1), 125-132.

Klein, F. (1908/1939). *Elementary Mathematics from an Advanced Standpoint. Part I: Arithmetic, Algebra, Analysis*. Part II: Geometry. Translated by E. R. Hedrick and C. A. Noble. New York: Dover Publications

Koichu, B., & Pinto, A. (2019). The Secondary-Tertiary Transition in Mathematics. What are our current challenges and what can we do about them?. *EMS Newsletter*, 6(112), 34-35.

Lacaz, T. M. V., Fernandes, J. A. S., & Carvalho, M. T. L. (2009). Contribuições da Educação Matemática para a análise das dificuldades dos alunos na aprendizagem da disciplina Cálculo Diferencial e Integral I. *In Anais do Congresso Nacional de Matemática Aplicada e Computacional-CNMAC* (pp. 1073-1079). Cuiabá: Universidade Federal de Mato Grosso. Retirado em 26 de setembro, 2020, de: http://arquivo.sbmac.org.br/cnmac.php

Marshall, C., & Rossman, G. B. (1999). *Designing qualitative research*. Thousand Oaks: Sage.

Martin-Martin, A., Orduña-Malea, E., Harzing, A. W., & López-Cózar, E. D. (2017). Can we use Google Scholar to identify highly-cited documents? *Journal of Informetrics*, 11(1), 152-163.

Meneghetti, C. M. S., do Amaral Rodriguez, B. D., & Pofffal, C. A. (2017). Gráfico de função polinomial: uma discussão sobre dificuldades de aprendizagem no Ensino Superior. *Ciência e Natura*, 39(1), 156-168.

Nardi, E., & Iannone, P. (2005). To appear and to be: acquiring the «genre speech» of university mathematics. *In M. Bosch, M. (Ed), Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education CERME* 4.

Nasser, L., Sousa, G., & Torraca, M. (2017). Desempenho em cálculo: investigando a transição do ensino médio para o superior. *Boletim Gepem,* 70(1), 43-55.

Oliveira, J. L. D. (2016). *A utilização de softwares dinâmicos no Ensino de Análise Real: um estudo sobre a construção do conceito de Integral de Riemann*. Dissertação de Mestrado Profissional em Educação Matemática. Ouro Preto: Universidade Federal de Ouro Preto. Retirado em 26 de setembro, 2020, de: https://repositorio.ufop.br/handle/123456789/6643

Otero-Garcia, S. C. (2011). *Uma Trajetória da Disciplina de Análise e um Estado do Conhecimento sobre seu Ensino*. Dissertação de Mestrado em Educação Matemática. Rio Claro: Universidade Estadual Paulista. Retirado em 26 de setembro, 2020, de: https://repositorio.unesp.br/handle/11449/91029

Palis, G. (2010). A transição do Ensino Médio para o Ensino Superior. *In Anais do X Encontro Nacional de Educação Matemática* (pp.10-21). Salvador: Universidade Federal da Bahia. Retirado em 12 de dezembro, 2015, de: http://www.lematec.net.br/CDS/ENEM10/artigos/PA/Palestra4.pdf.

Pinto, M. M. F. (1998). *Students' understanding of real analysis*. Tese de Doutorado em Filosofia. Coventry: University of Warwick. Retirado em 26 de setembro, 2020, de: https://oatd.org/oatd/record?record=oai%5C:pqdtoai.proquest.com%5C:3491803

Pinto, M. M. F. (2001). Discutindo a transição dos Cálculos para a Análise Real. M. M. F. Pinto (Org.), *A Prática Educativa sob o Olhar de Professores de Cálculo* (pp. 123-145). Belo Horizonte: Fumarc.

Pinto, M., & Tall, D. (2002). Building formal mathematics on visual imagery: A case study and a theory. *For the learning of mathematics*, 22(1), 2-10.

Pinto, M. M. F. (2009). Re-visitando uma teoria: o desenvolvimento matemático de estudantes em um primeiro curso de análise real. In M.C.R. Frota & L. Nasser (Orgs.), *Educação Matemática no Ensino Superior: pesquisas e debates* (pp. 27-4). Recife: Sociedade Brasileira de Educação Matemática.

Reis, F. D. S. (2001). *A tensão entre Rigor e Intuição no ensino de Cálculo e Análise: a visão de professores-pesquisadores e autores de livros didáticos*. Tese de Doutorado em Educação. Campinas: Universidade Estadual de Campinas. Retirado em 26 de setembro, 2020, de: http://repositorio.unicamp.br/bitstream/REPOSIP/253451/

Río, L. S. D. (2016). Enseñar y aprender cálculo con ayuda de la vista gráfica 3D de GeoGebra. *Revista Digital Matemática, Educación e Internet*, 17(1), 1-13.

Seco, G. M. D. S. B., Casimiro, M. C. S. M., Pereira, M. I. A. R., Dias, I. S., & Custódio, S. M. R. (2005). *Para uma abordagem psicológica da transição do ensino secundário para o ensino superior: pontes e alçapões*. Leiria: Instituto Politécnico de Leiria.

Silva, F.G.S. (2020). *Ensino de Estatística na Educação Básica em Países da América Latina: Uma revisão sistemática*. Dissertação de Mestrado em Ensino de Matemática. Rio de Janeiro: Universidade Federal do Rio de Janeiro. Retirado em 11 de outubro, 2020, de: http://www.pg.im.ufrj.br/pemat/mest_teses2020.htm

Sousa, M. A. D., & Diogenes, G. M. D. S. (2016). Juventudes e escola: os sentidos de transição no ensino médio. *Revista Encontros Universitários da UFC*, 1(1), 2148.

Tall, D. (1991). *Advanced mathematical thinking*. Hingham: Kluwer Academic Publ.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151-169.

Van Gennep, A. (1981). *Les Rites de Passages*. Paris: A. Picard.

Vieira, F. R., & Neto, H. B. (2011). Transição interna do cálculo em uma variável para o cálculo a várias variáveis: uma análise de livros. *Educação Matemática Pesquisa: Revista do Programa de Estudos Pós-Graduados em Educação Matemática*, 13(3), 597-626.

Weber, K., & Alcock, L. (2004). Semantic and syntactic proof productions. *Educational Studies in Mathematics*, 56(2-3), 209-234.

Wisland, B., Freitas, M. D. C. D., & Ishida, C. Y. (2014). Desempenho acadêmico dos alunos em curso de engenharia e licenciatura na disciplina de cálculo I. *Iberoamerican Journal of Industrial Engineering*, 6(11), 94-112.