



Teaching knowledge to teach Statistics: the teacher's view of students and pedagogical strategies

Conhecimentos docentes para ensinar Estatística: olhar do professor sobre os estudantes e as estratégias pedagógicas

Karla Priscila Schreiber¹

Mauren Porciúncula²

Abstract

This paper presents specific teaching knowledge about students and the pedagogical strategies identified in the narratives of teachers who participated of the Statistical Education Teacher Training Collaborative Group – MoSaiCo Edu. For this, seven Group meetings were recorded and transcribed, being analyzed using the Collective Subject Discourse technique, which led to the construction of seven speeches. One of them called “misunderstandings of the students and strategies for teaching statistical concepts” is highlighted in this paper. As a result of the analysis of this, the teachers’ knowledge about the specificity of learning, protagonism, interests and misunderstandings of the students, the appreciation of the context and everyday experiences, as well as the use of standardized tests, technological resources and teaching materials are highlighted. The characterization of teaching knowledge to teach Statistics, an object of study in the process of scientific development, is relevant, as it can guide initial and continuing education, and practice in the classroom.

Keywords: Statistical Education; Teaching knowledge; Collaborative teacher training.

Resumo

Este artigo apresenta conhecimentos docentes específicos sobre os estudantes e as estratégias pedagógicas identificados nas narrativas de professores que participaram do Grupo Colaborativo de Formação de Professores em Educação Estatística – MoSaiCo Edu. Para isso, sete encontros do Grupo foram gravados e transcritos, sendo analisados através da técnica do Discurso do Sujeito Coletivo, o que levou à construção de sete discursos. Um deles, denominado “incompreensões dos discentes e estratégias para o ensino dos conceitos estatísticos” é destaque deste artigo. Como resultados da análise deste, destacam-se os conhecimentos dos professores sobre as especificidades de aprendizagem, protagonismo, interesses e incompreensões dos estudantes, a apreciação do contexto e de experiências cotidianas, assim como a utilização de testes padronizados, recursos tecnológicos e materiais pedagógicos. A caracterização dos conhecimentos docentes para ensinar Estatística, objeto de estudo em processo de desenvolvimento científico, é relevante, pois pode orientar a formação inicial e continuada, e a prática em sala de aula.

Palavras-chave: Educação Estatística; Conhecimentos docentes; Formação Colaborativa de professores.

Sent on: 30/10/2020 – **Accepted on:** 28/02/2021 – **Published on:** 27/05/2021

¹ PhD student in Science Education: Life Chemistry and Health at the Federal University of Rio Grande – FURG, Brazil. E-mail: karla.pschreiber@hotmail.com. ORCID: <https://orcid.org/0000-0003-1681-0422>

² PhD in Informatics in Education at Federal University of Rio Grande do Sul – UFRGS. Associate professor at the Federal University of Rio Grande – FURG in the Graduate Program in Science Education: Life and Health Chemistry, Brazil. E-mail: mauren@furg.br. ORCID: <http://orcid.org/0000-0003-1161-8220>.

Introduction

This paper presents the scientific construction in the field of teaching knowledge regarding the teaching of Statistics. Specifically, it tries to highlight and systematize the knowledge about the students' understandings and difficulties, as well as the pedagogical strategies used for the teaching of Statistics. These findings were made from the analysis of the narratives of the teachers of the Collaborative Group for the Formation of Teachers in Statistical Education – MoSaiCo Edu. This group, headquartered at the Federal University of Rio Grande – FURG, aims to provide a space for collaborative training, where teachers working in Basic and Higher Education can socialize and discuss their pedagogical experiences, in the perspective of Statistical Education.

Bearing in mind that the teacher's knowledge is organized in a specific way for each disciplinary content (Shulman, 1986; 2014), identifying the understandings that guide his pedagogical practice is necessary, since with this Knowledge Base, it will be possible to expand the knowledge base discussions and base the training and practice of the teacher in relation to Statistical Education. As highlighted by Watson, Callingham and Donne (2008), in the course of the 1990s, there was a growing interest in the understanding and professional development of teachers in relation to the teaching of Statistics, which accompanied a growing appreciation of teacher training and knowledge related to successful teaching.

Among the researchers who were interested in investigating the knowledge teachers related to statistics, there is the work of Burgess (2008), which examined these understandings, taking into account the statistical investigations. For this purpose, he related the knowledge of the mathematics professor (Hill, Schilling & Ball, 2004; Ball, Thames & Phelps, 2005), with the model of Statistical Thinking by Wild and Pfannkuch (1999). When comparing the practice of two teachers, the researcher described the knowledge provoked during the activities, as well as the implications on the teaching and learning process when there were failures in the understanding of these professors.

Watson, Callingham and Donne (2008) evaluated the Pedagogical Content Knowledge – PCK of 42 teachers, using a teacher profile instrument. He considered elements about trust, beliefs, teaching practice, assessment practice and teaching history, also supported by the knowledge proposed by Shulman. The researchers interest involved “content knowledge, its reflection in knowledge of their students' content knowledge, and their PCK in using student responses to devise teaching intervention” (p. 1). As a result, in addition to identifying three levels of comprehension – low, medium, and high – they also indicated the importance of teacher training to emphasize, in a more targeted way, knowledge related to the students' understandings.

Although these researches make important contributions to the training and practice of teachers in relation to the teaching of Statistics, it is considered that these understandings are still explained in general and deserve a more precise description, as proposed in this paper. It is worth mentioning that the mastery over statistical content, although necessary for teaching

(Burgess, 2008; Godino, Ortiz, Roa & Wilhelmi, 2011), does not guarantee professional competence, since different knowledge is mobilized by the teacher in pedagogical practice, such as “to recognize the statistical objects and processes that intervene in the students’ statistical practices, be aware of the norms that support and condition learning, affect, resources and interactions in the classroom” (Godino et. al., 2011, p. 12).

To analyze and systematize the teaching knowledge related to the teaching of Statistics, it is assumed that these are also mobilized and built when teachers share pedagogical experiences with their peers, mediated by a collaborative work context. The emergence of groups, seen as collaborative, in which university and basic school teachers propose to work collectively, emerged in the last decade of the twentieth century, from changes related to the concepts of continuing education, since their practice teacher education could not be seen as a “field of application of academic theories” (Lopes, 2013, p. 231), but rather, the focus of training processes in relation to teacher professional development (Crecci & Fiorentini, 2018), as it is proposed in collaborative groups.

In this perspective, collaborative contexts, between professors and researchers, present themselves as an alternative for the professional development of teachers as the focus of teacher training itself, and not as a field of research/action of “trainers”, since they enable the construction of collective knowledge, through a reflective process of teachers about their pedagogical practices (Lopes, 2013). In fact, it is through “participation in the group’s reflective and investigative practices that teachers become legitimate members of the professional community, with professional development and the improvement of their teaching practice a consequence of this participation” (Fiorentini, 2010, p. 583).

In this collaborative context, teaching knowledge related to Statistical Education emerges, which is also the focus of the Group. To analyze them, we start from the studies evidenced by Shulman (2014), on the Knowledge Base for teaching, presented in the following section. In the methodological procedures, in addition to the research actions, a brief history of the MoSaiCo Edu Group is described. The teaching knowledge related to the teaching of Statistics is then presented and discussed, followed by the final considerations on these results.

Knowledge Base for teaching

This research is based on the approaches developed by Shulman and collaborators of Stanford University, carried out from the 1980s on, in the research program called “Knowledge Growth in a Profession: Development of Knowledge in Teaching”. These approaches influenced, in different parts of the world, research and policies related to the training and professional development of teachers (Shulman, 1986; 2014; Mizukami, 2004). In the wake of the movement for the professionalization of teaching, the program led by Shulman was interested in “investigating the development of professional knowledge during the training of teachers and how they transformed the content into didactic representations and used it in teaching” (Bolívar, 1993, p. 115).

In this perspective, two models were constructed with the purpose of delineating the categories and processes related to teaching, in their procedural components (phases or cycles of reasoning and action) and logical (seven categories of knowledge underlying the teacher's understanding that are necessary for the teaching) (Shulman, 2014). The first is close to the educational actions, that is, how the teaching knowledge is mobilized, related, and built in the teaching and learning process, being this cycle, composed of the following activities: understanding, transformation, instruction, evaluation, and reflection, that lead to a new understanding of the teacher (Shulman, 2014).

The logical components refer to the Knowledge Base for teaching, that is, a “body of understandings, knowledge, skills and dispositions that are necessary for the teacher to provide processes of teaching and learning, in different areas of knowledge, levels, contexts and teaching modalities” (Mizukami, 2004, p. 38). When analyzing classroom practices of beginning and experienced teachers, Shulman, in 1987, after expanding the knowledge proposed in 1986, presented seven categories, namely: a) Content Knowledge; b) General Pedagogical Knowledge, which refers to the most comprehensive principles and strategies for classroom management and organization; c) Curriculum Knowledge, which covers instructional materials and programs related to the teaching of specific subjects and topics at a given level of study; d) Pedagogical Content Knowledge; e) Knowledge of learners and their characteristics; f) Knowledge of Educational Contexts, which involves everything from the functioning of the group or the classroom, moving through the management and financing of educational systems, to the characteristics of communities and their cultures; and, finally, g) Knowledge of the Educational ends, Purposes, and Values, and their Philosophical and Historical grounds.

In the set of these categories, Shulman (2014, p. 207) describes the relevance of the PCK, which is the “blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction”. Even when comparing the practices of experienced teachers and beginners, it is possible to identify the influences that the PCK exercises in the exercise of teaching, since teachers with a more solid base of this knowledge present a larger and more diversified set of strategies, in addition to having more resources to determine the most appropriate method to approach a specific content (Gudmundsdóttir & Shulman, 1987).

Therefore, in the light of the theoretical reflection presented, it is considered that this knowledge makes it possible to analyze the set of compressions necessary for teaching performance, with a view to the learning and training of students in relation to Statistics. It is worth mentioning that the proposals of other researchers – who study teaching skills and/or Statistical Education – are close to the discussions, since they contribute to the structuring of a proposal related to the Knowledge Base for the teaching of Statistics.

Methodological procedures of the research

The methodological procedures of the research are presented in two subsections, starting with the presentation of the MoSaiCo Edu Group and the themes discussed in the course of the first year of activities, between 2018 and 2019. In the sequence, the research actions are characterized, especially the instruments of production of records and the data analysis process, using the Collective Subject Discourse technique.

Presentation of the MoSaiCo Edu Group and of the meetings

Different themes were the basis for the socialization and discussion of teaching practices and knowledge of the MoSaiCo Edu Group. These included, between August 2018 and June 2019, statistical skills (Campos, Wodewotzki & Jacobine, 2011), teaching narratives (Nacarato & Grando, 2013; Lopes & Mendonça, 2017), pedagogical strategies (Porciúncula & Samá, 2015) and the National Common Curricular Base – BNCC (Brazil, 2018).

Since the beginning of the Group, practices have been established based on a dynamic of work and research by collaborative groups, proposed by Fiorentini (2004) – voluntariness, identity, and spontaneity; shared leadership or co-responsibility; mutual support and respect. Despite the presence of these principles, the teachers still expected a direction of activities, which was acceptable, since the collaboration could not be imposed on the members of the Group, but built between the teachers, who leave the “posture of accommodation and subordination for autonomy to choose their own theoretical study topics and content to be further explored” (Coelho, 2010, p. 177).

In choosing the dates of the meetings, defined collectively, the personal and professional commitments of the Group’s members were considered, especially the evaluation periods in the educational contexts, academic events, meetings, and class councils. On a voluntary basis, 18 teachers participated in the Group in this first year of activities, of which 13 were graduated in Mathematics Degree courses, two were Oceanographers (who taught subjects related to Statistics at the university) and three were pedagogues (two of them were also psychologists, and one of the psychologists was a pedagogy student).

Research actions

Bearing in mind the assumptions and central ideas that guide this research – especially the references that point out the teaching knowledge and aspects related to Statistical Education – in this paper, the teaching knowledge about the students’ compressions and difficulties is highlighted and systematized, in addition to the strategies considered for teaching Statistics. It is worth mentioning that this is an ongoing study and, therefore, this paper presents an outline of a broader doctoral research by the first author, under the guidance of the second, which has the research scenario and unit of analysis, the MoSaiCo Edu Group. In this context, it was decided to conduct a research with a qualitative approach (Lüdke & André, 1986; Bogdan & Biklen, 1994), following the methodological steps of a Case Study (Yin, 2010).

The methodological option is justified, from the perspective of Yin (2010), in which Case Studies do not intend to generalize to a universe, that is, they do not make a generalization in extension, but rather for the theory. Therefore, they collaborate with the emergence of new references or to confirm or disprove the theories that already exist, that is, the teaching knowledge mobilized by teachers when they collaboratively socialize practices related to the teaching of Statistics.

In the initial phase of the Case Study, defined by Lüdke and André (1986) as “exploratory”, the object of study, the questions or critical points of the research are more precisely determined, as well as the contacts with the subjects are established, and delimited the data sources of the study. For this research, at first, teachers who worked in Basic and Higher Education were invited to participate in the MoSaiCo Edu Group. Also, during this period, procedures and instruments for the production of records were defined, which involved: audio recordings of the meetings, the researcher’s diary and the teachers’ identification card.

In the design of the study, the second stage of the case study, record productions were performed, using instruments and techniques, defined based on the characteristics of the object to be studied, as well as the focus of the research and the study outlines (Lüdke & André, 1986, p. 22). To achieve these purposes, seven meetings were recorded on audio, between August 2018 and June 2019, which correspond to the Group’s first year of activities, with a variable number of participants in each meeting, taking advantage of the spontaneous and volunteer from this collaborative training space.

Finally, in the third stage of the Case Study, the systematic analysis of the data is developed, as well as the final research report (Bogdan & Biklen, 1994). In order to carry out this analysis, the technique of the Collective Subject Discourse – DSC was considered, which “as a testimony processing technique, consists of gathering, in empirical social research, in the form of unique speeches written in the first person singular, contents of testimonies with similar meanings” (Lefèvre & Lefèvre, 2009, p. 1194).

It is worth mentioning that the testimonies or individual opinions are not annulled or reduced to common unifying categories but reconstructed in order to “express a given ‘figure’, that is, a given thinking or Social Representation of the phenomenon” (Lefèvre & Lefèvre, 2005, p. 19). For the construction of the DSC, four operations are considered, namely: key expressions – ECH; central ideas – IC; the anchorages – AC; and the Collective Subject Discourse. Each of these operations interferes with the understanding and composition of the collective discourse, which is formed by the ECH (Lefèvre & Lefèvre, 2005; 2012).

The key expressions are literal, continuous, or discontinuous excerpts, analyzed and selected by the researcher and that signal the essence of the testimony content, based on the question of the research (Lefèvre & Lefèvre, 2012). With regard to this study, ECH were selected from the transcripts of the audios of the MoSaiCo Edu Group meetings. These ECH received colored markings in order to demarcate the different themes addressed in the

meetings, as well as to identify the mobilization of teaching knowledge, in the perspective of Statistical Education.

In the second operation of the DSC methodology, the IC are named, which correspond to the concise description of the meaning of the statements, since, by means of a name or linguistic expression, “reveals and describes, in the most synthetic, precise and reliable way possible, the meaning of each of the analyzed speeches and each homogeneous set of ECH, which will later give birth to the DSC” (Lefèvre & Lefèvre, 2005, p. 17). In the analysis of the ECH selected for the discourse presented in this study, 16 IC emerged, namely: Contents/concepts taught repeatedly; Difficulties in understanding the concept/calculation of Probability; Interpretation of problems and statistical data/measures; Difficulties in understanding the concept/calculation of Central Tendency measures; Difficulties in understanding/building graphs and tables; Difficulties in understanding the concept/calculation of measures of dispersion and variability; Difficulties in understanding the concept/calculation of Percentage; Recognize the student; Activities for teaching dispersion and variability measures; Contextualization with reality; Probability teaching activities; Activities for teaching graphs and tables; Activities for teaching Inference/Hypothesis testing; Statistics/Probability through questions from the National High School Exam – ENEM; Project methodology; Technology methodology.

The third methodological operation of the DSC is the Anchorage, which consists of the indication of a certain theory or ideology that is identified by the researcher in the statements, based on explicit discursive marks (Lefèvre & Lefèvre, 2005). Regarding the records considered for this research, six anchorages were identified, related to the teaching knowledge, indicated by Shulman (2014), being: Content Knowledge; General Pedagogical Knowledge; Knowledge of Educational Contexts; Curriculum Knowledge; PCK; and the Knowledge of learners and their characteristics.

Finally, these three methodological operations converge to form the Discourse of the Collective Subject, which are organized based on the approximation of the ECH whose IC or AC have the same sense, equivalent sense, or complementary sense (Lefèvre & Lefèvre, 2005). To achieve this, basic editing techniques are employed, which should not affect the meaning of each isolated statement, as well as connectors are incorporated, signaled through the “underlined” tool, which allowed the text to cohesion, without, however, altering the reading in the semantic field of discourse (Ibidem).

Bearing in mind the methodological steps of the DSC, in this study, the analyzes referring to the collective discourse “misunderstandings of the students and strategies for teaching statistical concepts” are presented, built from the approximation of the 16 IC, previously indicated. For this, in a first moment, a brief narrative was created for each IC, which were approximated and constituted the final discourse.

It should be noted that, the volume of data analyzed in this research made it necessary to choose some ECH and to exclude others. For example, when teachers talk about students’ difficulties in understanding the concept / calculation of measures of dispersion and

variability, more than one situation is used by them to illustrate such an understanding. As a result, ECH selections were necessary, within the same IC, without compromising the analyzes presented. In this perspective, the next section will present the analysis of that speech, which made it possible to identify and discuss the teaching knowledge mobilized by the MoSaiCo Edu Group teachers, when they socialized practices related to the teaching of Statistics.

Presentation and analysis of results

In the speech “misunderstandings of the students and strategies for teaching statistical concepts” (presented in Chart 1), the teacher described the students’ understandings and misconceptions about statistical concepts, as well as presenting the pedagogical strategies used to approach these contents. The teacher also reported aspects related to the repetition of contents, as well as recognizing gaps in the basic knowledge of students, which could interfere in the teaching and learning process of Statistics.

Chart 1 – DSC: Misunderstandings of the students and strategies for teaching statistical concepts

I am passionate about the research projects themselves, for the construction of the concepts of Statistics. There are several denominations that are made, that it deals with the same question: to develop a statistical research. That is, that students participate in the stages of an investigative process. I've done many different ways, even without them, theoretically, having seen anything of the content, only what they saw on TV, what they have seen in life. I fell in love with the Learning Project strategy, which is to work with a student's authorial theme. However, you have to be very careful not to empty yourself of the concept of the mean, the concept of the median, of the other statistical concepts that are all there. By the way, we worry because they have arrived without the basics of the basics. So, you have to understand what you are proposing and what type of basis is necessary, as well as review these bases, because you cannot exclude that student who has no base. For example, one of the students came to ask me this: “I want to put a percentage on my graph, but you didn't let us use a cell phone. How is it that I solve this rule of three?”. Likewise, I feel that the people are needed when it comes to seeing probability, operating with fractions, factorial. So, in a little while, the person does not fail because he did not understand probability, but because he does not know how to operate with a fraction, so he cannot resolve the issue. I mean, we give such a basic course and we have to repeat all that, several times and we ended up redoing the work of Basic Education in Graduation. For a start, they don't know what double entry is. I asked two questions to them [students]: how many days they practiced physical activity and how many days they came to the university. I asked them to build a table and a double entry chart, but until I built my double entry table, they were unable to build it. At another time, I worked with them a pie chart, so we made a chart with “how many came to class today and how many didn't come. For each one that arrived, put a slice on the chart, until it formed the circle. Then who missed it, I put another color. I had already left a caption, more or less ready, I made with them, the color “black” who came, the color “orange” who did not come. Then I asked them what the title of the chart could be. In fact, I like this part and make them see the potential of Statistics in everyday life, working with context, that is, with the reality of your student. So, when I work on the theoretical question, we already do it with examples and research in the class itself. The exercise now was in relation to “Fantástico” (TV program) which says that for the first time the rate of Brazilians who smoke has increased, after 15 years decreasing. We are going to do the test comparing two population proportions using the data we saw in “Fantástico”: so, did it really increase or not? In addition, we work a lot on the issues of ENEM – a lot of graphics, a lot of questions about mean, mode, median, table analysis and probability issues. Even the Probability I work in a class that is given thumbtack and Monty Hall's theorem. The thumbtack, I take the one of two types of that upholstery tack – one flat top and the other round top. We play to know. In other words, the thumbtack falls, butt down; or it falls to the side. The odds are usually reversed, 2/3 for one and 1/3 for the other. Then everyone comes to the computer and places their data, after each one does it ten times. Anything can happen within 10 tries. Sometimes it doesn't even stabilize, but it gets very close to what was expected. Still, the probability at that level that we teach, I can't understand what the difficulty is, so I have a lot of

difficulty on helping. Every time there are some problems that have to bring the idea of combinatorics for you to know what the sample space is: “My God! You have to make combinations in the middle of probability”! So they also have no idea of what is mean, even though they see it even on a daily basis, one of the most basic of Statistics. What do they think the mean is? Add and divide by the quantity. Mean is even more common, but the median they do not know that it exists or what it is. What is median? And they, memorized: “is what is in the middle”. I said: So, what is the difficulty? “Okay, but isn’t the median value mean?” – No, that would be mean value! They assume that if you went through the door in the Statistics class it is another world. The word “mode” will have nothing to do with the word “fashion” outside. (Mode and fashion are the same word in portuguese: “moda”.) Furthermore, when you ask the question of standard deviation, variance, then ‘it screwed up’. Like, there was a problem that was a comparison between the price of gasoline and alcohol. They had to calculate mean and standard deviation in the letter ‘a’ and in the letter ‘b’ they had to explain what the fuel was with the most homogeneous price, but they couldn’t explain it, because when you put a little bit of interpretation it is an immense difficulty, since they want to memorize formulas and cannot see and think. By the way, I have an activity that I use in class, which never comes out with the word “standard deviation” and variance, but the concept of variability emerges a little, which is a sampling activity, with straws, where they will cut, put in a bag and count how many there are in each bag. That is, they have to make the sample, take, measure and put the straw back. We will compare, because, theoretically, straws were cut about the same size, but different people are cutting. By the way, do you know how I did it once, with people who were unmotivated to answer? I took everything and put it in the format of “QR Code”, that is, when they read by cell phone, the question arose. Kahoot is similar to the one in the QR Code, where you can create your quiz, set up your questions with videos, tables, graphs and you can put the time you want the student to think about the problem and answer. I do not see the potential with the issue of building learning, but I think it is very potent in relation to your review, because we can rescue if the student was able or not to understand the real meaning of the concepts of Statistics. So, you use your cell phone, it can be like a computer too, which is what is holding a student today, you have this idea of the competition and rescue the contents that were worked on during your practice. So, if I can propose this, which is an activity that enters technology, they will love it.

Source: Collection of authors (2020)

In order to optimize the presentation of the results of this research, the discussions related to the speech will be indicated in two stages. In the first, students’ understandings and difficulties in relation to statistical content will be exposed, socialized by the teachers of the MoSaiCo Edu Group. Then, the strategies and resources used for teaching Statistics will be shared and analyzed.

Understanding and difficulties of students about statistical content

In the speech, the teacher exposed possible “gaps” in the training of students, as well as the importance of content prior to the formalization of Statistics – “*they have arrived without the basics of the basics [...] you have to understand what you are proposing and what kind of base are needed*” (DSC clipping). This situation has caused repetition in the teaching of concepts, which for him were basic, also necessary so that the learning of subsequent contents would not be impaired – “*we teach such a basic course and we have to repeat all that, several times*” (DSC clipping).

These aspects are indicated by Grossman and Schoenfeld (2019, p. 182), who point out as essential to educational practice, that teachers “determine what types of understanding of the course their students already have and that create teaching practices that are appropriate for the level knowledge and development of students”. Regarding Statistics, Garfield and Ahlgren (1988) point out that the inadequate reasoning in relation to these concepts is generalized and persistent, similar at all age levels, including among experienced researchers, in addition to being quite difficult to be modified.

In view of this knowledge about learners, as well as about the concepts necessary for the understanding of statistical content, two examples were presented by the teacher to demonstrate the reflexes of the gaps in the training of students, in this case, in the construction of graphs and in the calculation of probability. In the first context, the teacher reported a situation in which, due to not knowing how to calculate a percentage with a rule of three, the student had difficulty with the graphs – *“I want to put a percentage on my graph [...] how is the order that do I solve this rule of three”* (DSC clipping).

In this case, the students’ lack of understanding of the percentage, which may be the result of previous learning, reflected on the construction of the graph, which is, therefore, a necessary mathematical concept when proposing activities related to Statistics. According to Parker and Leinhardt (1995), the percentage, despite being commonly used in the media and in the school and university curriculum, is a difficult concept to be learned and taught, due to its ambiguous and subtle character, which may be the result of calculation simplifications and conversions; for appearing, many times, to have different meanings at the same time; for using an extremely concise language; in addition to being “poorly taught such that students often have a limited view of the concept as meaning only part of a whole” (*Ibidem*, p. 473).

Another situation that illustrated how previous difficulties could hinder the learning of later contents, was related to mathematical operations, such as fractions and factorials – *“I feel that the people are needed when it comes to seeing probability, operating with fractions, factorial”* (DSC clipping). About this, O’Connell (1999) describes problems involving the learning of probability, such as difficulties in understanding text, concepts, processes, and arithmetic/computational calculations – which, when considered by the teacher, can contribute to the process teaching and learning this content. Thus, when students have difficulties in fraction and factorial, these misunderstandings have repercussions on the activities developed in the classroom, even though errors and misunderstandings are not directly related to the learning of probability.

Still about this subject, the professor described problems that involved combinatorial analysis, as a support to the learning of probability. According to Batanero, Godino and Navarro–Pelayo (1997), many errors in the calculation of probability are due to issues involving combinatorial reasoning, such as the erroneous enumeration of the sample space. Thus, in view of the importance of calculating clusters for the probability, understanding the possible combinations and in which specific situations they are used, are fundamental knowledge, since possible errors in probability may be related to difficulties in combinatorial reasoning, as described in teacher, when narrating the reaction of the students in the classroom – *“some problems that have to bring the idea of combinatorics so you can know what the sample space is: “My God! You have to make combinations in the middle of probability”* (DSC clipping).

In these examples, although the mathematical concepts (rule of three, percentage, fractions, factorial and combinatorial) are not the focus of teaching, this knowledge supports the development of Statistical Literacy, especially in the “generating certain statistical

indicators, as well as the mathematical connection between summary statistics, graphs, or charts, and the raw data on which they are based” (Gal, 2002, p. 14). Thus, it can be considered that the difficulties of students on mathematical knowledge reflect on the learning of Statistics, as in the case of the percentage and the rule of three, which are fundamental resources for the interpretation and construction of graphs and tables.

In addition to recognizing students’ misunderstandings about mathematical concepts and considering them in planning activities (Park & Oliver, 2008; Shulman, 2014), teacher knowledge about the vertical curriculum is necessary, that is, “familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the materials that embody them” (Shulman, 1986, p. 10). Such knowledge is required so that the teacher can understand which concepts should already be known by the students (such as combinatorial analysis, fractions, and factorial to solve probability problems), as well as the contents that will later be part of the students’ school education.

In addition to the difficulties in mathematical concepts, in Statistics there are specific misunderstandings, such as in graphical and tabular representations and in measures of central tendency and dispersion. About the double – entry graphs and tables, the professor described an activity that related the presence of students at the university and their physical activity practices, but that was only developed with their guidance and exemplification – “*I asked them to build a table and a double entry chart, but until I built my double entry table, they were unable to build it*” (DSC clipping).

In the course of this activity, the teacher recognized the students’ difficulties in relation to the construction of graphs and double – entry tables, a scenario close to that found by other researchers (Fernandes & Júnior, 2014). In this situation, when identifying the difficulties of the students on this content, the teacher mobilized pedagogical knowledge, when he considered exemplification as a means to contribute to the understanding of graphic and tabular representations. In fact, when the teacher recognized the students’ understandings and possible mistakes about these statistical concepts, his pedagogical decisions were impacted, since they needed to be reorganized in view of the students’ learning difficulties and specificities.

Another important aspect, observed in the proposal of this activity, was the choice of the theme to be researched, related to the students’ daily experiences that, possibly, could interest them to study. In this perspective, teaching Statistics requires that the teacher exposes real examples and illustrations, as well as knowing “how to use them to involve students in the development of their critical judgment”, through statistical thinking (Cobb & Moore, 1997, p. 803), this being, therefore, a teaching knowledge about the educational contexts and the interests of the students.

In the sequence of the speech, when entering the measures of central tendency, more specifically, in the description about the students’ understanding of the mean, one can identify with this teacher understood the “mean”, especially applicable in everyday situations

and a basic concept among the rest of Statistics students – “*they see it even on a daily basis, one of the most basic of Statistics*” (DSC clipping). In addition, the professor shared his students’ understanding of this measure of central tendency, which was based on the calculation algorithm – “*what do they think the mean is? add and divide by quantity*” (DSC clipping).

This understanding of the students about mean, may be the result of a decontextualized teaching and based on algebraic procedures (Mokros & Russell, 1995), which leads to conceive this measure as being direct and simple (Strauss & Bichler, 1988). In this way, strategies based on real and contextualized data, as well as involving the idea of “representativeness”, even before the formalization of the calculation algorithm, can be relevant means for understanding the concept of mean (Mokros & Russell, 1995).

Despite these difficulties, the average is the central measure chosen when students and teachers need to represent a set of data, although, in some cases, it is not the most appropriate measure (McGatha, Cobb & McClain, 2002). This situation corroborates the one exposed by the teacher, who described the lack of knowledge of the median and the confusion of this measure in relation to the mean – “*Mean is even more common, but the median they do not know that it exists or what it is. What is median? And they, memorized: “is what is in the middle”. I said: So, what is the difficulty? “Okay, but isn’t the median value mean?” – No, that would be mean value!*” (DSC clipping).

According to Boaventura and Fernandes (2004), the median is little used in everyday situations, in addition to being confused with the other central measures (as in the “mean value”) and being considered, among these, the most difficult, especially in relation to interpretation and the calculation algorithm. When recognizing these difficulties, as well as the specificities of the median, the teacher has the possibility to prioritize the teaching of this concept, as well as to develop proposals in which not only calculations are applied, but where students can evaluate and define the most appropriate measure to characterize a data set.

Regarding mode, the professor pointed out the students’ difficulty in relating this statistical concept to the idea of fashion (mode in Portuguese), commonly used in everyday life (“fashion car”, “fashion clothes” etc.), since they did not establish an association between what is taught in the formal space of the classroom and their everyday experiences – “*the word ‘mode’ will have nothing to do with the word ‘fashion’ outside*” (DSC clipping). In this situation, it is not possible to identify the reason why the students did not understand the concept of mode, since this measure, according to Boaventura and Fernandes (2004), is the one that generates less doubts for students, when compared to the median and mean. However, if the variable is of a qualitative type or when it is necessary to work with mode, relating it to the mean and the median, students tend to have difficulties (*Ibidem*). In this way, practices that prioritize everyday examples can help in understanding mode, as students start to relate their personal experiences to the activities developed in the classroom.

In addition to the mean, mode and median, difficulties were also indicated in the measures of dispersion – “*when you ask the question of standard deviation, variance, then ‘it*

screwed up” (DSC clipping). To exemplify this situation, the teacher described an activity in which it was necessary to identify and justify which fuel (gasoline or alcohol) had the most homogeneous price, having already calculated the mean and standard deviation – “*they had to calculate mean and standard deviation in the letter ‘a’ and in the letter ‘b’ they had to explain what the fuel was with the most homogeneous price, but they couldn’t explain it*” (DSC clipping).

This difficulty of the students in explaining the degree of variability in the price of fuel was justified by the teacher as problems of interpretation, since, for him, the students stopped at the formulas – “*putting a little bit of interpretation is an immense difficulty, since they want to memorize formulas and cannot see and think*” (DSC clipping). This situation is in line with what Garfield and Ben-Zvi (2008) point out, that is, although students know the algorithm of formal measures of variability, they have difficulty in understanding what these measures represent in a data set, they do not understand the importance of such a measure and its relationship with other statistical concepts. Therefore, it can be seen that the finding of this research is also present in other studies. Consequently, it is an indication that the difficulties of the students, with regard to interpretation, which limits them to the use of formulas at the expense of understanding the context, make up the teaching knowledge to teach Statistics. Elements like these corroborate for the delimitation, as well as for the detailing of the specific teaching knowledge regarding the difficulties of the students, for the development of statistical skills. Furthermore, they constitute a repertoire for the advancement of research in the area and the systematization of results.

Although the teaching knowledge regarding student difficulties, regarding interpretation, being restricted to the use of formulas, is evident, there are still some reflections. Although these have not been evidenced in the corpus of analysis, they are elements for further investigation, which have already been described in some studies. Then, the aforementioned activity stands out. Although “gasoline” and “alcohol” are terms, possibly known to students, such a topic may not have interested them – if they are considered underage students and/or who do not have motor vehicles – which can also help explain these difficulties interpretation, even though the results of calculating the mean and standard deviation have already been identified. In other words, it is necessary to delimit teaching knowledge in this environment, referring to which not only the context needs to be considered in pedagogical strategies, but especially the context that is relevant to the learner, so that he can build meanings on statistical concepts.

Strictly gathering the research findings regarding the restricted use of formulas, to the detriment of interpretation, combined with the context identified in the DSC, previously mentioned, and integrating this evidence with other studies in the area, it is possible to extrapolate to a teaching knowledge, derived from the student’s difficulty, that knowing the characteristics of the different scenarios that permeate pedagogical practice, from the classroom to the school community, are understandings related to the context and students, fundamental to the exercise of teaching (Shulman, 2014). In fact, the teaching and learning context “is formed from the interaction of teachers with different students, who come from

private social addresses and are inserted in school realities and specific classrooms” (Marcon, Graça & Nascimento, 2011, p 329). In other words, the theme can be a central point for movement in the sense: from the restricted use of formulas to interpretation. This subtlety found in the corpus, but endorsed by other previous research findings, presents itself as a teaching knowledge beyond the students’ learning problems, being characterized as a teaching knowledge for a possible mitigation of this difficulty.

In some moments of the speech, in spite of recognizing the problems involving Statistics, the teacher showed some concern about the reasons that led the student to not understand the mean, seen by him as one of the most basic concepts of Statistics, or else, the confusion between mean and median, through the expression “mean value”. This situation may indicate an underestimation of these students’ difficulties (Garfield, 1995), which may also reflect the teacher’s beliefs about how he teaches and how he sees the content to be taught (Danişman & Tanişli, 2017).

These teaching understandings, related to what you know about your subject and how you believe that knowledge is built in relation to a specific content, can modify the way it plans activities, selects materials for teaching, organizes the curriculum, as well as interacts with students (Grossman & Shulman, 1994). In this perspective, the figure of the teacher and the way he sees the students’ content and learning are elements that reflect on the practices in the classroom, as well as in the process of learning the statistical concepts.

Pedagogical strategies and resources for teaching Statistics and Probability

Project methodology was indicated by the professor as a strategy to address statistical concepts – “*i am passionate about the research projects themselves, for the construction of the concepts of Statistics*” (DSC clipping). Among the different nomenclatures that can be attributed to the projects, the professor described the importance he attributed to the Learning Projects – PA, which is characterized by the role of the student to research in view of his knowledge and interests – “*I fell in love with the Learning Project strategy, which is to work with a student’s authorial theme*” (DSC clipping).

The choice of the teacher by the PA, at the expense of other strategies that also show investigative processes for the teaching of Statistics, can be better understood when considering the texts discussed collectively in the meetings, especially in the third, in which the Group dedicated itself to reading and share understandings about the text by Porciúncula and Samá (2015). In this text, the authors present the references that support this strategy, especially Fagundes, Sato and Laurino-Maçada (1999), in addition to indicating the steps that can be considered for the development of PA and the importance of choosing the research problem by the student himself, based on their curiosities, desires, doubts and questions.

Still on the projects, the professor highlighted the different ways of developing them, including when starting the activities with the students’ previous experiences and knowledge – “*I’ve done many different ways, even without them, theoretically, having seen anything of*

the content, only what they saw on TV, what they have seen in life” (DSC clipping). The experience of the stages of a statistical investigation by the students, as in the case of Learning Projects, is an alternative to the construction of statistical knowledge, when their interests and individual styles are considered, a strategy that can also enhance the dialogue and the “construction of a living space that promotes interaction, cooperation, autonomy in the investigative process and criticality” (Porciúncula & Samá, 2015, p. 139).

When proposing work with projects, the teacher also explained, even without mentioning a specific pedagogical model, as he saw the students’ learning process. This, because learning through projects meets a Piagetian perspective of knowledge construction (Piaget, 1976), as described by Porciúncula and Samá (2015, p. 134), since in this strategy it is understood that “educating consists in providing paths of interaction that lead to the construction of knowledge, that have meaning for the student”.

Thus, when the teacher described the relevance he attributed to the projects, it can be considered that he was opposed to a dynamic of knowledge transmission, since he considered the students as protagonists of the teaching and learning process, so that they – *“participate in the stages of an investigative process”* (DSC clipping). Therefore, these teaching understandings are part of the General Pedagogical Knowledge of the teacher, since they describe “knowledge of theories of learning and general principles of instruction, an understanding of the various philosophies of education, general knowledge about learners, and knowledge of the principles and techniques of classroom management” (Grossman & Richert, 1988, p. 54).

In addition, when developing projects, doubts and conceptual confusions can be evidenced, as in the definition of the research question, in the construction of the research instrument, in the process of analysis and presentation of the results, which demands from the teacher, knowledge about the statistical content, the pedagogical aspects and the learning specificities of the students who, together, represent the Pedagogical Knowledge of Statistical Content – CPCE. Consequently, when proposing the projects, if the teacher has gaps in the Knowledge Base for the teaching of Statistics, he may miss opportunities to address the students’ understandings and misunderstandings, when they express their understandings and difficulties in classroom interactions (Burgess, 2008).

On the other hand, the sector charts were approached through an activity, using pedagogical material – *“we made a chart with “how many came to class today and how many didn’t come. For each one that arrived, put a slice on the chart, until it formed the circle”* (DSC clipping). In this case, the professor, based on his knowledge of graphic representations, in view of the students’ context and learning, as well as the curricular resources available to address such content, provided a space for the construction of knowledge about graphic representations. Therefore, the professor considered the student’s context as a means of motivating him to get involved in the activity, also reinforced by the use of pedagogical material, which “facilitates observation and analysis, develops logical,

critical and scientific reasoning, it is fundamental for experimental teaching and is excellent for assisting students in building their knowledge” (Turrioni & Perez, 2006, p. 61).

The pedagogical material was also used to work with the concept of variability, through an activity related to sampling – *“they will cut, put in a bag and count how many there are in each bag [...], they have to make the sample, take, measure and put the straw back. We will compare”* (DSC clipping). Thus, the students evaluated the dispersion around the average length of the plastic straws, although without formalizing the concepts of standard deviation and variance. This proposal was relevant for the students to construct a reasoning about variability, as suggested by Garfield and Ben-Zvi (2008), starting with the understanding of informal ideas (such as analysis of differences in data values) for the understanding and interpretation of measures formal variables of variability (such as interval, interquartile interval, and standard deviation).

When describing this activity, the teacher demonstrated a profound knowledge of the statistical and mathematical content, necessary so that, when using straws with the intention of promoting learning spaces, he developed strategies and materials appropriate to address dispersion measures, even before the formalization of this concept. In this perspective, such a situation requires deep and broad knowledge about the statistical content, which includes language, situations, concepts, propositions, procedures, and arguments, specific about what will be taught (Godino et. al., 2011).

In this context, the use of teaching materials also lacks specific knowledge from teachers, who need to define why, which, when and how to use them in the classroom, so that they do not become ineffective or harmful to learning (Lorenzato, 2006). In other words, the teacher needs to know how to use the curricular materials and resources that are available for teaching a specific content (Shulman, 1986; 2014). To explain this curriculum knowledge, Shulman (1986, p. 10) suggests an analogy: just as a doctor needs to know the different interventions and medications for the treatment of a disease, in the same way the teacher is expected to know the tools curricula available to teach their students.

In addition to the use of pedagogical material, in the activities described in the speech, there was an emphasis on the students’ daily experiences, as in the case of the proposal to compare two population proportions, based on statistical information published in the media – *“The exercise now was in relation to “Fantástico” (TV program) which says that for the first time the rate of Brazilians who smoke has increased, after 15 years decreasing. We are going to do the test comparing two population proportions”* (DSC clipping). It is worth mentioning that hypothesis tests are possibly among the contents related to statistical inference, “the least understood, most confused and the one that is most abused in all Statistics”, among university students and scientists (Batanero, 2001, p. 106).

Facing this problem, the teacher needs to look for different alternatives to approach this content, such as the use of contextualized situations, in which teaching is based on working with real data and problems, since the context provides meaning to the data analysis (Cobb & Moore, 1997). In the situation presented in the speech, the teacher once again

recognized the importance of the context for understanding the concepts related to statistical inference, a fact encouraged by the mastery of the content to be taught, when he then mobilized pedagogical aspects in the promotion of learning environments, for example, through the analysis of data published in the media.

In addition to pedagogical material and contextualized activities, in the speech the teacher described proposals related to ENEM issues – “*we work a lot on the issues of ENEM – a lot of graphics, a lot of questions about mean, mode, median, table analysis and probability issues*” (DSC clipping). In this case, problem solving becomes a means by which students begin to “understand the data of a problem, make decisions to solve it, establish relationships, know how to communicate results and be able to use known techniques” (Zuffi & Onuchic, 2007, p. 83).

Even though students may present an unsatisfactory performance on ENEM issues related to statistical content, problem solving based on this assessment, “encourages logical reasoning, interpretation, data analysis, tables and graphs, and makes it possible to develop mathematical concepts in an interesting, challenging and sometimes contextualized way” (Amorim, 2009, p. 60). In this case, the teacher not only recognized the importance of this evaluation process in education as a whole, but also included it as a means of addressing statistical concepts, this being a knowledge of the educational system, which integrates his curriculum knowledge, necessary for planning and developing classroom activities.

Technological resources have also been described as a means to promote Statistics learning spaces. Two examples were presented: the two-dimensional bar codes (QR Code), to answer questions in a more dynamic way – “*I took everything and put it in the format of “QR Code”, that is, when they read by cell phone, the question arose*”; and Kahoot, as a tool to review / resume students’ understandings of statistical content – “*Kahoot is similar to the one in the QR Code, where you can create your quiz, set up your questions with videos, tables, graphs and you can put the time you want the student to think about the problem and answer*” (DSC clipping).

In these cases, the planning and development of these activities demanded knowledge related to the technological tools available, about the content to be covered in the proposal, specific pedagogical issues for teaching with technology, as well as about the interest of students in using the cell phone to learn Statistics. In view of this different knowledge mobilized by the teacher, it is possible to present what Lee and Hollebrands (2011) called Technological Pedagogical Statistical Knowledge – TPSK, which integrates the knowledge of the statistical content and the technological statistical knowledge. In other words, teaching that aims to involve students, through the learning of Statistics with technology, requires that the teacher has a “depth of knowledge about statistics, technological tools for exploring statistical ideas, and of pedagogical issues related to teaching and learning statistics with technology” (*Ibidem*, p. 359).

Finally, among the elements of the statistical knowledge base, necessary for the promotion of Statistical Literacy, Gal (2002) describes the understanding of the basic notions

of probability, which were also exposed in the discourse, through activities with “tacks” and the Monty Hall problem. In the first case, two types of thumbtack were used for an experiment and, later, calculation and estimation of the probability – “*the thumbtack falls, butt down; or it falls to the side*” (DSC clipping).

In this activity, the teacher addressed the frequentist probability, through successive releases of the “thumbtack”, in which the frequency of occurrence in which, when falling on the ground, the tip was up or tilted was evaluated. It should be noted that “the perception of the object’s asymmetry instigates the analysis of the absence of equiprobability for the elementary events ‘tip up’ and ‘tip down’, which identifies the position of the object when touching the ground”, which differs from strategies, such as drawing colored balls of the same shape and size in a box (Oliveira & Pereira, 2012, p. 156). Therefore, for the proposition of this activity, a wide knowledge of frequentist probability was necessary, since he founded the pedagogical choices of the teacher who, in this proposal, extrapolated the conception of probability related to phenomena with equally probable results.

The second proposed activity involved the Monty Hall problem, in which it is possible to address conditional probability, since the presenter knows where the prize is and the door to open is conditioned on the participant’s initial choice. In addition, this activity “because it involves gains and losses, attracts attention, creating an environment in which the student feels challenged to make a decision that seems favorable to him” (Cordani & Satie, 2019, p. 2). Thus, this problem, which arose from an American television program and was adapted to Brazilian television as “The door of the desperate people”, allows students to analyze the possibilities of winning a prize, through experimentation.

When describing these activities, which involved experimenting with “thumbtacks” and the Monty Hall problem, different teaching knowledge related to curriculum resources, the content of Probability and the interests of students can be identified (Shulman, 2014). According to Danişman and Tanişli (2017), teachers’ knowledge about the content of probability affects their pedagogical prepositions, which also require teaching in addition to commonly used examples, which involve data, currencies, weather forecast, race winner and gamble. In the case presented in the speech, the teaching of probability did not address these typical pedagogical representations, since the teacher sought, through materials and games, to encourage the discussion of non-equiprobable situations (thumbtacks) and that involved decision-making, at Monty Hall.

In view of the analysis presented here, a synthesis of the identified knowledge that constitutes the Knowledge Base for the teaching of Statistics is highlighted in Figure 1. In this case, the Pedagogical Knowledge of Statistical Content is built as a result of the confluence of other knowledge of the base, such as students, contexts, teaching strategies and methods, statistical content and curriculum.

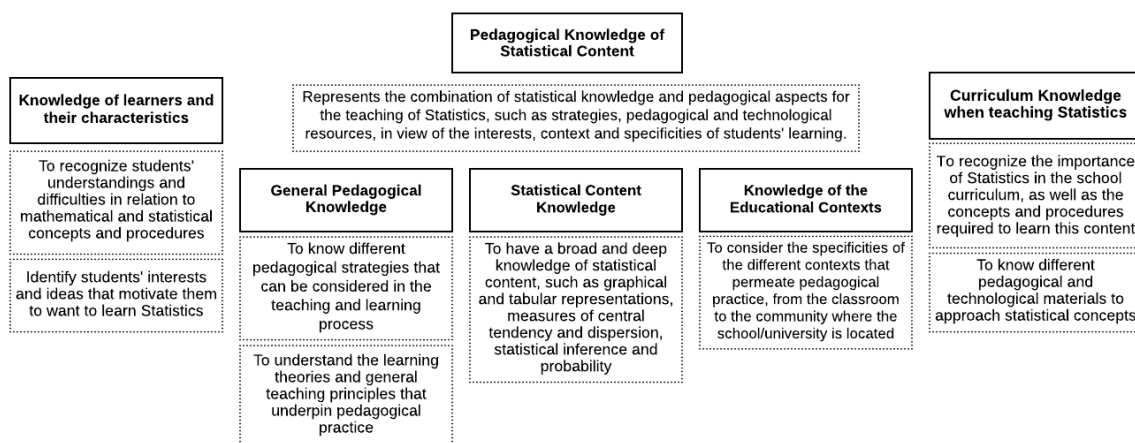


Figure 1 – Knowledge mobilized by the teachers of the MoSaiCo Edu Group for the teaching of Statistics

Source: Collection of authors (2020)

The knowledge shown in Figure 1, emerged from the socialization of the experiences lived by the teachers of the MoSaiCo Edu Group, which involved the students' understandings and misunderstandings (about mathematical concepts – percentage and rule of three, fractions, factorials, and combinatorics; and statistical – graphical and tabular representations; measures of central tendency and dispersion). The curriculum knowledge, on the other hand, was related to mathematical and statistical concepts and procedures, pedagogical, technological, and evaluative resources. Finally, the development of research projects, activities with teaching materials (graphs and variation), analysis of data exposed in the media and in the students' daily lives (statistical inference and probability), standardized tests (ENEM), technologies (Kahoot! and QR Code), and experimental activities (with “thumbtacks” – probability) in the teaching of Statistics were also shared.

Final considerations

In this paper, the teaching knowledge, identified from the narratives of the teachers of the MoSaiCo Edu Group, was analyzed, especially with regard to the students' understandings and difficulties, and the pedagogical strategies considered for the teaching of Statistics. This knowledge involved understandings related to the specificities of learning, protagonism, interests and mistakes of the students, to the appreciation of the context and everyday situations in the activities proposed in the classroom, in addition to the use of technological resources, teaching materials and standardized tests.

The knowledge of learners included the recognition of statistical and mathematical misunderstandings, as well as the motivations and interests in learning Statistics. In other words, the teacher not only assumed the difficulties and gaps in the training of students, but he also proposed contextualized activities, with experimentation in which evidenced their protagonism, through the development of statistical research, pedagogical and technological resources, in addition to working with real data and problems.

The Statistical Content Knowledge supported the discussions presented in the speech, since it was provoked in the interactions with the students, when they explained their misunderstandings and mistakes, as well as in the proposal of the strategies, materials, and resources, which, to be defined, were based by the facts and concepts inherent to Statistics. In addition, the Curriculum Knowledge enabled the teacher to understand the previous concepts, necessary for learning Statistics, as well as the curricular materials that can be used in classroom activities.

The General Pedagogical Knowledge involved, not only the most general teaching strategies, but also the theories of learning and the principles that structure the actions and planning of the teacher on the activities developed in the classroom, with a view to the learning of the students. In addition, it is possible to highlight aspects related to the Knowledge of Educational Contexts, especially about the role of the context in the learning of statistical concepts, this scenario being linked to the social and cultural particularities of the community where the school/university and students are inserted.

Based on the analysis of this knowledge presented here, the Pedagogical Knowledge of Statistical Content is constituted, mobilized, and built on the practices of teaching and learning Statistics in the classroom, especially in the preposition and development of research projects, experimentation activities, problem solving, contextualized activities and technologies. This knowledge represented the combination of Statistical Content Knowledge and pedagogical aspects for the teaching of Statistics, such as strategies, didactic and technological resources, in view of the interests, the context and the specificities of the students' learning. Therefore, this knowledge is built with and about the Statistical Content Knowledge, the General Pedagogical Knowledge, the Knowledge of the Educational Contexts, the Knowledge of the learners and the Curriculum Knowledge when teaching Statistics.

Finally, he points out that other knowledge can be identified in this text, as these underlie the teacher's practice in the classroom. Therefore, future analyzes on the mobilization of teaching knowledge for the teaching of Statistics, in the collaborative scope of this training space, may contribute to the systematization of these and other understandings, which are part of the Knowledge Base for the teaching of Statistics.

Acknowledgments:

This paper was carried out with the support of the Coordination for the Improvement of Higher Education Personnel – Brazil (CAPES) – Financing Code 001.

References

Amorim, L. D. (2009). *Estratégias utilizadas por estudantes na resolução de questões do Exame Nacional do Ensino Médio (ENEM)*. TCC do curso de Licenciatura em Matemática. Porto Alegre: Universidade Federal do Rio Grande do Sul. Retirado em 20 de setembro de 2020, de: <https://www.lume.ufrgs.br/handle/10183/18225>

- Ball, D. L., Thames, M. H., & Phelps, G. (2005). *Articulating domains of mathematical knowledge for teaching*. Online: www-personal.umich.edu/~dball/.
- Batanero, C. (2001). *Didáctica de la Estadística*. 1. ed. Granada: GEEUG, Departamento de Didáctica de la Matemática, Universidad de Granada.
- Batanero, C., Godino, J. D., & Navarro-Pelayo, V. (1997). Combinatorial reasoning and its assessment. In Gal, I., & Garfield, J. B. (Eds.). *The assessment challenge in statistics education* (pp. 239–252). Amsterdam: IOS Press e International Statistical Institute.
- Boaventura, M. G., & Fernandes, J. (2004). Dificuldades de alunos do 12.º ano nas medidas de tendência central: o contributo dos manuais escolares. In *Actas do I Encontro de Probabilidades e Estatística na Escola* (pp. 103–126).
- Bogdan, R. C., & Biklen, S. K. (1994). *Investigação qualitativa em educação*. Tradução de Maria João Alvarez, Sara Bahia dos Santos e Telmo Mourinho Baptista. 1. ed. Porto: Porto Editora.
- Bolívar, A. (1993). “Conocimiento didáctico del contenido” y forma–ción del profesorado: El Programa de L. Shulman. *Revista Interuniver–sitaria de Formación del Profesorado*, 16, 113–124.
- Burgess, T. (2008). Teacher knowledge for teaching statistics through investigations (pp. 1–6). In Batanero, C., Burrill, G., Reading, C. & Rossman, A. (Eds.), Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. *Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference* (pp. 1–6). Monterrey: ICMI and IASE. Retirado em 03 de julho de 2020, de: https://www.stat.auckland.ac.nz/~iase/publications/rt08/T2P12_Burgess.pdf.
- Brasil. (2018). Ministério da Educação. Secretaria de Educação Básica. *Base Nacional Comum Curricular*. Brasília: MEC/SEB.
- Campos, C. R., Wodewotzki, M. L., & Jacobine, O. R. (2011). *Educação Estatística: teoria e prática em ambientes de modelagem matemática*. Belo Horizonte: Autêntica Editora.
- Cobb, G. W., & Moore, D.S. (1997). Mathematics, statistics, and teaching. *American Mathematical Monthly*, 104, 801–823.
- Coelho, M. (2010). *Os saberes profissionais dos professores: a problematização das práticas pedagógicas em estatística mediadas pelas práticas colaborativas*. Doutorado em Educação. Campinas: Universidade Estadual de Campinas. Retirado em 05 de fevereiro de 2020, de: https://bdtd.ibict.br/vufind/Record/CAMP_c62bbb18f6727e03e27146126de283ba
- Cordani, L. K., & Satie, D. (2019). Uma abordagem didática do problema de Monty Hall. In Contreras, J. M., Gea, M. M., López–Martín, M. M. & Molina–Portillo, E. (Eds.). *Actas del Tercer Congreso Internacional Virtual de Educación Estadística* (pp. 1–10). Retirado em 10 de setembro de 2020, de: www.ugr.es/local/fqm126/civeest.htm.
- Crecci, V. M., & Fiorentini, D. (2018). Desenvolvimento profissional em comunidades de aprendizagem docentes. *Educação em Revista*, 34, 1–20, e172761.
- Danişman, S., & Tanişli, D. (2017). Examination of Mathematics Teachers’ Pedagogical Content Knowledge of Probability. *Malaysian Online Journal of Educational Sciences*, 5 (2), 16–34.

- Fagundes, L. C., Sato, L. S., & Laurino–Maçada, D. L. (1999). *Aprendizes do futuro: as inovações já começaram*. Brasília: MEC, s/d. Retirado em 15 de outubro de 2020, de: <http://www.proinfo.mec.gov.br/>.
- Fernandes, G. J. R., & Junior, S. G. (2014). O ensino e aprendizagem de gráficos e tabelas para os anos iniciais do ensino fundamental. *Anais do Congresso Iberoamericano de Ciencia, Tecnología, Innovación y Educación* (pp. 1–14). Buenos Aires: Congreso Iberoamericano de Ciencia, Tecnología, Innovación y Educación. Retirado em 20 de setembro de 2020, de: <https://www.oei.es/historico/congreso2014/memoriactei/1390.pdf>.
- Fiorentini, D. (2004). Pesquisar práticas colaborativas ou pesquisar colaborativamente? In M. C. Borba & J. L. Araújo (Orgs.), *Pesquisa qualitativa em Educação Matemática* (pp. 47–76). 1. ed. Belo Horizonte: Autêntica.
- Fiorentini, D. (2010). Desenvolvimento profissional e comunidades investigativas. In A. Dalben, J. Diniz, L. Leal & L. Santos (Orgs.), *Convergências e tensões no campo da formação e do trabalho docente: Educação ambiental, Educação em ciências, Educação em espaços não escolares, Educação matemática* (pp. 570–590). 1. ed. Belo Horizonte: Autêntica.
- Gal, I. (2002). Adults Statistical Literacy: meanings, components, responsibilities. *International Statistical Review*, 70 (1), 1–25.
- Garfield, J. (1995). How students learn statistics. *International Statistical Review*, 63, 25–34.
- Garfield, J., & Alhgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. *Journal for Research in Mathematics Education*, 19 (1), 44–63.
- Garfield, J. B., & Ben–Zvi, D. (2008). Learning to reason about variability. In Garfield, J. B., & D. Ben–Zvi (Eds.), *Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice* (pp. 201–214). Springer.
- Godino, J. D., Ortiz, J. J., Roa, R., & Wilhelmi, M.R. (2011). Models for statistical pedagogical knowledge. In C. Batanero, G. Burrill & C. Reading (eds.), *Teaching Statistics in School Mathematics – Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study* (pp. 271–282). Berlin: Springer.
- Grossman, P. L., & Shulman, L. S. (1994). Knowing, believing, and the teaching of English. In T. Shanahan (Ed.). *Teachers thinking, teachers knowing: Reflections on literacy and language education* (pp. 3–22). Urbana, IL: National Council of Teachers of English.
- Grossman, P., & Schoenfeld, A. (2019). O ensino do conteúdo da disciplina (pp. 170–196). In L. Darling-Hammond & J. Bransford (Orgs.), *Preparando professores para um mundo em transformação: o que devem aprender e estar aptos a fazer*. 1. ed. São Paulo: Penso editora.
- Gudmundsdóttir S., & Shulman, L. S. (1987). Pedagogical Content Knowledge in Social Studies. *Scandinavian Journal of Educational Research*, 31 (2), 59–70.
- Hill, H. C., Schilling, S., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal*, 105 (1), 11–30.
- Lefèvre, F., & Lefèvre, A. M. C. (2005). *O Discurso do Sujeito Coletivo: um novo enfoque em pesquisa qualitativa (desdobramentos)*. Caxias do Sul: Educs.

- Lefèvre, F., Lefèvre, A. M. C., & Marques, M. C. C. (2009). Discurso do sujeito coletivo, complexidade e auto-organização. *Ciência & Saúde Coletiva*, 14 (4), 1193–1204.
- Lefèvre, F., & Lefèvre, A. M. C. (2012). Pesquisa de representação social: um enfoque qualitativo – a metodologia do Discurso do Sujeito Coletivo. 2. ed. Brasília: Liber Livro. (Série Pesquisa; 20)
- Lee, H. S., & Hollebrands, K. F. (2011). Characterizing and developing teachers' knowledge for teaching statistics. In C. Batanero, G. Burrill & C. Reading (Eds.), *Teaching statistics in school mathematics – Challenges for teaching and teacher education: A joint ICME/IASE study* (pp. 359–369). New York: Springer.
- Lopes, C. E. (2013). O Desenvolvimento Profissional de Professores em Educação Estatística nas Pesquisas Brasileiras. In A. Salcedo (Org.). *Educación Estadística em America Latina: tendências e perspectivas* (pp. 229–253). Caracas: Universidad Central de Venezuela.
- Lopes, C. E., & Mendonça, L. O. (2017). *Trilhas investigativas em educação estatística narradas por professores que ensinam matemática*. (Orgs.). 1. ed. Campinas, SP: Mercado das Letras (Série Educação Estatística).
- Lorenzato, S. (2006.). *Laboratório de Ensino de Matemática na formação de professores*. Campinas: Autores Associados.
- Lüdke, M., & André, M. E. D. A. (1986). *Pesquisa em educação: abordagens qualitativas*. São Paulo, Editora Pedagógica e Universitária. 99p
- Marcon D., Graça, A. B. S., & Nascimento, J. V. (2011). Reinterpretação da estrutura teórico-conceitual do conhecimento pedagógico do conteúdo. *Revista Brasileira de Educação Física e Esporte*, 25 (2), 323–39.
- McGatha, M., Cobb, P., & McClain, K. (2002). An analysis of students' initial statistical understanding: Developing a conjectured learning trajectory. *Journal of Mathematical Behavior*, 21, 339–355.
- Mizukami, M. G. N. (2004). Aprendizagem da docência: algumas contribuições de L. S. Shulman. *Revista do Centro de Educação da UFSM*, 29 (2), 01–13.
- Mokros, J., & Russell, S. J. (1995). Children's concepts of average and representativeness. *Journal of Research in Mathematics Education*, 26, 20–39.
- Nacarato, A. M., & Grando, R. C. (2013). *Estatística e probabilidade na Educação Básica: professores narrando suas experiências*. 1. ed. Campinas, SP: Mercado de Letras.
- O'Connell, A. A. (1999). Understanding the Nature of Errors in Probability Problem-Solving. *Educational Research and Evaluation*, 5(1), 1–21
- Oliveira, P. C., & Pereira, J. C. (2012). Planejamento e delineamento de experimentos probabilísticos para o Ensino Fundamental I. *Linhas Críticas*, 18 (35), 151–170.
- Park, S., & Oliver, S. (2008). Revisiting the conceptualization of pedagogical content knowledge (PCK): PCK as a conceptual tool to understand teachers as professionals. *Research in Science Education*, 38, 261–284.
- Parker, M., & Leinhardt, G. (1995). Percent: A privileged proportion. *Review of Educational Research*, 65 (4), 421–481.

- Piaget, J. (1976). *A equilibração das estruturas cognitivas – o problema central do conhecimento*. Rio de Janeiro: Kahar Editores.
- Porciúncula, M., & Samá, S. (2015). Projetos de Aprendizagem. In M. Porciúncula & S. Samá (Orgs.), *Educação Estatística: Ações e estratégias pedagógicas no Ensino Básico e Superior* (pp. 133–141). Curitiba: CRV.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, 15 (2), 4–14.
- Shulman, L. S. (2014). Conhecimento e ensino: fundamentos para a nova reforma. *Cadernos Cenpec*, São Paulo, 4 (2), 196–229. Tradução: Leda Beck. Revisão técnica: Paula Louzano.
- Strauss, S., & Bichler, E. (1988). The development of children’s concepts of the arithmetic average. *Journal for Research in Mathematics Education*, 19 (1), 64–80.
- Turrioni, A. M. S., & Perez, G. (2006). Implementando um laboratório de educação matemática para apoio na formação de professores. In S. Lorenzato (Org.). *Laboratório de Ensino de Matemática na formação de professores* (pp. 57–76). Campinas: Autores Associados.
- Watson, J. M., Donne, J., & Callingham, R. A. (2008). Establishing PCK for teaching statistics. In C. Batanero, G. Burrill, C. Reading & A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference* (pp. 1–6). Monterrey, Mexico: International Commission on Mathematical Instruction and International Association for Statistical Education
- Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review*, 67 (3), 223–265.
- Yin, R. K. (2001). *Estudo de caso: planejamento e métodos*. Tradução Daniel Grassi. 2. ed. Porto Alegre: Bookman.
- Zuffi, E. M., & Onuchic, L. R. (2007). O Ensino–Aprendizagem de Matemática através da Resolução de Problemas e os Processos Cognitivos Superiores. *Unión Revista Iberoamericana de educación Matemática*, 11, 79–97.