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Specialised knowledge of an early childhood education teacher when teaching geometric solids

Conocimiento especializado de un profesor de Educación Infantil al enseñar cuerpos geométricos

Ana María Escudero Domínguez[1]

María de la Cinta Muñoz-Catalán[2]

José Carrillo Yáñez[3]

Abstract

This work is part of a broader investigation that aims to understand the knowledge of an Early Childhood Education teacher when teaching geometric solids. To give theoretical support to the research shown here, we use the *Mathematics Teacher's Specialised Knowledge* model (MTSK), with which we obtain indicators of specialised knowledge from the Early Childhood Education teacher. It is an instrumental case study, qualitative in nature, focused from an interpretive paradigm. The results show the complexity of the teacher's knowledge in the Early Childhood Education. We show evidence and indications of the specialised knowledge of our informant, highlighting the depth of the mathematical knowledge and pedagogical content knowledge he possesses, as well as the relationships between knowledge of different kinds.

Keywords: Knowledge of the teacher; Geometric solids; Early Childhood Education; Case study

Resumen

Este trabajo forma parte de una investigación más amplia que tiene como objetivo comprender el conocimiento de un profesor de Educación Infantil al enseñar cuerpos geométricos. Para dar sustento teórico a la investigación aquí mostrada usamos el modelo *Mathematics Teacher's Specialised Knowledge* (MTSK), con el cual obtenemos indicadores de conocimiento especializado del profesor de Educación Infantil. Se trata de un estudio de caso instrumental cualitativo enfocado desde un paradigma interpretativo. Los resultados ponen en evidencia la complejidad del conocimiento del profesor en la etapa de Educación Infantil. Mostramos evidencias e indicios del conocimiento especializado de nuestro informante, poniendo de manifiesto la profundidad de los conocimientos matemáticos y del conocimiento didáctico del contenido que posee, así como las relaciones entre conocimientos de distinta índole.

Palabras clave: Conocimiento del profesor; Cuerpos geométricos; Educación Infantil; Estudio de caso

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[1] Doctora en Didáctica de la Matemática, Profesora-Investigadora de la Universidad de Sevilla. España. Email: aescudero1@us.es. ORCID: https://orcid.org/0000-0002-1620-3852

[2] Doctora en Didáctica de la Matemática, Profesora Contratada Doctora de-Investigadora de la Universidad de Sevilla. España. Email: mcmunozcatalan@us.es. ORCID: https://orcid.org/0000-0003-2329-7612

[3] Doctor en Didáctica de la Matemática, catedrático de la Universidad de Huelva. España. Email: carrillo@uhu.es. ORCID: https://orcid.org/0000-0001-7906-416X

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Introduction and background

In recent years there has been a growing interest in research related to the Early Childhood Education stage, assuming that the first years of schooling constitute the foundation for future mathematical learning (McCray & Chen, 2012). There is still little research regarding the teacher at this educational stage (Parks & Wager, 2015), as it is often thought that mathematics in Early Childhood Education can be taught by anyone (Castro & Castro, 2016). Although professional associations such as the *National Association for the Education of Young Children* and the *National Council of Teachers of Mathematics* (NAEYC & NCTM, 2013) establish the need for teachers of the Early Childhood stage to have a solid mathematical education through which they can show the basis for the development of early mathematical training in these students (Muñoz-Catalán, Liñán & Ribeiro, 2017). Along these lines, Perry and Dockett (2002) state that these teachers must know the deep mathematical ideas on which the tasks performed at this stage are based, even if these seem simple (Pitta-Pantazi & Christou, 2011). The Early Childhood Education teacher is not a specialist in any area, so we cannot speak of the mathematics teacher in Early Childhood Education. Although the Spanish curriculum does not establish mathematics as a curriculum area, this professional needs to have a solid knowledge to identify the fundamental mathematics that serve as a foundation for mathematical learning at later stages.

One of the main characteristics of Early Childhood Education teaching is that school contents are approached in a playful way (Wernberg, Larsson & Riesbeck, 2010), as work is done with guided play with the aim of nurturing children's inquiry, engagement and interest in mathematics. Another characteristic of this teacher's professional practice is that he/she must keep in mind that this is a language-generating stage (Escudero-Domínguez, Escudero-Ávila, Aguilar-González & Vasco-Mora, 2019). Therefore, we understand that teachers have to offer children opportunities to think and express themselves, with their own language, and once they have internalized the concept, offer them the formal word. On the other hand, it must be taken into account that the treatment of concepts that is being worked on at this stage is done in a helical way.

The learning of geometry is essential in these early ages due to the help it provides for children to understand the space in which they move (Clements & Sarama, 2011). The acquisition of geometric knowledge contributes to the development of mathematical competence and logical reasoning, which in turn enhances the development of visualization, critical thinking, intuition, problem solving and conjecture skills (Gamboa & Ballestero, 2010). However, despite the role of geometry in student learning, for years it has not been given the attention it deserved (Clements & Sarama, 2011).

Reviewing the proceedings of the main congresses in the area from the period 2010-2018, it is observed that research on professional knowledge of the Early Childhood Education teacher who teaches mathematics is still incipient. In one of them, *Mathematical Knowledge for Teaching* [hereafter MKT] is taken as a theoretical model of analysis (Ball, Thames & Phelps, 2008) but it is concluded that this framework must be adjusted to the Early Childhood Education stage in order to be used in that setting (Mosvold, Bjuland, Fauskanger & Jacobsen, 2011). In recent years, we found another research about the knowledge revealed by an Early Childhood Education teacher about teaching Geometry in a 5-year-old classroom (Hundeland, Erfjord & Carlsen, 2017). This study uses the *Knowledge Quartet* lens [hereafter

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KQ] (Rowland, Huckstep & Thwaite, 2005) as a tool to observe the knowledge that comes into play in the classroom and concludes that the KQ develops differently in the context of Early Childhood Education, but that it is useful to characterize the knowledge of the teacher at this stage. Since 2017 we find different works (Policastro, Almeida & Ribeiro, 2017; Muñoz-Catalán et al., 2019; Escudero-Domínguez, Muñoz-Catalán & Carrillo, in press, among others) that try to advance in the understanding of the specialized knowledge of the Early Childhood Education teacher in the teaching of mathematics. In these investigations it is clear how the *Mathematics Teacher's Specialized Knowledge* [hereafter MTSK] framework (Carrillo, et al., 2018) is a useful tool to reflect on the specialized knowledge of the Early Childhood Education teacher. Different analyses show aspects such as the density and cohesion that define the specialized knowledge of the Early Childhood Education teacher (Muñoz-Catalán et al., 2017). These works show how MK elements are important in this professional and how these sustain to the development of PCK dimensions (Policastro et al., 2017; Escudero-Domínguez et al., in press, among others). For example, knowledge of different representation systems and the complementarity between them (KoT) is key to make decisions about didactic resources (KMT). And knowledge of comparison as a mathematical practice that promotes mathematical knowledge at the stage (KPM) is used as a teaching strategy (KMT) (Muñoz-Catalán et al., 2019). These relationships give us a glimpse of the existence of knowledge elements of different kinds, as well as the diversity of relationships between the different subdomains (Zakaryan & Ribeiro, 2016).

Having as a horizon to deepen the development of MTSK in Early Childhood Education, in this article we aim to understand the specialized knowledge mobilized by an Early Childhood Education teacher in relation to the teaching of geometric solids.

Theoretical Foundation

Teacher knowledge has been studied for decades, being Shulman (1986) who proposed the need to consider the discipline itself to characterize professional knowledge and introduced a distinction between *subject matter knowledge* and *pedagogical content knowledge*. On this basis, Ball, Thames, & Phelps (2008) developed the *Mathematical Knowledge for Teaching* model which poses difficulties for the Early Childhood stage because its application is strongly conditioned by the pedagogical approach of the cultural context in which it was created. Thus, in the American context, where the pre-school stage is oriented as a preparation for Primary school, the tasks that define the teaching work of the mathematics teacher (such as *presenting mathematical ideas or finding an example to illustrate a specific idea*) allow the model to be applied directly. In other contexts, such as the Norwegian, for example, with a more social pedagogical approach, with another role attributed to the teacher, this application is more problematic as they detected (Mosvold, et al., 2011). The above, coupled with our consideration that the specificity of the teacher's knowledge in relation to mathematics teaching is not limited to a single subdomain, led us to use the *Mathematics Teacher's Specialized Knowledge* [MTSK] (Carrillo, et al., 2018). This model is characterized by considering the specialization of the mathematics teacher's knowledge in each of the subdomains that compose it (Scheiner, Montes, Godino, Carrillo & Pino-Fan, 2019). The model conceives the teacher's knowledge in an integrated way, but for its analytical purpose, it subdivides it into three domains: *Mathematical Knowledge* [MK], *Pedagogical Content Knowledge* [PCK] and *Beliefs* that permeate each domain. The latter are not the subject of study in this article. Each domain has distinct subdomains, three in the case

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of the first two knowledge domains (Figure 1).

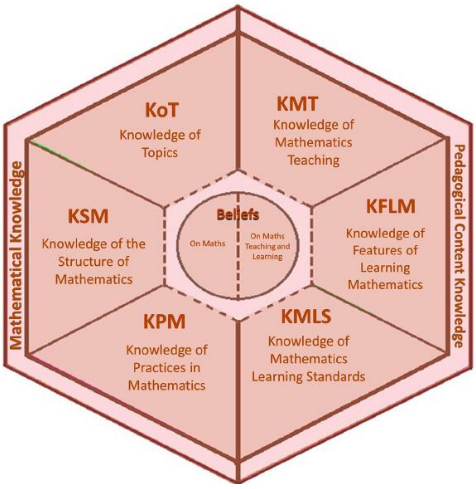

Figure 1 - Subdomains of the *Mathematics Teacher's Specialized Knowledge* [MTSK]
Source: Carrillo et al., 2018

On the one hand, the *Mathematical Knowledge* domain studies different aspects of the mathematical knowledge of the mathematics teacher (Carrillo et al., 2018 and its adaptation to the stage in Muñoz-Catalán, Joglar, Ramírez & Codes, in press, b):

KoT (*Knowledge of Topics*) is more than knowledge of mathematics as a discipline. In addition to school mathematics, we consider here all mathematical knowledge that is useful to the teacher in his or her work. It contains the procedures, the characterization of concepts, properties of mathematical objects and their comprehensiveness, different ways in which a topic can be represented, and knowledge of uses and applications of a topic. It would be examples of KoT to know the relevant and irrelevant attributes of geometric solids, as well as their definitions and the registers that can be used to represent the decomposition of solids, as well as the specificities of each of them and their complementarity.

KSM (*Knowledge of the Structure of Mathematics*) is formed by the teacher's knowledge of the connections that allow an overview of mathematical knowledge. This subdomain is formed by three types of connections: auxiliary (that allow making an instrumental use of a content in the work with another content), transverse (that underlie different mathematical contents) and connections based on simplification or on increased complexity, that allow both advanced knowledge from an elementary perspective, and elementary content from an advanced perspective. While the role of the first two types does not seem clear in children, the connections based on simplification and on increased complexity seem to have an important role in characterizing the specialized knowledge of this professional. An example would be the knowledge that the teacher has of the fact that the task of sorting by size in Early Childhood Education will help in working with scales.

KPM (*Knowledge of Practices in Mathematics*) consists of the teacher's knowledge of those ways of producing and proceeding in mathematics, including aspects of mathematical

Zetetiké

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communication, argumentation, and proof that are brought into play in the performance of a mathematical practice. An example of KPM would be to know that the comparison of geometric solids is a valid mathematical process to identify properties of each body.

On the other hand, in the domain of *Pedagogical Content Knowledge* is the characterization of the work of teaching mathematics. The following three subdomains are considered:

KMT (*Knowledge of Mathematics Teaching*) encompasses the teacher's knowledge of how to teach a mathematical content. It includes knowing different strategies, that allow the teacher to foster the development of procedural or conceptual mathematical skills, the theories of mathematics teaching, and knowing the useful characteristics of different tools for teaching mathematics (physical and digital). It would be an element of KMT to know the importance of working from different representation systems (concrete material and photos from different perspectives), a certain geometric composition (cube and cylinder), to enhance the process of spatial visualization in students.

KFLM (*Knowledge of Features of Learning Mathematics*) which refers to the knowledge of how a mathematical content is learned. It contains knowledge of the strengths and weaknesses in learning mathematics, as well as knowledge of how students learn a certain content. In addition to the learning theories associated with a mathematical content and the interests and expectations of students about a mathematical content. Thus, for example, elements of KFLM would be the knowledge that Early Childhood students require real and relevant situations that give meaning to the manipulation of objects or the fact that they are not able to understand mathematical messages without visual support.

KMLS (*Knowledge of Mathematics Learning Standards*) constituted by all those referents that indicate at what time each content should be learned and at what level of depth. It contains what a student is expected to know at that particular school moment, the level of conceptual or procedural development expected and the sequencing with previous and subsequent topics. An example of KMLS would be to know what properties of geometric solids can be understood at the Early Childhood Education stage.

Methodology

In this research we set out to understand the specialized knowledge mobilized by an Early Childhood Education teacher to teach the properties of geometric solids, identifying elements of specialized knowledge and relationships between them. This is carried out from practice as a privileged place to understand how this knowledge sustains the teacher's actions. We approach this objective from an interpretive approach (Bassey, 1999) and using an instrumental case study (Stake, 2005) that allows us to understand in depth a singular reality. Our informant is José (pseudonym), an active teacher with more than a decade of experience at this stage. José is concerned about improving his practice, which led him to receive specific training in Mathematics Education.

DOI: 10.20396/zet.v29i00.8661819

The gathering information was carried out through non-participant classroom observations (video recordings) and semi-structured interviews carried out at the end of the implementation of the unit as a support to complement the information provided by the video recording. Eight recordings were made during two consecutive school periods in an Early Childhood Education classroom of a public school in the province of Seville (Spain). In this study we present the analysis of the sixth recording dedicated to geometry, with students of 4 years old. We also consider an interview with José to clarify the knowledge that underpins his actions. With this group-class he had worked since the previous year, in addition to the name of the solids, some elements such as faces, edges and vertices. He approaches this work through different strategies: direct manipulation of the solids, stamping with paint and representation of some of their faces on the blackboard. In the course of the sessions, classroom agreements were established on the names of their elements, with the aim of gradually detaching them from this everyday language and orienting them towards mathematical vocabulary (e.g., "little paths" for edges and "little beaks" for vertices). The chosen session is initially a review of the properties of geometric solids, although it focuses specifically on their faces.

The data analysis is carried out on the basis of the transcription of the information, where we apply the interpretative approach (Kvale, 1996). For this analysis, units of information are selected according to whether we consider that they constitute a resounding proof of the teacher's knowledge framed in a category of an MTSK subdomain (evidence) or a signal that leads us to a certain category but that needs to be further probed to be evidence (indication) (Flores-Medrano, 2015). To analyze we use the characterization of subdomains and categories (Carrillo, et al., 2018) presented above, which help us to approach the data.

Description of the classroom session under study

In this paper we present the description of a class session in relation to geometric solids in which he works on their characteristics. He tries to deepen on the number of faces and the geometric figures that make them up. The teacher begins by reviewing some characteristics: *"We are going to take out pieces and you have to tell me what they are. Let's remember how many faces they have, how many edges they have. We are going to remember if they roll, which ones look alike, what their characteristics are..."*. As usual in his classes, the teacher uses a manipulative material (wooden pieces) selecting the solids in figure 2, without naming them.

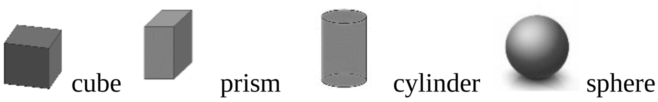

Figure 2 - Parts and specific names of the solids used during the session.
Source: Own elaboration

José asks in a playful way *"Which hand is it in? left or right?"* and when they visualize the piece, he asks them *"What is it?"*. He begins by pulling out a cube and, once the name is verbalized by the students, he goes on to write it on the board. Through questions,

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José focuses their attention on their faces: *"Let's put a sticker on each face so we can count more easily".* He does this each time they say a number and at the end, he comments: *"How many faces did we say it has? Let's put it here (pointing to the board). To know that it has a face, we are going to put a face on it"* and he draws a face on the board and places the number six underneath, thus beginning to make a table for recording it. Next, he indicates: *"In a cube, what are the faces? How many squares does it have? Does it have any rectangles?".* In the table, for each body, he indicates number of faces (he designates it with a pictogram) and the number of faces that are squares and rectangles (designated with squares and rectangles, respectively, in standard form) (Figure 3). In the course of the session there are students who focus on the edges, but José tells them *"yes, it does have edges, but we are not going to count them. It has many".* The teacher continues counting the sides of a square. Next, the same thing happens with the vertices, where José comments *"each edge ends in a pinch, as we call it, how many pinches does each square have?"*

Figure 3 - Representation on the blackboard made by the teacher.
Source: Classroom video recording

Using the same procedure, José focuses on the prism. Once the students visualize the piece, name it and the teacher writes the name on the board, they count the number of faces it has, also writing it on the table. After this, the teacher points out: *"Six, just like the cube. But now let's see how many squares it has and how many rectangles. The cube had six squares, but no rectangles. Let's see how many squares the prism has. Let's put a sticker on each square."* After counting the squares and recording them in the table, José proposes a challenge: *"Now I ask, let's see who knows this, because it is quite difficult [...] The prism has six faces, let's put the six faces, as if they were children (using pictograms he represents the six faces on the board, Figure 4) and how many faces are squares?".* A student answers two and the teacher crosses out two faces from the representation on the board, and asks them, *"How many faces will you have that are rectangles?".* Faced with the students' incorrect answer, José insists again: *"Two are squares, the rest, up to six, are rectangles. Let's see, kids, how many rectangles does a prism have?".* The students answer correctly and then count the rectangular faces on the wooden prism. Finally, a review is made based on the table, where, through questions, he compares the cube and the prism, making them see the similarities and differences between them based on what is shown in the table.

DOI: 10.20396/zet.v29i00.8661819

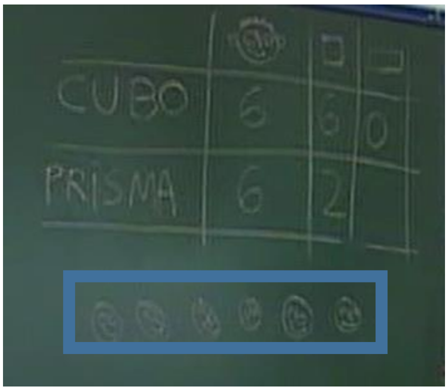

Figure 4 - Representation on the blackboard of the six faces (as if they were children's faces).
Source: Classroom video recording

He then focuses the students' attention on the sphere by asking them if it has faces, squares and rectangles, reflecting everything in the table. He then clarifies *"because if it has no faces it cannot have rectangles or squares, does it have edges?"* and after the students' refusal, he asks them: *"So, what is the only thing we can say about the sphere?",* and they answer that it has a wheel.

To end the session, he focuses his attention on the cylinder, although this is no longer represented on the blackboard, as the students are tired. After identifying the solid, he asks them to indicate some of its characteristics. The first thing they point out is that they roll, whereupon the teacher intervenes by asking them *"but which way?"*. Next, one of the students indicates that it has circles, and the teacher emphasizes: *"it has two circles"*. Immediately, José asks them about the number of faces of the cylinders and whether they are squares or rectangles. Finally, José asks about the geometric figures that make up the faces of the cylinders.

Analysis and Results

In this paper we present the analysis of the specialized knowledge mobilized by an Early Childhood Education teacher about geometric solids. José puts into practice what he has learned in the training courses, which leads him to work on geometry starting from geometric solids, instead of flat figures as usual. He had already worked with his students on this topic the previous year, trying to move from the identification of the solid to the observation of some properties: type and number of faces, edges, and vertices. In an interview he told us that her goal in teaching geometry is for children to discriminate the solids, to be able to provide properties of the solids and not just to identify their names. He started using the manipulation of the solids and, progressively, he incorporated other approaches such as stamping and representation on the board of the most representative face for him, so Jose believes that in this course, students should move towards the identification of properties (KMLS, expected learning outcomes), based on his knowledge that they must be helped to advance from their knowledge of the figures as a whole (Level 1, Van Hiele) to

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an analysis of them (Level 2, Van Hiele) (Jaime & Gutiérrez, 1990) (KFLM, theory of mathematical learning).

In his classes José works with the usual solids at this stage (prism, cylinder, and sphere) since he considers that they are the ones to learn (KMLS, expected learning outcomes). For him his notion of prime is linked to three types: cube, square-based prisms and rectangular prisms), therefore, he considers that only the square and rectangle can be part of the faces of these (KoT, definitions, properties). Of these three, in class he uses only the first two: one by its particular name (cube) and the other by the generic name of the class it represents (prism), which corresponds to the fact that he emphasizes the differences between the two figures more than the similarities. José values as something important at this stage, learning certain solids and their properties (KMLS, expected level of conceptual or procedural development).

José knows that for these students it is fundamental to manipulate the solids in order to understand their properties (KFLM, ways pupils interact with mathematical content) and for this he uses different strategies, oriented to build a rich image of each geometric solid and uses a specific wooden material (KMT, resources). He starts by direct manipulation, passing through stamping and projection, that is, using different types of approach (KMT, strategy) until he considers that the students can already work on them without the need for direct manipulation (KFLM, theory of mathematical learning). This occurs in the analyzed session (closing activity), where we observe how it is the teacher who manipulates the solids and shows them to the students, since he considers that the students have manipulated enough through the different strategies used and are already able to answer the questions with the mere visualization of the solid (KFLM, ways pupils interact with mathematical content). He proposes the activity to help students advance from Van Hiele's Level 1 to Level 2 (Jaime et al., 1990), since he knows that it is difficult for students to notice, distinguish and recognize the faces of a solid (KFLM, weaknesses in learning mathematics), commenting in an interview that, when he showed a solid to her students, they did not notice if it had squares or rectangles, but they noticed the color, the appearance, so that *"the faces are something that children are not able to distinguish".*

The teacher wants students to review properties such as edges and vertices and, above all, to enrich their knowledge about the faces of solids. To this end, José formulates a task in which, for each type of geometric solid, they count the total number of faces and the number of faces depending on the type of plane figure (square and rectangle) (KMT, task). José intends that students at this age are able to know how many faces each worked solid has and the shape each face has (KMLS, expected learning outcomes). In addition, in this procedure he intends that the students become accustomed to the vocabulary worked on (names of the solids), both through an oral and written representation register, since he considers it content to be worked on for this age group (KMLS, expected learning outcomes).

The type of task he implements reflects his knowledge about the use of different forms of representation as a vehicle for the development of the session, using it as a teaching

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strategy (KMT, strategy). José shows his knowledge of different registers of representation in the field of geometry (manipulative [wooden pieces], oral, written, symbolic [when he writes the number 6] and graphical [table]) (Lesh, Post, & Behr, 1987) and the complementarity between them (KoT, register of representation). In addition, he values the table as a useful register to collect information in an orderly manner and to promote later interpretation (KoT, register of representation).

In the table, he uses different types of designation (pictogram and symbol "6"), aware of the importance of designation in the genesis of mathematical contents (KoT, properties). These aspects are an indication that the teacher knows that the use of pictograms and everyday language brings them closer to the mathematical concept, helping them in their mathematical learning (KMT, strategy). When José is asked in the interviews about definitions of concrete solids, he usually provides them on the basis of a list of constituent elements, which are far from the characteristics of a formal definition of them. This way of defining corresponds to how he conceives that work on definition begins in infants, which is helping students to identify their properties (KPM, definition), knowing how children reason geometrically at this stage (KFLM, theory of mathematical learning).

Once the information is recorded in the table, the teacher promotes through questions the comparison of the properties of the solids (KMT, strategy). The comparison becomes a practice through which, by means of the differences and similarities between the solids, the students deepen their knowledge of them, identifying their properties (KPM, epistemological genesis form). The teacher knows that it is easier for them to start from an object they already know, drawing similarities and differences with the new object (KFLM, ways pupils interact with mathematical content). José knows the importance of showing the type of faces of the solids as he considers that it helps students to differentiate the cube from the regular quadrangular prism (KMT, strategy). In his classes, José starts using the child's own vocabulary to refer to some properties of the solids, for example, he calls "little paths" to the edge, "little beaks" to the vertex and comments "what wheel" when he refers to the fact that it has a curved face (KMT, resources). In the course of the session, we observe how José values the acquisition of specific geometric vocabulary and the use of correct and precise language (KMLS, expected learning outcomes).

As we have said, in the session José proposes the study of geometric solids by counting their faces. The teacher intends to work on the faces of different solids (cube, prism, sphere, and cylinder) using the same teaching procedure, starting by showing the piece, placing the name on the blackboard, counting the total number of faces, and counting the number of faces that are squares and rectangles (KMT, technique). The teacher knows the needs of his students and adapts to them, so he makes decisions in his classes based on his knowledge of his students (KMT, strategy). For example, for the cylinder work, he chooses not to represent on the blackboard because he finds the students already quite tired. For counting faces, he places a sticker each time they say a number, which shows a conversion between two registers of representation of the same concept (pictorial-symbolic of quantity) (KoT, registers of representation). In addition, it reflects his knowledge of enumeration as a

Zetetiké

DOI: 10.20396/zet.v29i00.8661819

procedure for counting (KoT, procedures). And, on the other hand, he knows that sticking stickers serves as a strategy to count the number of equal faces in a geometric solid, associating a sticker to each face to facilitate counting (KMT, resources).

In counting the faces of the prism, he uses stickers to count the totality of faces and the square faces, but for the counting of rectangular faces he leaves aside the manipulative material, encouraging them to abstraction. He proposes another way to count the number of faces, presenting verbally a part-whole additive problem with the unknown in one of the parts (KoT, procedures). The teacher knows the importance of problem solving in school mathematical activity (KMLS, expected learning outcomes) and that students have difficulties in handling numerical quantities without the support of manipulative material (KFLM, weaknesses in learning mathematics), so he ends up switching to a pictorial-graphic register, where he represents on the blackboard the six faces and crosses out two of them (KoT, registers of representation). With this approach we can also say that he knows the importance of using different contexts (KoT, phenomenology). Next, the teacher proposes the verification of the result by physically counting the rectangular faces on the piece of wood (KPM, verification), valuing it as a relevant phase in problem solving (KPM, problem solving).

Although the teacher intends to work in this session only on faces, the students are used to dealing also with edges and vertices when working on solids, so they try to focus on these. In the case of edges, the teacher tells them *"we are not going to count them. There are many of them"*, possibly because he considers that the numbers handled at this age do not exceed ten (KMLS, expected level of development). On the other hand, we can say that it is because he does not intend to dedicate this session to deepen on it, but they are going to work on it later, and it could be just a teaching decision (KMLS, sequencing of topics). He then asks them about the number of sides of a square, referring again to the faces, which makes us think that he does so, not because it has many but because he is not interested in focusing on these properties. This way of redirecting the class to what he is interested in shows us another teaching strategy (KMT, strategy). With the expression *"each edge ends in a little bit"* we observe how the way of focusing on the edge is dynamic because it refers to the magnitude length (KoT, properties) and this in turn leads to teaching (KMT, strategy). And finally, we can also say that we have indications of knowledge about the consideration of counting sides and vertices of the square as content for age (KMLS, expected learning outcomes).

Conclusions

Research on mathematics at the early childhood education stage is a growing topic (Charalambous & Pitta-Pantazi, 2016). McCray, et al. (2012) reveal that the knowledge acquired at that stage forms the basis for future mathematical learning, which highlights the relevance of good mathematical training at this level. The mathematics of the stage contains deep ideas (Perry et al., 2002) so the teacher needs to possess a solid and cohesive mathematical knowledge (Muñoz-Catalán et al., 2017).

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Although this professional is different from Primary, Secondary, Baccalaureate or University teachers, the distinction of the MTSK model in the three major domains and even in each of their respective subdomains can be applied to him. This work allows us to draw conclusions that support and specify the specialized knowledge of an Early Childhood Education teacher in the teaching of geometric solids. The use of different simultaneous methodological tools such as categories, MTSK subdomains and the constructs evidence and cue has helped us to obtain a better understanding of the specialized knowledge of this teacher.

As we pointed out in Muñoz-Catalán, et al. (2019) the teacher's knowledge to teach geometry is different from that required to teach numbers. The analysis reflects evidence and indications of almost all knowledge subdomains, except for KSM, since it is a difficult subdomain to find in a classroom observation (Escudero-Domínguez, et al., 2019). In the analysis, we have observed how their aim is to advance towards the identification of properties of geometric solids, highlighting their knowledge about the notions they consider that students should learn in this course (KMLS), as well as their knowledge about how to help students in the advancement of their mathematical learning, bringing them closer to the concepts (KFLM) and making various teaching decisions (KMT). On the other hand, we have observed the important role played by the different representation registers in the session (KoT) and how they are decisive in her teaching (KMT) to favor the understanding of a concept. In this session he uses the table as a register that allows organizing the information and then being able to interpret it. Language is another important representation system in the teaching of mathematics in kindergarten (KoT), since it is a language-generating stage. José starts using everyday language terms that convey the idea associated with the concept (KMT), but once they have already grasped the meaning of the notion, he progressively introduces mathematical language, so he values the acquisition of specific geometric vocabulary and the use of correct and precise language (KMLS).

As a result of the analysis of this case we still have the need to value the KPM subdomain. As stated in Muñoz-Catalán et al. (in press, b) to know how to reason and proceed to reach mathematical results and how to validate it is necessary for a good professional performance. However, it requires us to take a new look at mathematical practices in the early childhood classroom that allow us to conceive them as *forms of epistemological genesis* (Muñoz-Catalán et al., in press, a). Thus, we could highlight comparison as a way of generating geometric knowledge at this age and which is used as a teaching strategy (KMT). We consider that a good knowledge in this subdomain would provide more depth in the design of tasks and the management of the mathematical activity that occurs in the classroom.

This work reaffirms the need for a solid specialized knowledge in the mathematical domain, as well as the relationship of these with elements of didactic knowledge of the content, i.e., we know that what they do in their classes is supported by their mathematical knowledge (Escudero-Domínguez, et al., in press). Among the diversity of relationships between different subdomains of knowledge existing in this analysis, we highlight the KPM -

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KMT relationship. For example, knowledge of comparison as a mathematical practice that is used as a teaching strategy; knowledge about the use of correct and precise language in geometric learning that is used as a teaching tool. These relationships support the complex and integral nature of specialized knowledge. The type of task proposed, the manipulable material, the interaction in the classroom, the counting of elements of the solids and the establishment of relationships between them are promoters of spatial reasoning processes (Gamboa, et al., 2010). We have also felt difficulty in distinguishing whether the elements of specialized knowledge identified were evidence or clues, especially in the mathematical domain (Muñoz-Catalán, Joglar-Prieto, Ramírez-García & Liñán-García, 2019).

Research addressing the knowledge of the Early Childhood Education teacher is scarce and concludes that the characteristics specific to the stage should be taken into account (Mosvold, et al., 2011). In addition, this work, along with others such as Muñoz-Catalán et al. (2019), emphasizes the importance of the general pedagogical knowledge of the teacher at this stage, for the cognitive and physical development of the students. For example, José uses pictograms to bring the concept closer to them; he gradually uses more mathematical information; he adapts to the needs of his students (fatigue, mathematical comprehension, ...) in each of his classes. As a prospective, we think that it would be interesting to delve into the conjunction of general pedagogical knowledge with specialized knowledge to gather more information that will help us to better understand the knowledge that this professional needs to exercise his profession.

The contributions of this work also focus on strengthening the mathematical preparation of these professionals, with the aim of building the consistency of the knowledge they need to manage student learning. The great weight that the mathematical knowledge part acquires contrasts with the perspective held of the Early Childhood Education teacher as throughout their professional career they are not usually required to have a deep and specific training in mathematics (Muñoz-Catalán, et al., 2019). Some associations (e.g. NAEYC & NCTM, 2013) have pointed out the need to provide solid mathematical training at this stage, whereby teachers must understand what kind of mathematical problems they have to solve and what kind of mathematical resources to handle to solve those problems (Bass, 2005).

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