Zetetiké

DOI: 10.20396/zet.v29i00.8661842



# Knowledge of the Mathematics teacher educator who being a researcher of teaching

## Conhecimento do formador de professores de Matemática que é investigador da docência

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**Abstract**

In this work, the objective is to understand the knowledge of the Mathematics teacher educator who is a researcher of teaching and takes teaching and teacher education as an object of study. The research is configured as a qualitative case study, in which, through the analysis of a narrative of professional development experiences, we tried to identify the knowledge of a higher education teacher to work in the training of Mathematics teachers. The results show that the content knowledge of training includes subdomains of Mathematics Teachers' Specialized Knowledge, and that being a trainer requires a specialized type of knowledge, focused on the pedagogical aspect of the content of Mathematics teacher training.

**Keywords:** Mathematics teacher educators; Teacher educator's knowledge; MTSK; Narrative.

**Resumo**

Neste trabalho, o objetivo é compreender o conhecimento do formador de professores de Matemática que é investigador da docência e toma a docência e a formação de professores como objeto de estudo. A pesquisa se configura como um estudo de caso de caráter qualitativo, em que, pela análise de uma narrativa de experiências de desenvolvimento profissional, buscou-se identificar o conhecimento de uma docente do ensino superior para atuar na formação de professores de Matemática. Os resultados denotam que o conhecimento do conteúdo da formação inclui subdomínios do conhecimento especializado para o ensino do professor de Matemática e que ser formador exige um tipo especializado de conhecimento, voltado para o aspecto pedagógico do conteúdo da formação do professor de Matemática.

**Palavras-chave:** Formador de professores de Matemática; Conhecimento do formador; MTSK; Narrativa.

## Introduction

Research on the teacher educator is often cited for being few in number (European Commission, 2013), especially when compared to research on the teacher (Smith & Flores, 2019). Only since the new millennium has there been a growing interest in constituting a body of knowledge about the teacher educator (Kelchtermans, Smith & Vanderlinde, 2018),

**Sent on**: 30/10/2020 – **Accepted on**: 06/04/2021 – **Published on**: 27/05/2021

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DOI: 10.20396/zet.v29i00.8661842

which has reverted into studies that take as their research focus the teacher who works in teacher education.

We observe a similar scenario with regard to the Mathematics Teacher Educator (MTE). As identified in Coura and Passos (2017), the 11 theses and 19 dissertations analyzed represent 3.5% of the production considered in national mapping of research on the teacher who teaches Mathematics, produced from 2001 to 2012 (Fiorentini, Passos & Lima, 2016). As for the period of production, the studies also fall within the mentioned movement, since the search for reports of investigations carried out in Brazilian stricto sensu graduate programs in the areas of Education and Teaching until the year 2015, made in Coura (2018), added only one more study (Gonçalves, 2000) to the 30 previously identified. Thus, besides being reduced in number, the Brazilian studies on the Mathematics teacher educator were carried out from the beginning of the new millennium, which also occurs at the international level (Contreras, Montes, Muñoz-Catalán & Joglar, 2017).

It is in this movement that the study presented here is inserted, in which we seek to present an understanding of the specialized knowledge of this specific trainer - in this text, the Higher Education teacher who works in the training of Mathematics Teachers (MT. Among these, we turn our gaze to the teacher educator[3] we qualified in Coura (2018) as researchers of teaching, for taking as objects of their investigations, besides their professional field, also teaching and teacher training.

Although they are a minority contingent in relation to the others, we understand that these investigations can provide elements to understand the expertise of this teacher educator in a perspective that advances in relation to the studies already produced, since they focus their professional performance on second-order teaching (Murray & Male, 2005), that is, on teaching about teaching, as we highlighted in Coura and Passos (2018). With this study, we can offer contributions to the open question indicated by Rediske de Almeida and Ribeiro (2019, p. 146) about "whether and how the PCK [pedagogical content knowledge] of a trainer who is a mathematician differs from the PCK of a trainer with another profile," as well as to the question Contreras et al. (2017) ask about what typologies can characterize the expertise of a Mathematics teacher educator.

The analysis of the narrative of professional development experiences of one of the participants in the study presented in Coura (2018), enabled us to understand the expert knowledge of this teacher educator. To this end, we will discuss her knowledge according to our reading of the model proposed by Carrilo, Montes, Codes, Contreras, and Climent (2019). Next, we will expose the methodological aspects according to which we conducted the study. We will present the results in the analyzed narrative, when we identify the types of expert knowledge of the trainer under study here. At the end, we will briefly discuss the specificity of the identified knowledge, including from the perspective of differentiating it from the

[3] Although the participating teacher educators are women, as the gender issue did not serve us as a parameter, in some parts of the text we kept the word trainers to designate a group of female and male professionals.

DOI: 10.20396/zet.v29i00.8661842

specific knowledge of the Mathematics teacher.

## The Mathematics teacher educator's knowledge

As agents in teacher education, trainers can be taken as key figures in the professional performance of teachers and, thus, they would occupy a strategic place in the current educational policy scenario, as they are involved in the relationship that is currently established between the quality of school education and teacher training (Vanassche & Kelchtermans, 2014). Thus, acting in teacher education requires the trainer to promote a set of actions that require from him a complex understanding of teaching and learning to ground his teacher preparation programs (Korthagen, Loughran & Lunenberg, 2005).

Indeed, it is not enough for the trainer to know the content, he or she must also help teachers learn how to teach (Cochran-Smith, 2005). This means that undergraduate mathematics students need support to transform their content-specific knowledge into knowledge that can be grasped by the students they have or will have in school. For this reason, it is complex to define a knowledge base for trainers - including those in mathematics teacher education - because it involves a dual character: the knowledge needed to teach the students of their students, future teachers, and the knowledge concerning teacher education (Dal-Forno & Reali, 2009).

However, if research on teacher training has already broken - as in Shulman (1986), for example - the barrier of deeming it sufficient, for students to learn, that one knows the content to teach, the same has not happened in relation to trainers. As Vaillant (2003, p. 28) has pointed out, there still remains the "belief that the only knowledge that is required to teach is knowledge of what is being taught - the content or subject matter to be taught." There is some change as of the beginning of this millennium (Kelchtermans et al., 2018), with increasing advocacy that a body of knowledge is required for the professional activity of the trainer and, therefore, this knowledge is also specific to these professionals (Koster, Brekelmans, Korthagen & Wubbels, 2005).

Beswick and Goos (2018), referring to mathematics teacher education, argue that, just as it was important to understand and articulate the specific knowledge of teachers of this curricular component, it is currently necessary to focus on what the trainers of these teachers need to know and how this knowledge is distinct from that of teachers and other trainers, including mathematicians.

In this sense, some studies (Contreras et al., 2017; Rediske de Almeida & Ribeiro, 2019) adopt perspectives that emerge from Mathematics Teachers' Specialized Knowledge (MTSK[4]) (Carrillo et al., 2018), to analyze Mathematics teacher educator's knowledge. So

[4] The MTSK is a model dedicated to teachers' knowledge of mathematics as an object of teaching and learning. According to Carrillo et al. (2018), it is divided into two domains: Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK), each of which is organized into three subdomains. The MK is composed of: Knowledge of Topics (KoT), Knowledge of the Structure of Mathematics (KSM), and Knowledge of Practices in Mathematics (KPM). PCK is divided into Knowledge of Mathematics Teaching (KMT),

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did Carrilo et al. (2019), in their reflection on the knowledge of this professional, which we take as a theoretical reference for the analysis exposed here.

*Knowledge of the content of the training*

Indeed, Carrillo et al. (2019) argue that the content of mathematics teacher education is not limited to mathematical knowledge. Based on Ponte (2012), they understand that such content is composed of three major domains in which a teacher must develop: knowledge, identity, and professional skills. Thus, the content knowledge of the Mathematics teacher educator, which is linked to the training of these teachers certainly includes Mathematical Knowledge, but is not limited to it, since it is composed of both mathematical knowledge and pedagogical knowledge of the mathematical content that he intends his students - the future mathematics teachers - constitute. Backed by Zopf (2010), Carrillo et al. (2019) assert that the trainer's mathematical knowledge should be broader and deeper than the teacher's, contemplating

> the connections between different aspects of the content, linked to knowledge of the structure of mathematics, as well as in their knowledge of mathematical syntax. Thus, a trainer should be aware of how the different properties of concepts interrelate, articulating with the processes of constructing new mathematical knowledge giving him a holistic view of mathematical content (Carrillo et al., 2019, p. 331, **our translation).**

Although the mathematical knowledge of the trainer has similarities with the Mathematical Knowledge (MK) of the mathematics teacher, it is characterized, in its three subdomains, by a greater density of connections and the depth of this knowledge. Thus, it transcends what a teacher-in-training is expected to learn (Carrillo et al., 2019).

When it comes to pedagogical knowledge of mathematics content (PCK), these researchers indicate that a trainer should be familiar with the elements of Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM), and Knowledge of Mathematics Learning Standards (KMLS), namely the three domains that make up the mathematics teacher's PCK, according to Carrillo et al. (2018). Put differently, they argue that such a teacher's PCK should be part of the content knowledge of mathematics teacher education, that is, the trainer needs to understand the knowledge pertaining to teaching the mathematical content that he or she aims to help future teachers constitute.

The researchers also argue that the Mathematics teacher educator should know about the different professional skills that a teacher - of any curricular component - should develop in his/her training, such as: student learning in school, lesson planning, or the design of assessment tools.

In short, for these authors, the content knowledge of this trainer refers to the content of teacher training, and we consider that it includes the Mathematics Teachers' Specialized

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Knowledge of Features of Learning Mathematics (KFLM) and Knowledge of Mathematics Learning Standards (KMLS).

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Knowledge (MTSK) (Carrillo et al., 2018), but it extrapolates it, in terms of both the connections that the teacher educator must establish between the different aspects of the mathematical content and the depth of this knowledge, and also the skills linked to the professional practice of the teacher.

*Pedagogical knowledge of the content of the training*

By understanding that the content of teacher training is composed of knowledge, identity, and professional skills, Carrilo et al. (2019, p. 330, **our translation**) assure that the pedagogical knowledge of the training content is that which "will allow the trainer to transform the previously described training content into a form that is more accessible to his students."

However, it would not be a replica of the teacher's PCK, although the authors take it as a reference to organize the teacher educator's PCK, based on the trainer's knowledge about teaching, learning and teacher education curriculum, into: Knowledge of teaching the content of mathematics teacher education, Knowledge of the features of the professional development of mathematics teachers and Knowledge of the standards of mathematics teacher education.

Knowledge of teaching the content of mathematics teacher education is what the Mathematics teacher educator has about how to organize the training and encompasses theories and approaches to training, including dynamics and tasks to design training scenarios for future mathematics teachers. The trainer's knowledge of the features of the professional development of mathematics teachers includes knowing how professional knowledge and identity are constructed and also the different degrees of evolution of professional skills. Finally, knowledge of the standards of mathematics teacher education is most directly related to the educational level at which the teacher trainees will work, although it must also include a more global view that encompasses the other levels. This knowledge also includes the ability to set, justify and evaluate the learning goals of future teachers and, in this sense, also involves the teacher education curriculum.

For the authors, the trainer's PCK can have multiple sources, which contemplate their own experience, their readings on teacher professional development, and the discussions they undertake with other teacher trainers. We conjecture that another source of the PCK of the Mathematics teacher educator may be the research that he, the trainer, carries out along his professional trajectory, taking teaching and teacher education as an object of study, as we will try to show in the discussion of the results.

## Methodological aspects

In the presented research, we seek to understand the knowledge of Mathematics teacher educators who are researchers of teaching, in order to contribute with an answer to the question of the specificity of their knowledge. To do so, we conducted a qualitative case

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study research (Stake, 2009), with the perspective of understanding human experience through interpretation in an instrumental perspective, that is, from a particular situation - the case of a researcher of teaching -, to better understand a phenomenon of broader spectrum: the specialized knowledge of the Mathematics teacher educator (Alves-Mazzotti, 2006).

The analyzed material is a narrative of professional development experiences, produced from a dialogical in-depth interview, with a biographical-narrative character (Domingo Segovia, 2014), which records part of the life and training history of Adair Mendes Nacarato - a Mathematics teacher educator who we understand to be a researcher of teaching, with more than 30 years of performance and research in this specific training. Produced as recommended by Clandinin and Connelly (2011), the narrative presented is a temporal sequence of experiences that we identified in the trainer's speech and was composed preserving her voice pronounced in the interview and in her own academic production[5]. The text transcribed in the next section refers to the period that begins with the entry into the teaching profession until her performance as a research supervisor with the teacher who teaches mathematics. In view of the use of the bibliographical production of the teacher educator, we referred to her in two ways: sometimes by her first name, when reporting what she revealed in the interview, recorded in italics; and sometimes according to the technical norms for document citation, when referring to her speech, present in her work.

This narrative expresses the way Adair developed other ways of being in teacher education, which allowed her to produce knowledge that grounded and guided her actions and practices as a teacher educator. To identify the types of knowledge revealed, we analyzed the narrative in a way also called "paradigmatic analysis of narratives" (Bolívar, 2002): the text was examined with support from Carrillo et al.'s (2019) reflections on the knowledge of the Mathematics teacher educator, in order to associate the content of the narrative to pre-established categories.

Throughout the narrative presented below, we record in brackets the type of knowledge revealed in a paragraph or part of it which, in this case, is highlighted in bold. We use the English acronyms of the six subdomains of the MTSK that, according to the previous section, make up the knowledge of the Mathematics teacher educator with regard to the content knowledge of training. As for pedagogical training content knowledge, we indicate the three subdomains in full, since in the referential used (Carrillo et al., 2019) there are no acronyms to designate them. Subsequently, in the part where we record our considerations about Adair's expert knowledge as a Mathematics teacher educator, we organize the content of this knowledge according to the subdomain to which it refers.

## Narrative of professional development experiences of Adair Mendes Nacarato

[5] The use of the trainer's academic production and the assumptions that guide narrative research made it necessary to identify the participant, who approved the narrative presented in Coura (2018) and agreed to be identified in the research, which was authorized by the UFSCar research ethics committee.

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Adair graduated in Mathematics from the Pontifícia Universidade Católica de Campinas (PUC-Camp) in 1975, and began working as a Math teacher in the São Paulo State System in 1977, where she remained until the 1990s, including as a teacher at the Specific Center for Training and Improvement of the Teaching Profession (CEFAM). She was also a Math teacher and coordinator in private schools in Campinas. From 1991 to 1994, she earned a Master's degree in Education from the State University of Campinas (Unicamp) and presented a dissertation on the construction of the number concept in school education. Between 1996 and 2000, she received her doctorate, also at the Graduate Program in Education at Unicamp, where she presented her thesis entitled "Continuing Education from an action-research perspective: curriculum in action of a group of female teachers learning to teach geometry". Since 1999, she has been a professor at Universidade São Francisco (USF), in Itatiba, São Paulo, where she joined the Graduate Program in Education in 2001. She was part of the coordination of GT7 (Working Group): "Training for teachers who teach mathematics", of the Brazilian Society of Mathematics Education (Sbem), for two terms, from 2001 to 2006.

*From Math Teacher to Researcher of Teaching: "I become a teacher educator as a teacher"*

In 1977, Adair started working as a math teacher in a school of the São Paulo state school system, located in the outskirts of Campinas. During this first year of teaching, she worked only in this school, which was "a learning experience". For her, as for any teacher, the first year in the profession was remarkable (Nacarato, 1995, 2000a):

> Besides the initial difficulty of relating to a classroom, what struck me most was the students' difficulty with mathematics. They could not successfully solve the four operations with Natural Numbers. And did I know how to teach? No. During my undergraduate studies, I studied a series of theoretical subjects, but none of them discussed the curricular and pedagogical issues of elementary school. Some contents I even knew, at least theoretically, but Natural Numbers operations? This was content for 1st to 4th grade, I thought at the time. (Nacarato, 2000a, p. 93) **[Knowledge of the features of the professional development of mathematics teachers]**

In search of solutions to difficulties such as the one mentioned, Adair started using another textbook and other materials, such as the kits of the High School Expansion and Improvement Program (PREMEN[6]) distributed by the Institute of Mathematics, Statistics and Scientific Computing (IMEC) of Unicamp. She also used the geometry kits in the private school where she started working in 1978, Colégio Progresso.

For her, working at this school was fundamental to rethink teaching, to the extent that she became part of a group that, on a weekly basis, discussed issues related to teaching, which pushed her to **seek new ways of working with mathematics [KMT]** and, as a consequence, to assume the production of her own didactic material as a pedagogical stance,

[6] The High School Expansion and Improvement Program was instituted by Decree No. 63,914, of December 26, 1968.

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even if at first this process of creation was restricted to **adaptations of existing materials** (Nacarato, 1995). **[KMT]**

As a mathematics teacher, Adair worked in other spaces besides the classroom. Starting in the 1980s, she became involved in discussions about the curriculum proposal of the State of São Paulo for Elementary School, which led her to work as a monitor in the discussion groups of the curriculum with the teachers of this network. For her, this was "a phase of acquisition of much theoretical and methodological knowledge for teaching Mathematics from 5th to 8th grade" (Nacarato, 2000a, p. 94).

Adair worked at both schools until 1984, when she resigned from the private institution. She worked only at the public school from 1985 to 1987. In mid-1987, she was invited to be the coordinator of the mathematics area of the Elementary School at Colégio Rio Branco, another private school in Campinas.

At this point in the narrative of her story, Adair interrupted the chronological order of her speech with a question: "*What mathematics did I know to be coordinator of the whole elementary school*?" With this, she made reference to the doubt raised by this invitation - "My initial attitude was one of refusal because I had no experience teaching 1st to 4th grade" (Nacarato, 2000a, p. 96-97) - which, despite this, she accepted.

> *I said, "I'll take it on one condition. I don't know anything about the early years. If you accept that I stay this second semester accompanying the work of the teachers, listening, getting to know them and I will study. Next year, I will effectively take over as coordinator. The Elementary II school was quiet. And the school agreed. And so, I did my best. I studied all the AMs [Mathematical Activities], I studied everything that the CENP produced. ...] The CENP was the Coordination of Studies and Pedagogical Norms, which was a fantastic organization within the Secretariat of Education of the state of São Paulo [...]* ***I read everything that the CENP produced. It was the height of constructivism. So, mathematics, who did you have? Constance Kamii. I read everything by Constance Kamii and other Piagetians of the time****. The textbook of Manhúcia's group, the Solution Group*[7]*, they started to create other material aimed at the early years. And it was a fantastic material. A lot of what we do today was already there in Manhúcia's textbook from the 1980s.* **[KFLM]**

In 1988, she started to work as Mathematics area coordinator and, under the influence of these materials, "conceived under constructivist assumptions, which reflected the trends of the time" (Nacarato, 2000a, p. 97), she formed her conceptions about teaching and learning Mathematics. **Probably the fact that I examined and studied these materials so hard was a determining factor in the configuration of my conception of mathematics teaching and learning, which, in later years, would guide my professional activity" [KFLM]** (Nacarato, 2000a, p. 98) - and guided her work with the teachers of Colégio Rio Branco.

Complementing theory with teachers' practice was what enabled her to understand "**the importance of a work with understanding in Mathematics in the early grades and**

[7] Textbook Doing and Understanding Mathematics, by Manhúcia P. Liberman and Regina Wey, Editora Solução (Nacarato, 2000a, p. 97)

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**the importance of the Decimal Numbering System, built with meaning, as the foundation for all Arithmetic" [KMT]** (Nacarato, 1995, p. 4). This represented an answer to doubts and restlessness that he had been carrying with him since the first year of teaching: "**I believe that with the consolidated counting and the concept of number built with meaning, it will become easier to build, with meaning, the decimal numbering system**" (Nacarato, 2000b, p. 105). **[KMT]**

Regarding this period, she stated, "*I then start working as an area coordinator for mathematics. And that's where I went to learn mathematics of the early years*." As recorded in Nacarato (2000a), it was a period of searching for and producing knowledge about teaching-learning situations in the initial grades of elementary school, which made her reflect and change her practice as a teacher in the final years. According to her: "without a doubt, this was a period that contributed to my professional development" (Nacarato, 2000a, p. 98).

This path made her understand that "the mastery of specific content does not guarantee efficiency in the classroom" (Nacarato, 1995, p. 4) and that the

> My undergraduate degree in Mathematics provided, without a doubt, important theoretical elements in the area. However, all my knowledge as an elementary school mathematics teacher was built based on my experience in the classroom, my anguish when faced with students' difficulties in understanding certain topics of the subject and, mainly, by my constant search for solutions to the challenges that were presented to me. (Nacarato, 2000a, p. 92) **[Knowledge of the features of the professional development of mathematics teachers]**

Adair first placed herself as a teacher educator when she referred to this period when she acted as coordinator of the Mathematics area, from mid-1987 to late 1990, in Colégio Rio Branco - "*And, until then, I was just a teacher. I didn't have any specialization. So, I became a teacher educator as a teacher".* About her "Career as a teacher trainer " (Nacarato, 2000a, p. 96), she pointed out that:

> Today, looking back on this period, which lasted from 1987 to the end of 1990, I can say that my work with these teachers was not concerned with preparing their practice based on ready-made theories. We always started from the practical needs of the group. There was no imposition of any guiding theory, nor of any didactic material to be used. Everything was done within a negotiation, having as main reference the knowledge of each one's experience. My role, as a coordinator and as an external element, was to discuss with the group exclusively issues of pedagogical practice, without much concern for "theories" - even because I thought I didn't have them yet - but with emphasis on the daily routine of the classroom. (Nacarato, 2000a, p. 98) **[Knowledge of teaching the content of mathematics teacher education]**

In 1991, she started her Master's degree and returned to work as a Math teacher at Colégio Progresso, the first private educational institution where she had worked. In 1993, she became the coordinator of the Mathematics area from 1st to 4th grade at this school and started to work with teacher training groups in municipal education networks, in the countryside of Minas Gerais and in the state of São Paulo.

If she had no difficulties to take on a coordinating position again - "I already had a knowledge built up from the first experience and also had a theoretical foundation that was

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being acquired with the Master's degree" (Nacarato, 2000a, p. 104) - according to Adair, the work with public school teachers, "was very different from everything I had done so far" (p. 111). Although this led her to "more studies and reflections" (p. 111), the focus remained on what was happening in the classroom:

> If, on the one hand, I recognized the existence of a group of teachers, many of them from rural areas and some lay teachers who did not have the minimum mathematical knowledge to teach and, therefore, needed content knowledge, I also recognized the need to value the pedagogical practice of these teachers, which was brought to me by my readings. But how to reconcile all this? I tried in the few meetings we had - an average of four meetings a year, eight hours each - to work/discuss articulated theoretical aspects and classroom situations. (Nacarato, 2000a, p. 112) **[Knowledge of teaching the content of mathematics teacher education]**

In 1994, she finished her Master's degree and, in the following year, she started to participate in PraPeM[8] "*to be able to maintain a link with the university*". During 1995, she came into contact with issues related to the reflective teacher and the challenges to the paradigm of technical rationality, based on her readings of Schön's texts.

> These theoretical discussions led me to reflect on my own practice. And this left me in conflict: how to free myself from the paradigm of technical rationality? It seems to be very strong in my training and in my trajectory as a teacher educator. How to take the teachers' practice as a starting and ending point? And what about the issue of theory? How to make a teacher reflective about his/her own practice? Reflect from what? A reflection without theoretical foundations? (Nacarato, 2000a, p. 118) **[Knowledge of the features of the professional development of mathematics teachers]**

Adair took these questions with him to his doctorate, which he started in 1996, when he also studied other theoretical constructs:

> *I kept improving myself, getting more notions of what it would be like to work with formation and then I started to do doctoral research in the field of formation. So, it was the peak. What was coming to Brazil? Schön's, Zeichner's, Elliott's reflective teaching. So, it was this effervescence that I experienced during my doctorate and that I ended up using a lot in my thesis*. **[Knowledge of the features of the professional development of mathematics teachers]**

During the doctorate, the theoretical discussions in the research group and the confrontation with practice gave rise to new conflicts - "How to discuss pedagogical knowledge without the specific knowledge of the subject? How to discuss the valorization, by the teacher, of some mathematical themes to the detriment of others?" (Nacarato, 2000a, p. 120) -, such as new perceptions - "I realized that, in a continuing education process, the conception of the teacher-trainer inevitably ends up prevailing (Nacarato, 2000a, p. 120) - and reflections - "I began to reflect on the need to consider, in a continuing education work, the working conditions of the teacher (Nacarato, 2000a, p. 121). **[Knowledge of the features of the professional development of mathematics teachers]**

[8] PraPem: Mathematics Pedagogical Practice Group.

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Although Adair had been working as a trainer since 1987, when she was coordinator of the Mathematics area in Elementary Education for the first time, this trajectory contemplated primarily the training of in-service teachers. It was in 2000, after she finished her doctorate and joined the Universidade São Francisco (USF), that she started working in the initial formation of teachers in Higher Education, as a teacher responsible for courses such as Foundations and Methodology of Mathematics Teaching in the Pedagogy course; and Pedagogical Practice, Supervised Internship, Didactics, Mathematics Teaching Laboratory, and Trends in Mathematics Teaching in the Mathematics Undergraduate course.

The publication of the guidelines for basic education teacher training courses (BRASIL, 2001a, 2001b) provoked a restructuring in the curriculum of the Mathematics undergraduate course at the institution where Adair worked, as it highlighted "the need to rethink the place and the role of the internship in the undergraduate course, seeking a rupture of the dichotomous poles: theory and practice and emphasizing the role of research in initial training" (Nacarato, 2006a, p. 1) **[Knowledge of the standards of mathematics teacher education].** In view of this, she focused her teaching activities on the curricular units with a workload dedicated to pedagogical practice and on the Supervised Internship of the Mathematics course.

> Involved as I am with studies and research about and with teachers who teach Mathematics, when I took over, in the second semester of 2005, the subject of Supervised Internship with a 4th semester class of USF's Mathematics Undergraduate course, whose internship regulation foresees the realization of an intervention project, I considered that this would be a fertile field for future teachers to be inserted in the research field. (Nacarato, 2006a, p. 4) **[Knowledge of the features of the professional development of mathematics teachers]**

Regarding this period, during which he oriented the supervised internship in the Mathematics Undergraduate course, he said that

> *was when I got to know what an internship was. And we were already doing something very different at that time, working with intervention projects. So, the students would go for observation, they would diagnose the problem together with the teacher, and the teacher would give them a hint. Sometimes, it was even a tutoring class that the teacher asked for. But we would set it up, and always in the form of a project, always thinking about the idea of research as the core of teacher education.* **[Knowledge of teaching the content of mathematics teacher education]**

For Adair, the realization of intervention projects in the internship is a formative strategy and a fertile field for future teachers to be inserted in the field of research, because it allows undergraduates to understand the practice and problematize the situations they observe and involves a movement of defining themes, choosing theoretical references, analyzing and selecting situations for regency (Nacarato, 2006a). **[Knowledge of teaching the content of mathematics teacher education]**

The constant concern, as a teacher educator, with the production of a repertoire of teaching knowledge (Nacarato, Grando, Toricelli & Tomazetto, 2008) also led her to

DOI: 10.20396/zet.v29i00.8661842

implement a proposal, together with professor Regina Célia Grando[9] , that took care of rescuing mathematical contents from Basic Education, because her experience in Mathematics Undergraduate had been indicating that, every year, students enter Higher Education with more gaps in basic contents from the previous schooling stage (Nacarato, 2006b).

> ***What do we know? That the student comes to the university to do a degree course and knows nothing about Geometry. So... And how is he going to be a Geometry teacher? We created the so-called Geometry Workshop, which was in their pre-class, from five to seven. Seven fifteen, the class would start [at graduation].*** **[Knowledge of the features of the professional development of mathematics teachers]** *And then, we had the great idea of calling teachers to participate. So, this workshop became GRUCOMAT, which has existed since 2003, when we created the famous geometry workshops. So, we spent years with geometry, then we changed. Each period, GRUCOMAT is then creating research projects.*

According to Nacarato, Grando and Eloy (2009), the constitution of this group was based on theoretical assumptions that guided the formators' conception of teacher education: **taking the contributions of collaborative dimension groups and the adoption of formative strategies as elements that enhance teacher learning [Knowledge of the features of the professional development of mathematics teachers].** In the group, composed by the two formators, students of the Mathematics Degree of USF and mathematics teachers, the formative **process adopted involved and still involves elaboration, application, analysis and systematization of classroom activities in basic school. The authors considered that, with this dynamic, the school teachers expanded their teaching knowledge about Geometry, the undergraduates started to build knowledge about teaching, and the teachers built a teaching training methodology. [Knowledge of teaching the content of mathematics teacher education]**

As a teacher trainer, Adair has taken her experience as an object of reflection and research, seeking to systematize some training practices that she has prioritized (Nacarato, 2010, p. 906). With this, she constituted principles that have guided her performance:

> We have argued that the teacher's professional knowledge is constituted in and from pedagogical practice. It is from the problematization of the practice that the teacher starts to reflect and produce meanings for the events he or she experiences. The specific knowledge of the content - often acquired in undergraduate courses - undergoes (re)significations when worked in the classroom, as it becomes intertwined with pedagogical and curricular issues. Thus, they constitute a unit in which it is no longer possible to separate the specific content, the pedagogical content, and the curricular content.
>
> We also argue that the contexts that privilege the problematization, analysis, and reflection of pedagogical practice are enhancers of teachers' professional development. (Nacarato et al., 2008, p. 200) **[Knowledge of the features of the professional development of mathematics teachers]**

[9] Prof. Dr. Regina Célia Grando (http://lattes.cnpq.br/6878232320203358)

DOI: 10.20396/zet.v29i00.8661842

For Adair, the production of knowledge by teachers can develop when they assume an investigative posture, resulting in knowledge of practice (Cochran-Smith & Lytle, 1999). From this perspective, she understands that in the university-school partnership,

> university teachers learn and investigate with basic school teachers, recognizing them as protagonists of their own practice and curriculum development; and these, in turn, can become critical consumers of the theories produced by academic research and also become researchers in everyday school life - they can, at the same time, produce knowledge and make research a tool for their teaching practice. (Nacarato; Grando & Mascia, 2013, p. 27) **[Knowledge of the features of the professional development of mathematics teachers]**

Weaving her research with that of real teachers and in their real working conditions, giving visibility and legitimacy to the investigations they carry out in their classrooms (Nacarato; Grando, 2015), has represented for Adair another way of keeping in touch with school, something she understands as fundamental to being a trainer.

> *To be a trainer, it is not enough to be a good teacher. Of course, being a good teacher helps. But I think that the trainer needs something more. He needs to know the reality of the school. So, those of us who have already left, who are at the university, do you know the school's daily routine? I don't. But if I listen to the teacher, I do. Indirectly, I know. If I produce a video of the school, as our group usually does, I get to see what a school is like, what a classroom is like. So, for me, today, everything is very distant. So, for me, the school is the school that the teachers narrate. So, this is the school that I know. I'm not there in the classroom anymore.* **[Knowledge of the standards of mathematics teacher education]**

In her trajectory as a teacher educator, she has dedicated herself to highlighting the importance of a teaching practice based on reflection, on the systematization of the productions of the students of the Pedagogy course (Nacarato, 2010), of the students of the Mathematics Degree, of the teachers of Basic Education and in the movement of listening to what these teachers in formation have to say: about themselves, their training trajectory and their learning. As she registered: "the possibility of looking at these productions and being able to analyze them and reflect on my own practice, systematizing some of these reflections (as in this article), is fundamental to my professional constitution" (Nacarato, 2010, p. 927).

## Considerations about Adair's specialized knowledge as MTE

Adair is one of a group of Mathematics teacher educators who have moved from first order teaching, which was about mathematics in Basic Education, to second order teaching, which is about teaching about teaching (Murray & Male, 2005). She began to work as a trainer of mathematics teachers, without formal preparation for this role, while teaching mathematics in Basic Education. As a beginner teacher trainer, she worked primarily in her specific area - Mathematics -, based on a knowledge generated largely in her professional practice as a specialist teacher (Coura & Passos, 2018).

The knowledge more directly related to acting as a Mathematics teacher is shown in the beginning of the narrative, predominantly composed of two subdomains of the MT's **PCK**: the **KMT** and the **KFLM**. The search for new ways of working Mathematics, the need

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to adapt existing teaching materials, and the importance she attributes to the concept of number and to the Decimal Numbering System for learning Arithmetic in the early years are indications of Knowledge of Mathematics Teaching (**KMT**). By mentioning constructivist assumptions as those that configured her conception of teaching-learning as an underlying theory for teaching, she shows indications of Knowledge of Features of Learning Mathematics (**KFLM**). The narrative shows that Adair's knowledge as MTE is composed of those that she aims to help teachers and future teachers to constitute as it relates to teaching mathematics content. This result denotes that the **knowledge of the content of the training** as MTE includes subdomains of the Mathematics Teachers' Specialized Knowledge, as stated by Carrillo et al. (2019).

The **pedagogical knowledge of the content of the training**, a specific dimension of MTE knowledge (Carrillo et al., 2019), is most present in the narrative, with examples of its three subdomains. The assumptions that originated when Adair was acting as a MT guided her from the beginning of her work as an MTE and comprised a theory to which she drew to ground her proposal for teacher education, i.e., her **knowledge of the features of the professional development of mathematics teachers**. In the narrative, this learning about how the identity and knowledge of the MT are constructed is what is most present, manifested in all the times that Adair shows he knows the aspects of teachers' professional development, which he does by indicating some theoretical constructs that underlie his work as MTE - influence of technical rationality, relationship between theory and practice, reflective teacher, contributions of collaborative dimension groups, constitution of professional knowledge from practice, investigative posture resulting in knowledge in practice. This subdomain of pedagogical knowledge of the content of the training is also revealed when she mentions the difficulties of the process of constitution of the specific knowledge of the MT - "How to discuss pedagogical knowledge without the specific knowledge of the subject?" -, when indicating the ways, she considers most appropriate for the MT to build the knowledge necessary for her professional exercise - "It is from the problematization of practice that the teacher starts to reflect and produce meanings for the events she experiences." And also when he considers the difficulties that future MT's bring from their previous schooling to the Mathematics Degree, which he did when he created the Geometry Workshop.

The **knowledge of teaching the content of mathematics teacher education** is revealed in the various activities that Adair proposed to develop the MT's professional knowledge and skills: starting "always from the practical need of the group" of teachers in which she acted as a trainer for the first time, valuing the pedagogical practice of lay teachers, to "work/discuss articulated theoretical aspects and classroom situations" and to organize the supervised internship as an intervention project, involving "elaboration, application, analysis and systematization of classroom activities in basic school".

**Knowledge of the standards of mathematics teacher education** is present in Adair's narrative when she mentions the guidelines for teacher training (Brasil, 2001b), also contemplating knowledge of the school and curriculum of the level of education in which the

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trainee teachers work or will work when they finish their course.

The **pedagogical knowledge of the content of MT training** revealed in the narrative, which Carrillo et al. (2019) associate with the trainer's PCK and, as these authors claim, extrapolates the MT's specialized knowledge, denotes a knowledge that Adair constituted in the practice of training these teachers: with the studies of Mathematics Education and research on teacher training, she constituted specific knowledge for the objectives and activities proper to her work as a trainer.

These results reinforce the existence of the specialized knowledge of the MTE, which, in Adair's case, includes especially the knowledge about how teachers learn and how they become competent - the teacher educator's PCK. Although there are similarities between teaching mathematics and teaching about the teaching of this subject, moving from first to second order teaching requires acquiring new types of knowledge, especially when one realizes that, in order to work in MT training, it is not enough to transmit knowledge, be it mathematical or about teaching practices.

## Acknowledgements:

We register our thanks to Coura's (2018) research participants, who granted us the privilege of listening and telling their life stories, especially, Adair, whose narrative we brought to this text.

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