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How much of High School mathematics do Brazilian future mathematics teachers master?

Quanta matemática escolar é conhecida pelos egressos dos cursos brasileiros de Licenciatura?

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Abstract

Our article aims at better understanding the meaning of future mathematics teachers' scores on the Brazilian National Higher Education Exam (ENADE). Our approach is based on the proportion of right answers on the multiple choice questions about mathematical knowledge applied to future mathematics teachers at ENADE 2017. First, we considered the proportion of right answers to questions whose content were identified by the creators of the exam as part of the curriculum of High School. Second, we complemented the analysis by comparing the knowledge required by a few questions in ENADE with the content of questions of the National Exam for High School (ENEM). The conclusions show that most of future mathematics teachers conclude their initial education without developing knowledge on topics taught in High School.

Keywords: Teacher Education; Mathematics; ENADE; Content Knowledge;

Resumo

Neste texto, buscamos interpretar o significado do desempenho de estudantes de licenciatura em matemática no ENADE para além dos conceitos divulgados pelo INEP. O caminho escolhido se baseou nos índices de acerto em questões objetivas de conteúdo matemático presentes na prova aplicada em licenciandos em matemática no ENADE 2017 de acordo com dois critérios. Consideramos as questões descritas pelo INEP como questões de Ensino Básico e complementamos a análise associando o conteúdo de algumas das questões dessa edição do exame com conteúdos de questões que compuseram o ENEM. As conclusões apontam que a grande maioria dos futuros professores de matemática chegam ao final da sua formação inicial sem domínio adequado sobre conteúdos notadamente de Ensino Médio, com preocupante nível de acertos a perguntas que, espera-se, seus futuros alunos sejam capazes de resolver.

Palavras-chave: Formação de Professores; Matemática; ENADE; conhecimento de conteúdo;

Introduction

All national and international assessment tests on Basic Education in Brazil (*Prova Brasil*, PISA, ENEM, etc.) coincide in identifying a huge gap between the expectation of

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learning and proficiency effectively acquired in all areas of knowledge. Considering PISA (as it is a widely adopted external reference), the gap in mathematics is significantly greater than in reading or science. Table 1 shows the average score of both Brazilian and OECD countries' students.

Table 1: Average scores for Brazil and OECD countries in the different areas assessed by PISA

	Reading		Sciences		Mathematics	
	Brasil	OCDE	Brasil	OCDE	Brasil	OCDE
PISA 2015	407	493	401	493	377	490
PISA 2018	413	487	404	489	384	489

Source: authors' own elaboration based on data from ENADE 2017

Considering the results of PISA 2018 (Brazil, 2019), 41% of Brazilian students were assessed below Level 1 of proficiency in mathematics and less than 5% assessed at proficiency levels 4, 5 and 6 (the upper levels), compared to rates of 23.2% (below Level 1) and 9.3% (levels 4, 5 and 6) regarding reading proficiency. The standard deviation of PISA is 100 points, which means that the gap in the mathematics average of the 2018 exam (105 points) is greater than the standard deviation, whereas the reading gap (74 points) is way below.

Without minimizing the relevance of factors of socioeconomic and family nature in the educational process, international studies show that the specific knowledge of the mathematics teacher is the most relevant variable regarding the influence on learning when the analysis is carried out considering variables related to the teacher (Hill; Rowan; Ball, 2005). In some scenarios, its influence is very close to variables of socioeconomic nature (Charalambous, Hill, Chin and McGinn, 2019). Therefore, it is reasonable to expect that teacher training, specifically for mathematics teachers, have a relevant role in the results obtained in Basic Education.

Initial teacher training has been intensely discussed in Brazil due to efforts by the public authorities to propose and implement regulations to positively influence it. A broad study on the topic, developed by Gatti, Sá Barreto and André (2011), points out several problems, including the excessive proliferation of distance courses and the structure of curricular organization with little emphasis on practical aspects of the profession. According to the authors, “the theory-practice relationship so emphasized in documents and standards, with the proposed integrated curriculum design, is not seen in the daily life of different licentiate degree course” (Gatti et al, 2011, p. 114). The criticism above refers to licentiate courses in general, but with a distinction regarding licentiate degree courses in mathematics:

> The licentiate degree courses in mathematics are different, as they present a greater balance between the subjects related to specific knowledge in the area and the specific knowledge for teaching, although the majority of public institutions maintain a much higher workload for disciplines related to specific knowledge, mirroring the idea of a bachelor's degree rather than a licentiate degree. (p. 115 – own translation)

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The distinctions pointed out by Gatti et al. (2011) between specific knowledge of the area, specific knowledge of teaching, theoretical and practical knowledge are related to a large area of research in Mathematics Education that seeks to understand and characterize the knowledge necessary for mathematics teachers that teach mathematics need for their practice professional, and the different facets that lead to learning outcomes. This question was raised by Shulmann (1986) for teachers in general and has been studied by several other authors. Specifically to mathematics teachers, we emphasized the work of a group of researchers from the University of Michigan, in the United States (Ball, Thames & Phelps, 2008) and from a university in Germany (Baumert et al. 2010). In general, the knowledge needed by the teacher is differentiated between content knowledge (CK), pedagogical knowledge (PK) and pedagogical content knowledge (PCK)[3]. If there are disputes about the possible ways to characterize and categorize knowledge and about the possible effect or usefulness of these different categories, some consensus is beginning to be formed on some points. In a recent preface to a major publication on mathematics teacher's knowledge, some researchers linked to the Michigan group summarized these points as follows:

> From several decades of research, we propose what we see as a few related emerging lessons: a) Teaching teachers additional standard disciplinary mathematics beyond a basic threshold does not increase their knowledge in ways that impact teaching and learning; b) Providing teachers with opportunities to learn mathematics that is intertwined with teaching increases their mathematical knowledge for teaching; c) The focus of the content, tasks, and pedagogy for teaching such knowledge requires thoughtful attention to ways of maintaining a coordination of content and teaching without slipping exclusively into one domain or the other. (Hoover, Mosvold, Ball & Lai, 2016, p. 11-12)

The duality of the statement of the first point is especially important for our article. We want to emphasize that if, on the one hand, it is explicitly stated that advanced knowledge of mathematics has no apparent influence on the teaching actions of a teacher, the statement also raises the existence of some type of baseline level of content knowledge. This duality is emphasized a little later in the text:

> […] a certain threshold level of knowledge of the subject is essential, but preparing teachers by requiring mathematics courses that are not directly connected to the content being taught or to the work involved in teaching that content is misguided. (Hoover, Mosvold, Ball & Lai, 2016, p. 12)

In fact, this statement by Hoover et al. (2016) is in line with the conclusion of Begle (1972), who found a significant correlation between the performance of elementary school students and the knowledge of teachers in basic algebra of real numbers, but not with their knowledge of modern algebra (sets, rings and bodies).

The statement is also consistent with Baumert et al. (2010). The authors analyzed the

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[3] *Content Knowledge (CK), Pedagogical Knowledge (PK) and Pedagogical Content Knowledge (PCK).*

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influence of the mathematics teacher's knowledge on their students' learning considering two dimensions, content knowledge and pedagogical content knowledge. A close look at the questions that were used in the research instrument shows that the content sought by the first dimension is strictly linked to the content typically taught in high school and not to the content typically taught in higher education mathematics courses.

Based on these findings and considering the composition of the issues present in the ENADE, National Student Performance Exam, applied to students of undergraduate courses in mathematics throughout Brazil, our study seeks to analyze the knowledge of future mathematics teachers about contents that are part of the Basic Education curriculum.

Objectives

The question that guides our study is: **How much do future mathematics teachers master the mathematical content of Basic Education that their students are expected to master?**

This question requires some clarification. By "mathematical content that their students are expected to master" we mean that mathematical content that has become consensual in school practice in Brazil, whether by textbooks approved by the National Textbook Plan (PNLD), by the National High School Exam (ENEM), by the Common National Curricular Base (BNCC) or by the ENADE organization itself. That is, we do not wish to discuss the relevance of these contents and the various possible meanings about teaching and learning, but we adopt a pragmatic stance of studying what is adopted, in fact or in law, as school mathematical content in Basic Education.

Coherently, we adopt the same pragmatic stance regarding assessing the mastery of such content by mathematics teachers: we accept, although aware of the limitations pointed out by authors such as Primi, Silva and Bartholomeu (2018) regarding ENADE as an assessment tool for higher education institutions, that knowledge expectations are those assessed in standardized and normative exams.

That said, to try to answer the question posed above, we will use the microdata from ENADE 2017, a data source that includes all 10,861 possible graduates of 2017 in all mathematics licentiate degree courses in Brazil. Thus, we can refine our question around a more specific objective: to qualitatively understand the meaning of the scores attributed to licentiate degree courses in mathematics at ENADE 2017 regarding the correct percentages of the multiple-choice questions of specific knowledge that make up this assessment, with an emphasis on questions that address Basic Education content according to the classification made by the ENADE organization. Later on, we will show that this classification is in line with the expectations of high school learning from the point of view of both the Brazilian Ministry of Education and mathematics teachers.

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Before presenting the adopted methodological procedures, we briefly present the structure of ENADE and make a brief review of studies using this data.

ENADE

The National Higher Education Assessment System (SINAES) was established in the Brazilian Ministry of Education (ME) in 2004, with Law 10.8611, with the objective of ensuring a national process for the assessment of higher education institutions (HEIs), of undergraduation courses and academic performance of their students. Student performance is assessed using the National Student Performance Exam – ENADE, which replaced the National Course Exam, applied between 1996 and 2003.

In Article 5, paragraph 1, of the SINAES' creation law, it was established that ENADE should assess students' performance regarding the syllabus contents provided for in the curricular guidelines of the respective undergraduate courses, besides the skills to adjust to the requirements arising from the evolution of knowledge and skills to understand topics outside the scope of the profession. ENADE has the exclusive objective of evaluating courses and institutions (individual results are not public) and was established with the aim of guiding HEIs on their academic effectiveness and subsidizing government agencies in the formulation of public policies.

Each course has been evaluated by ENADE every three years and participants are entering and graduating from the course, where graduates, in the case of licentiate courses, are considered students that have already completed more than 80% of the workload of the course in the period in question. The ENADE tests contain a part of general training common to all courses (corresponding to ¼ of the test questions and the final score) and a part of specific contents of the course (corresponding to ¾ of the test questions and the score).

The National Institute of Educational Studies and Research Anísio Teixeira (INEP), the ME's autarchy responsible for SINAES, makes available on-line a large number of well-organized data about ENADE in raw format. A systematic review of the literature by Lima, Ambrosio, Ferreira and Brancher (2019) compiled 39 academic papers on ENADE made public by 2016, grouping them into six categories. The categorization proposed by the authors was not informative in terms of the question discussed in our study, but we used these references as a starting point to understand what has already been studied about and from the ENADE microdata.

These studies adopted different foci. Several studies attempt to relate student performance to socioeconomic variables or course structure and teaching staff (for example, Moriconi and Nascimento 2011, Silva and Vendramin 2010). Others analyze the evidence considering the content and skills grid prescribed in the exam notice or in normative course documents (see Costa and Martins, 2014 on matrices of physics content). Other studies focus

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on the effects and influences of ENADE on the organization and curriculum of specific courses (see Gontijo, 2014 for pedagogy courses).

In this sense, none of the studies identified by Lima et al. (2019) addressed issues that dialogue explicitly with our study question. However, we identified two studies that establish a slightly closer relationship to that we intend in this article, and that are not included in the review mentioned.

Rothen and Nasciutti (2011) compared the performance of newcomers and graduates in the ENADE tests in the years 2005 and 2006. The authors present data on Architecture and Urbanism, Biomedicine, Accounting Sciences, Computing, Engineering, Physics and Teacher Training courses. The data are presented according to the different constituent parts of the evidence. In the objective questions of the specific part of the test, an average increase of 20% (compared to the performance of the freshmen) is verified in the average of the grades in the seven courses studied. Considering that specific questions must assess the knowledge acquired during professional training, this variation is extremely small and calls attention to some serious problem in the education offered at the HEIs, in the ENADE methodology or both. When establishing a parallel between the performance of freshmen and graduates, Rothen and Nasciutti (2011) approach our goal of qualifying the knowledge of graduates of a specific course. However, the authors do not further discuss this aspect, limiting themselves to presenting descriptive data that suggest that some higher education courses are adding little specific knowledge to their students.

Barros, Campos, Teixeira and Cabral (2020) approach the same objectives of our study when analyzing the performance of Physical Education students at ENADE 2014 regarding the content covered in part of the questions. Thus, the authors go beyond the presentation of the rates of right answers and suggest what they can mean, from a qualitative point of view, regarding the expected knowledge and practices of these professionals. Barros et al. (2020) mention the low performance in the specific training component, which had an average of 46.2% correct answers and argue, based on an analysis of the content of these questions, that this result suggests serious deficiencies regarding the future professional practice of these students.

Our study aims at addressing this issue more explicitly regarding the licentiate degrees in mathematics: what is it possible to infer, besides the scores attributed to institutions, from ENADE data about the mathematical knowledge of future mathematics teachers?

Data processing

The ENADE test consists of ten questions of general training, two of which are discursive and eight of multiple choice, thirty questions of specific training, three of which

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are discursive and twenty-seven are multiple choice and, at the end, a questionnaire on the perception of the test. The test is accompanied by a socioeconomic questionnaire and a questionnaire on the student's perception of the course. In the case of a licentiate degree in mathematics, there is also a questionnaire with thirteen specific items about the course. These questionnaires must be completed online by students before the exam.

Both newcomers and graduates must register to participate in the test; however, after the first years of implementation of ENADE, newcomers have been dismissed. In the microdata used for this text, there is no field that suggests this distinction between the individuals and the distribution of the variable "year of admission" indicates that the data contains only information about graduates[4].

Estimating the ENADE score of a course

The detailed procedure for estimating the ENADE score of a course can be found in Brazil (2017). In short, it is estimated as follows, considering as universe the courses participating in a given area (for example, all mathematics licentiate degree courses):

a) the weighted average is estimated between the gross grade in questions of general training (25%) and the questions of specific knowledge (75%) of all students in the course of a given HEI;
b) this average is normalized to the standard scale for all participating courses and HEIs;
c) the range between the lowest and highest average is converted to a scale of 0 to 5,
d) this interval is then divided into 5 intervals of the same amplitude so that the grades in each of them are converted into scores from 1 to 5.

ENADE's score estimating system is exemplified bellow, using data referring to the 426 licentiate degree courses in mathematics evaluated in 2017. The graph in Figure 1 shows the distribution of grades after step (c) and before step (d) described above.

The graph shows that the notes behave in a manner close to a normal distribution (mean equal to 2.20 and standard deviation equal to 0.88). Similar behavior is observed if we consider the ENADE score obtained after step (d), which rounds up the values by grouping them into five scores.

The distribution of courses between scores changes when we consider, instead of the number of courses, the number of students in these courses.

[4]In the data from ENADE 2017, among the students of the licentiate degrees in mathematics, there were only two whose year of entrance was 2017, possibly students that re-entered to complete a few missing subjects in their curriculum.

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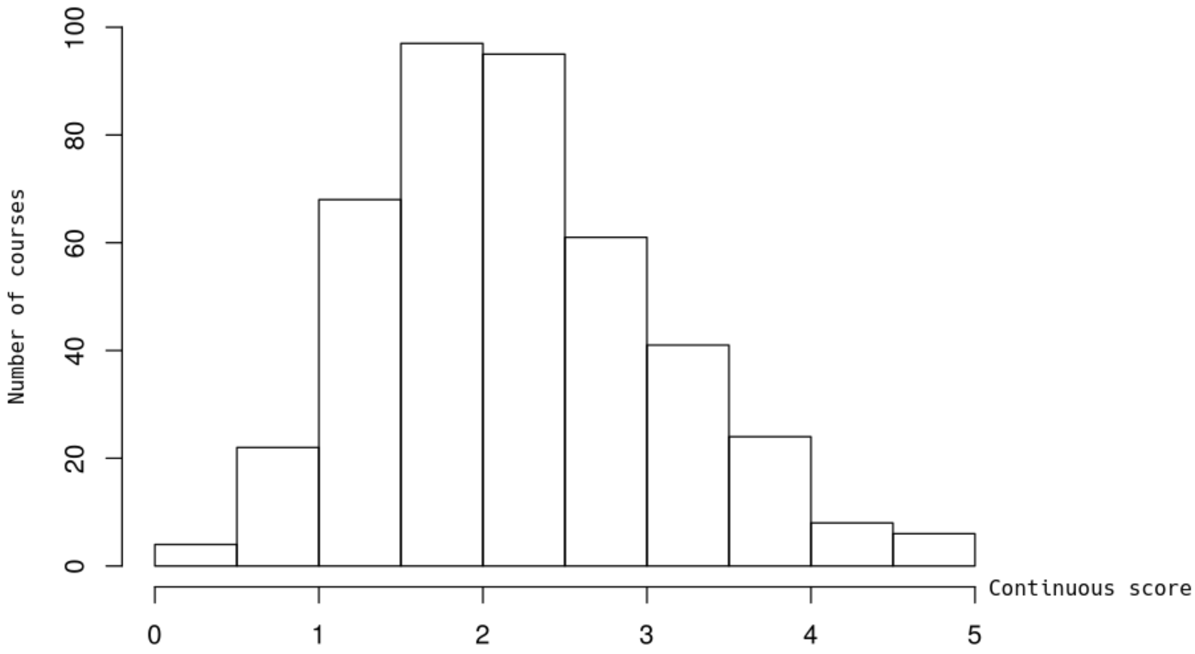

Figure 1: Histogram of the continuous ENADE score of licentiate degree courses in mathematic

Source: authors' own elaboration based on data from ENADE 2017

The distribution of courses between scores changes when we consider, instead of the number of courses, the number of students in these courses.

Table 1: Concentration of students by courses

Score	Number of Courses	Total of students	Accumulated percentage of courses	Cumulative percentage of students in courses	Average number of students per course
1	22	412	5%	3.79%	18.73
2	160	5618	43%	55.42%	35.11
3	158	3506	80%	87.64%	22.19
4	71	1133	96%	98.05%	15.96
5	15	212	100%	100%	14.13

Source: authors' own elaboration based on data from ENADE 2017

Table 2 shows that courses with score 2 concentrate the largest number of students. This difference in the concentration of students means that there are many students in courses with the lowest scores in ENADE. If we consider the students of all courses that have individual performance equivalent to that of the courses evaluated with score 1 or 2, this proportion will increase even more.

The question that motivated our study refers to the mathematical knowledge of mathematics teachers, and not to teacher training courses. Thus, to try to answer this question using the ENADE data, we decided to use the criterion used by INEP to define the *ENADE score of the participant*, which we explain below.

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The estimating of a student's ENADE score

To determine the ENADE score of students of licentiate degree courses in mathematics, a procedure was adopted to obtain an individual ENADE score for each student[5], which is very close to that used by INEP for estimating the score of courses (described in the previous session). The steps taken are as follows:

- We consider the weighted average between the gross grade in general training questions (25%) and specific knowledge questions (75%) for each of the students of licentiate degree in mathematics that took ENADE. In this step, INEP considers the grades of the students of each course;

- this average was converted to the standard scale in relation to the average and standard deviation of this group, considering the 10,861 students as universe (instead of the courses, as INEP);

- the range between the lowest and highest average is converted to a scale of 0 to 5. However, in this step, instead of using the minimum and maximum values of the students, we decided to use the same that were used by INEP in estimating the score of each course. Thus, the analogy between the scores of the courses and the participants is more direct[6]. We observed that the minimum grade for courses and individuals is the same (zero), but the maximum average for courses is slightly lower than that of individuals;

- The fourth step converts the individual continuous note into discrete scores from 1 to 5.

Thus, we obtain a grade for each student that can be interpreted in a similar way to the grade assigned to the courses by INEP.

Data extraction and processing

The two data sources used in this text are the ENADE 2017 score report and ENADE 2017 microdata made available by INEP. Both are available on the INEP website. All other files referred to in the description below are available at https://osf.io/qpt8c/.

[5] We recognize here an intrinsic limitation to this approach, as ENADE is an exam done with the objective of evaluating the courses, without significant consequences for the graduates, the students' commitment to taking this test can be questionable. Nevertheless, this is an intrinsic limitation to ENADE, which is the only scale examination conducted with students about to conclude undergraduate courses.

[6] This is the reason why there was a very small number (n = 5) of students with a grade higher than 5 for having a higher grade than the highest grade of an institution. To simplify, these students were treated as having obtained a grade 5.

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The first one brings the scores of all the courses evaluated in ENADE 2017 (last to evaluate licentiate degree courses in mathematics) of each Brazilian HEI. Based on this file, we generated (via spreadsheet software) a new data set with all the licentiate degree courses in mathematics in the document, its gross score in general and specific training, its standardized score in general and specific training, its continuous ENADE score, its ENADE score and the IES code.

The processing performed on the microdata was more laborious. Due to our objective to qualitatively understand the meaning of each score regarding the rates of right answers in each of the 27 objective questions contained in the specific test. This information is presented in the microdata in three formats: a string with the alternatives indicated by the respondent for each of the 27 questions, a string with the correct alternatives for each question and a string formed by 0's and 1's indicating error or correctness for each of the questions. To access the rates of right answers for each question, a script written in C language was used to read the microdata, copy the fields of interest to the researchers and break the string of correct answers into 27 independent variables, facilitating the subsequent analysis. The result of this process is a new set of data containing only the participants that were attending a licentiate degree course in mathematics and had a regular presence in the period of the application of the exam (n=10,861) with the vector of right answers separated in independent variables.

Once these steps were completed, the two new data sets were imported into the R software for a final round of processing. Thus, a new table with the frequency of correct answers for each question for students that were grouped in each of the five scores of ENADE was generated. This table is the basic reference for the discussion in the next section and can be replicated from the official documents available on the INEP website, the C script and the R commands available at https://osf.io/qpt8c/ along with the table.

Data analysis

In this section, we present some descriptive information and analyze the data generated from the ENADE microdata to answer the proposed question.

Distribution of students by ENADE score

First, we describe below the concentration of students by score. Note that this distribution is different from that shown in Figure 1. This difference was already expected when we consider the concentration of students in courses with the ENADE 2 and 3 score shown in Table 2.

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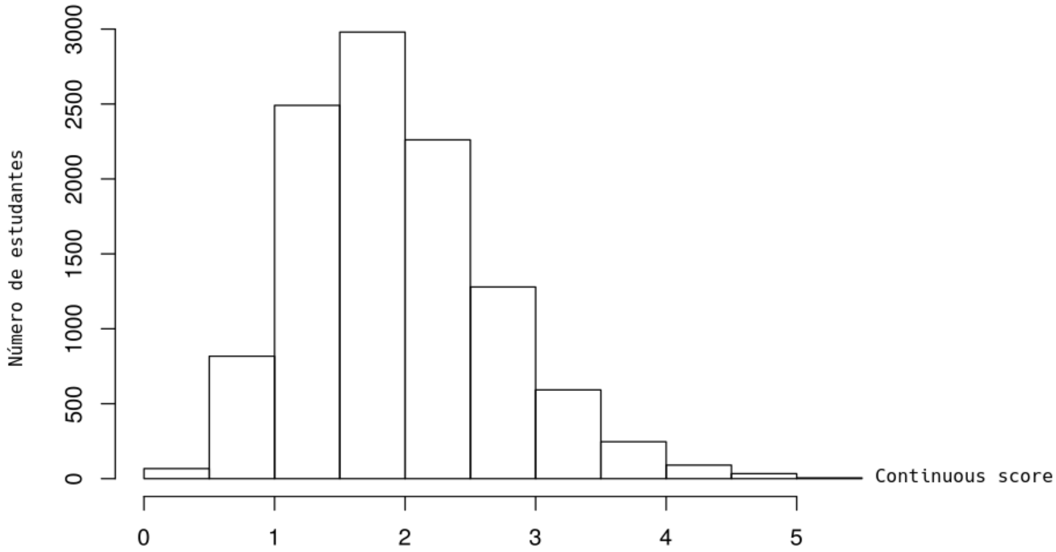

Figure 2: Histogram of the continuous ENADE scores for students

Source: authors' own elaboration based on data from ENADE 2017

The bar graph below shows the frequency of each score, once the rounding used in estimating the ENADE score of a course has been applied.

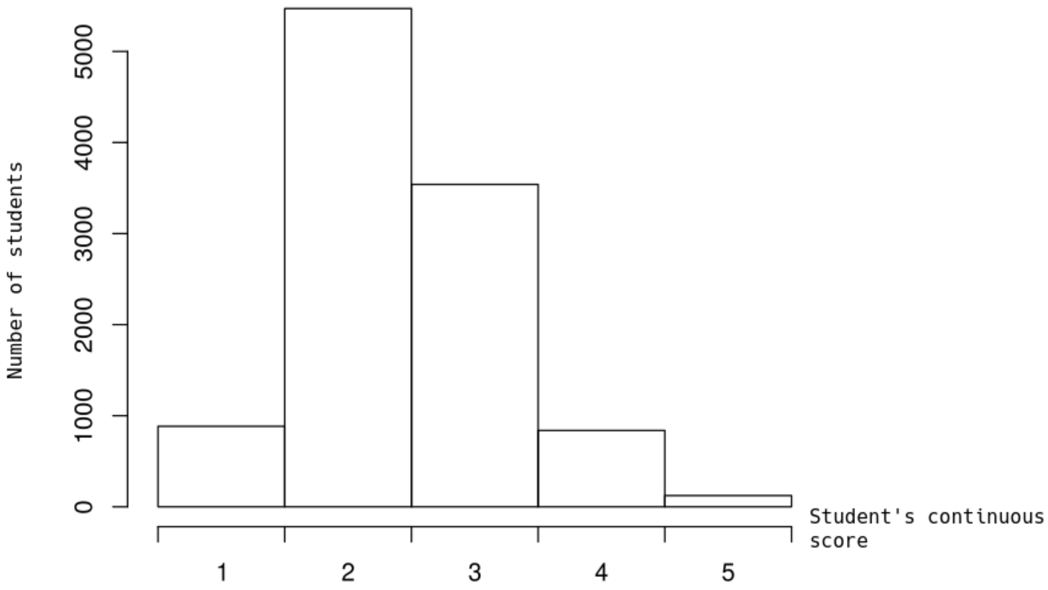

Figure 3: Number of students per ENADE score.

Source: authors' own elaboration based on data from ENADE 2017

The histogram behaves in a manner close to a normal curve (mean equal to 1.93 and standard deviation equal to 0.75) with a mean lower than the histogram in Figure 1. The graph in Figure 3

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shows the frequency of each score, once the rounding used in the estimation of the ENADE score of a course has been applied. Besides, this graph is even more concentrated in score 2 than that shown in Figure 1, as a result of the higher concentration of students in institutions with low scores in ENADE. In the table below, we can compare the number of students in courses with a given score (as shown in Table 2, data estimated directly by INEP) and the number of students that, as individuals, would obtain the given score. This is the data estimated above.

Table 3: Comparison between the concentration of courses and students in each score.

Score	Percentage of students in courses with a given score	Cumulative percentage of students in these courses	Percentage of students with a given score	Cumulative percentage of students with a given score
1	3.79%	3.79%	5.99%	5.99%
2	51.63%	55.42%	52.32%	58.31%
3	32.22%	87.64%	34.86%	93.17%
4	10.41%	98.05%	6.31%	99.48%
5	1.95%	100.00%	0.52%	100.00%

Source: authors' own elaboration based on data from ENADE 2017

Observing the individuals, the concentration increases in the lowest scores and decrease significantly in the highest scores: the number of students with score 5 is about 1/4 of the number of students in courses with score 5.

Up to this point, a statistical treatment has been performed, estimating and comparing the ENADE score by student, which seems more significant than the ENADE score of courses if we want to infer something about the mathematical knowledge of recently graduated teachers. In the next session, we go into the merit of the question proposed in this text, searching for tangible meanings for each of the scores, adding to the statistical data an analysis of the mathematical content of the objective questions of the specific part of the ENADE 2017 test.

A possible interpretation for the meaning of each score

Large-scale assessments such as ENADE serve to establish a comparison parameter between institutions, both for making public decisions and for informing society as a whole about the quality of services provided or offered by the different HEIs. However, since it does not use methodologies such as TRI, ENADE does not allow analysis over time; either for specific courses (has this course improved in relation to the last assessment?) or for the group as a whole (the mathematics courses in Brazil are improving over the years?).

Since it is an exam with standardized results, making a qualitative analysis of the contents covered can be a way to apprehend what each score means in terms of what its students demonstrate to know.

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ENADE is composed of two sets of questions, those of general knowledge, which deal with text interpretation and problem solving not specific to the areas of knowledge, and those of specific knowledge, which are developed specifically for each course considering the National Curriculum Guidelines for undergraduate courses, the National Catalog of Higher Technology Courses and other relevant official documents (BRASIL, 2017). We will focus our analysis on the objective questions of specific knowledge.

For the licentiate degree course in mathematics, these questions are created by INEP in two large groups, the first with questions about mathematics (18), which also figure in the evaluation of bachelor's courses in mathematics, and those of a more pedagogical nature (9), which only appear in the test of the licentiate degree in mathematics (BRASIL, 2017).

An inspection of the first group of questions allows us to notice a second distinction pertinent to our objectives: questions whose content needed for resolution are addressed throughout Basic Education and questions that demand knowledge typically addressed in Higher Education.

As previously discussed, although there may be discussions about what content is beyond the threshold in which the domain, on the part of the teacher, does not generate learning results, it is agreed that the teacher should at least know what he is going to teach (Lorenzato, 2006; Hoover et al, 2016). In this sense, it seems pertinent to us to analyze the knowledge of future mathematics teachers evaluated at ENADE regarding performance in questions that demand only mathematical content from Basic Education.

Mathematics questions of Basic Education and Higher Education

INEP does not considere questions when they present a biserial point below 0.2 (BRASIL, 2017). In these cases, answers are omitted from the microdata, which forced us not to consider them in our analysis. In 2017, this occurred with four questions (all of them with mathematical content), which left us with nine questions of a pedagogical nature (PE) and 14 about mathematical content.

In Annex IXb (BRAZIL, 2017), INEP reports the elements that characterize each of the questions used in the test, namely: characteristics of the professional profile, skills and curriculum content. For the curricular content element, one of the categories present is "mathematical content of Basic Education". Among the questions present in the 2017 edition that were classified in this category (8), one was canceled and another is in the group of pedagogical nature, which are outside the scope of this paper.

Regarding the mathematics teacher's knowledge models mentioned in the introduction, the remaining six questions are consistent with the questions used in Baumert et al (2010) to evaluate the component called *content knowledge* and what Ball et al (2008) call *common content knowledge*, that is, that knowledge that could be used in other contexts other

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than educational, being therefore not specific to the mathematics teacher. Although the authors of both works admit that knowledge of this nature does not seem to be that with the greatest influence on student learning, both admit that it is part of the arsenal of knowledge necessary for teaching practice.

Knowing that the staff of professionals that work at INEP are linked to Higher Education and not necessarily to Basic Education, the authors found it pertinent to pragmatically verify this classification by subjecting it to the appreciation of mathematics teachers working in Basic Education. Thus, we would be reinforcing the main characteristic that we seek in these issues: reflecting the content that students are expected to master at the end of their schooling.

For such purpose, we invited 11 mathematics teachers with at least 2 years of experience in high school to classify the questions based exclusively on the content required for the resolution, and not on their difficulty, in two categories: Basic Education and Higher Education. The sample of participating teachers was made by proximity, without the intention of making it representative, since we understand that the mathematics curriculum of Basic Education is widely known, well established and quite homogeneous. The participants were not aware of the content of the study to the point that it could influence their answers. The classification was made via an online form and the results are shown in Table 4.

Table 4: Classification of questions categorized as "mathematical content of Basic Education" made by mathematics teachers

	Question 10	**Question 15**	**Question 17**	**Question 18**	**Question 22**	**Question 24**
Higher Education	2	1	0	0	0	3
Basic Education	9	10	11	11	11	8

Source: authors' own elaboration

The table showed that there was a good degree of agreement between the classification performed by active mathematics teachers and the categorization reported in the INEP report. Since the objective of our article is not to problematize this categorization, but to confirm pragmatically that the content covered in some questions of ENADE 2017 for mathematics licentiate degree course is compatible with the content that is expected to be taught to students of Basic Education, we are not concerned with the statistical significance of the analysis presented, since three distinct elements seem to converge on this categorization: the information reported by INEP, the classification by active mathematics teachers and the qualified perception of the authors.

Thus, to perform the analysis presented below, six questions were considered (10, 15, 17, 18, 22 and 24, following the numbering of the ENADE test) as questions involving Basic Education (BE) content. The others were considered as involving higher education content

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(HE) and are included in the table below, along with questions of a pedagogical nature (PE), for informational purposes only.

Students' performance on different types of questions

The following table shows the percentage of right answers among students that obtained each of the five scores.

Table 5: Rate of correct answers of the questions of each type: Pedagogical questions (PE), questions about mathematical content of Basic Education (BE) and Higher Education (HE)

	BE Questions	**HE Questions**	**PE Questions**
Score 1	13.0%	12.6%	16.1%
Score 2	22.0%	25.1%	33.4%
Score 3	36.0%	40.3%	56.0%
Score 4	61.2%	65.6%	72.8%
Score 5	86.6%	88.4%	86.7%

Source: authors' own elaboration

It is important to note that these are the multiple-choice questions, with five alternatives. In this sense, it is noteworthy the fact that students with score 2 have shown a frequency of right answers just above the rate of 20% (expected value in the case of random guessing) in questions of BE. This scenario is worrying, since these future teachers will be qualified to work in high school and 58.31% of the students concluding the licenciate degree in mathematics that participated in ENADE 2017 scored 1 or 2 and 93.17% score up to 3, as shown previously in the Table 3.

Considering our goal of understanding the meaning of each ENADE score regarding the mathematical knowledge of future mathematics teachers in Brazil, we chose two questions for a more detailed analysis, the questions 15 and 20. They were chosen due to their great similarity with two other questions that appeared in recent editions of ENEM (see Appendix 2). This similarity with ENEM reinforces the representativeness of questions 15 and 20 to address the question that guides our study: high school students are expected to be able to solve them correctly.

These two questions were identified randomly, having arisen from the first author's familiarity with these two assessments. Despite the different guidelines of ENADE and ENEM, and their objectives and consequences for the participants, the similarity between the questions mentioned is clear both when analyzing the statements, commands and alternatives that make up each of them and when considering plausible solutions. The fact that there are such similarities between questions from two exams that are applied to students of different educational levels is not surprising, since the mastery of the content to be taught is one of the dimensions of the specific knowledge that is expected from a math teacher (SHULMANN,

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1986; BALL et al, 2008; BAUMERT et al, 2010; HOOVER et al, 2016). This statement can be simply put as: “nobody can teach what they don’t know” (Lorenzato, 2006, p.3).

The first pair, formed by question 15 of ENADE 2017 and question 155 of ENEM 2009 (gray worksheet), asks for the algebraic expression of a quadratic function that models a situation described in the context of sales. The second pair, formed by question 20 of ENADE 2017 and question 138 of ENEM 2017 (gray worksheet), asks for the probability of an event composed of a series of independent events related to weather forecast.

The overall rate of right answers for these two ENADE questions was 22.7% and 20.3%, respectively, which are quite worrying figures. We can also look at this rate of right answer given among students in each score, as shown below.

Table 6: Rate of correct answers the two questions selected by score

	Question 15	**Question 20**
Score 1	12.4%	11.4%
Score 2	18.5%	18.4%
Score 3	31.8%	25.4%
Score 4	58.7%	59.4%
Score 5	91.1%	91.1%

Source: authors’ own elaboration

These data explicit even more the aforementioned problem. In the case of these two questions, which are very similar to questions asked in recent editions of ENEM, the rate of right answers of the students that obtained grade 2 is below 20%[7]. Not only that, but the rate observed among students of score 3 is slightly higher than this percentage, which at least raises uncertainties about the proficiency acquired by future teachers in most of the mathematics degree courses in Brazil, a concern that must be extended to the quality of the courses in question. ENEM is an exam that is taken by most of Brazilian students that complete high school and has been established as almost the only criteria for access to higher education. In this sense, using it as a reference for what students at this level of education should learn seems reasonable.

As mathematics teachers, we also do not see objective reasons for considering that these two questions are especially challenging, which could explain the low rates of right answers shown in Table 6. The first one asks only for the algebraic expression of a quadratic function, one of the types of function most worked on during high school, and the context (involving sales and discounts) is frequently used since middle school. The second question, using a common context for probability, does not use any device that induces the error neither

[7] The rates of correct answers given significantly below 20% are noteworthy and we can only conjecture that they are due to some distractor present in the questions. We could not evaluate this hypothesis from the available data, since they only present a correct answer or an error, without indicating the alternatives marked.

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demands a resolution strategy that may be considered unusual. Therefore, both can be considered good representatives of the kind of mathematical knowledge that is expected from students in the last years of high school and, consequently, from their teachers.

Conclusions

Our study aimed at understanding one of the components usually pointed out as fundamental, although not the only one, for the practice of future mathematics teachers: their knowledge of specific contents. Our study was based on the microdata of ENADE 2017, considering a universe of 10,861 possible graduates across Brazil.

In a first step, we found that if we used the methodology used by INEP to assign a score to individuals instead of courses, 93.17% of the students in mathematics licentiate degree would receive a score less than or equal to 3.

To apprehend something substantive about the knowledge of these future teachers, we first analyzed the rate of right answers of the participants in the multiple-choice questions of mathematical content.

We analyzed the performance of the participants in six test questions whose content was classified by INEP, by a sample of teachers working in basic education, and by the two authors as being compatible with Basic Education. The 10,120 students that would obtain an individual grade of 3 or less in the 2017 edition of ENADE had, on average, a 27.0% rate of right answers on these questions. We emphasize that these are multiple-choice questions with five alternatives, which means that random guesses would result in 20% right answers.

Then, we focused on the rate of right answers in two of these questions that we identified as being very similar to two questions in recent editions of ENEM. In these specific questions, the rate of right answers was even lower among students with a score equal to or less than 3: 22.7% and 20.3%, respectively. These questions were not only classified as being high school content, but they are also very similar to questions that were effectively part of an exam aimed at high school students, that is, whose content is certainly part of the repertoire that is expected to have developed at the end of a licentiate degree course in mathematics.

Although the authors are aware of the differences between ENADE and ENEM regarding the test format, development methodology, objectives and guidelines, the similarity between the questions and the performance levels identified make the parallel drawn very relevant regarding the object of study of our article, the knowledge of content of future mathematics teachers.

The international literature has made a great effort to characterize more clearly what types of knowledge are necessary for a teaching practice in mathematics that effectively results in learning. Studies like those carried out by Ball et al. (2008), Baumert et al. (2010),

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Carrillo-Yañez et al. (2018) and Charalambous et al. (2019) are among those that seek these answers. Despite some small divergences that eventually arise due to specific choices and emphases, the authors agree that the teacher must know, at least, the mathematical content that his students are expected to learn or, simply, "no one can teach what they don't know" (Lorenzato, 2006, p.3).

Our results point out that this baseline level of mathematical knowledge, based on references actually developed for students, does not seem to be achieved by 93% of graduates in licentiate degree courses in mathematics close to completing their courses.

Although the authors are aware that the results of standardized tests are only indicators of reality, the results of this analysis are quite striking and raise at least two urgent questions from the point of view of public policies: how is it possible to change the initial training of mathematics teachers substantially and effectively? Considering that it is inevitable that a significant part of these students will effectively teach in Basic Education, what support do they need to receive to develop the knowledge necessary for teaching?

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Appendix 1

Table 7: Percentage of correct answers for each of the objective questions of the specific component among students in each score.

	Type of question	Score 1	Score 2	Score 3	Score 4	Score 5
Q9	HE	12.44%	21.64%	27.18%	51.68%	91.07%
Q10	BE	22.12%	43.27%	71.00%	90.07%	98.21%
Q11	HE	11.83%	22.95%	37.64%	70.95%	87.50%
Q12	HE	9.52%	21.33%	43.58%	79.27%	92.86%
Q13	HE	13.36%	27.96%	51.48%	72.70%	85.71%
Q14	HE	9.06%	14.52%	18.89%	39.12%	76.79%
Q15	BE	12.44%	18.48%	31.80%	58.69%	91.07%
Q16	Cancelled	-	-	-	-	-
Q17	BE	11.06%	21.59%	32.75%	51.53%	67.86%
Q18	BE	7.22%	11.54%	19.55%	39.71%	87.50%
Q19	Cancelled	-	-	-	-	-
Q20	HE	11.37%	18.39%	25.36%	59.42%	91.07%
Q21	Cancelled	-	-	-	-	-
Q22	BE	12.29%	19.69%	40.28%	73.87%	96.43%
Q23	Cancelled	-	-	-	-	-
Q24	BE	12.60%	17.26%	26.41%	53.58%	96.43%
Q25	HE	12.14%	25.07%	40.99%	60.00%	82.14%
Q26	HE	20.89%	48.74%	77.58%	91.39%	100.00%
Q27	HE	14.75%	34.19%	60.43%	80.15%	96.43%
Q28	PE	15.21%	32.68%	53.41%	67.45%	69.64%
Q29	PE	15.67%	27.27%	43.42%	60.58%	80.36%
Q30	PE	14.75%	30.95%	60.83%	82.48%	92.86%
Q31	PE	13.52%	29.49%	48.18%	61.02%	76.79%
Q32	PE	15.21%	30.25%	55.12%	74.74%	92.86%
Q33	PE	16.90%	32.31%	57.69%	77.81%	94.64%
Q34	PE	14.13%	27.34%	43.77%	61.02%	83.93%
Q35	PE	24.58%	56.27%	81.25%	89.78%	92.86%

Source: authors' own elaboration

Note: Questions 1 to 8 are common knowledge. This table also does not include the five discursive questions.

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Appendix 2

QUESTÃO 15

O gerente de um posto de combustíveis observou que, na primeira semana do mês em que definiu o preço do litro de gasolina a R$ 3,70, foram vendidos 15 000 litros diários. Com isso, o posto fez uma promoção e percebeu que, para cada centavo de desconto que concedia por litro, eram vendidos 200 litros de gasolina a mais por dia.

Representando por p a quantidade de centavos correspondente ao desconto dado no preço de cada litro de gasolina, e por F o valor, em reais, faturado por dia com a venda de gasolina, a expressão que descreve essa situação é

A. $F = 15\,000 + 590p - 2p^2$
B. $F = 15\,000 + 590p + 2p^2$
C. $F = 55\,500 - 590p - 2p^2$
D. $F = 55\,500 + 590p - 2p^2$
E. $F = 55\,500 - 590p + 2p^2$

Figure 4: Question 15 of ENADE 2017 – Licentiate Degree course in Mathematics
Source: authors' own elaboration

Questão 155

Um posto de combustível vende 10.000 litros de álcool por dia a R$ 1,50 cada litro. Seu proprietário percebeu que, para cada centavo de desconto que concedia por litro, eram vendidos 100 litros a mais por dia. Por exemplo, no dia em que o preço do álcool foi R$ 1,48, foram vendidos 10.200 litros.

Considerando x o valor, em centavos, do desconto dado no preço de cada litro, e V o valor, em R$, arrecadado por dia com a venda do álcool, então a expressão que relaciona V e x é

A. $V = 10.000 + 50x - x^2$.
B. $V = 10.000 + 50x + x^2$.
C. $V = 15.000 - 50x - x^2$.
D. $V = 15.000 + 50x - x^2$.
E. $V = 15.000 - 50x + x^2$.

Figure 5: Question 155 of the gray test of ENEM 2009
Source: authors' own elaborationSource: authors' own elaboration

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QUESTÃO 20

Durante o final de temporada de um evento de corrida automobilística, é comum chover nos dois dias de treino, sexta-feira e sábado, e no dia da corrida, domingo. Suponha que a previsão meteorológica para esses dias indique 80% de chance de chuva para cada um dos dias de treino e 30% de chance de chuva para o dia da corrida.

Considerando as informações do texto acima, avalie as afirmações a seguir.

I. A chance de não chover em nenhum dos três dias é de 2,8%.

II. A chance de chover em pelo menos um dos três dias é de 97,2%.

III. A chance de chover sexta-feira e sábado é de 80%.

É correto o que se afirma em

(A) I, apenas.
(B) III, apenas.
(C) I e II, apenas.
(D) II e III, apenas.
(E) I, II e III.

Figure 6: Question 20 of ENADE 2017 – Licentiate Degree in Mathematics
Source: authors' own elaboration

QUESTÃO 138

Um morador de uma região metropolitana tem 50% de probabilidade de atrasar-se para o trabalho quando chove na região; caso não chova, sua probabilidade de atraso é de 25%. Para um determinado dia, o serviço de meteorologia estima em 30% a probabilidade da ocorrência de chuva nessa região.

Qual é a probabilidade de esse morador se atrasar para o serviço no dia para o qual foi dada a estimativa de chuva?

(A) 0,075
(B) 0,150
(C) 0,325
(D) 0,600
(E) 0,800

Figure 7: Question 138 of the gray test of ENEM 2017
Source: authors' own elaboration