



## Planning to teach and learn math: Opening of the lesson study

### Planejar para ensinar e aprender matemática: abertura de um estudo de aula

---

*Carolina Cordeiro Batista<sup>1</sup>*

*Rosa Monteiro Paulo<sup>2</sup>*

#### Abstract

The objective of this text is to bring aspects of the teacher's learning that is involved with the planning of a mathematics class, preparing tasks to be carried out with GeoGebra. The theme of the class is “relative position of lines” and the planning context is that of a group of three mathematics teachers from a full-time school in the state public system of São Paulo, constituted for the development of lesson study, a theme that we investigated in a doctoral research. Group meetings took place weekly for a year and a half. The teachers analyzed the possibilities of exploring the content with the GeoGebra and evidenced learning, such as the understanding that, with the application, students can mobilize knowledge to solve tasks and constitute their own way of understanding mathematical ideas. This understanding leads them to recognize the importance of giving the student the opportunity to carry out investigations, having, in the software, a possibility.

**Keywords:** Mathematical Education; Teacher Education; GeoGebra.

#### Resumo

O objetivo deste texto é trazer aspectos das aprendizagens do professor que se envolve com o planejamento de uma aula de matemática, elaborando tarefas para serem realizadas com o GeoGebra. O tema da aula é “posição relativa de retas” e o contexto do planejamento é o de um grupo de três professores de matemática de uma escola de Tempo Integral da rede pública estadual paulista que foi constituído para o desenvolvimento do estudo de aula, tema que se investigou em uma pesquisa de doutorado. As reuniões do grupo ocorreram semanalmente durante um ano e meio. Os professores analisaram as possibilidades de exploração do conteúdo com o GeoGebra e evidenciaram aprendizagens, como a compreensão de que, com o aplicativo, os alunos podem mobilizar conhecimentos para resolver as tarefas e constituir um modo próprio de entender as ideias matemáticas. Essa compreensão leva-os a reconhecer a importância de dar ao discente a oportunidade de fazer investigações tendo, no software, uma possibilidade.

**Palavras-chave:** Educação Matemática; Formação de professores; GeoGebra.

---

**Submetido em:** 04/01/2021 – **Aceito em:** 12/11/2022 – **Publicado em:** 30/12/2022.

<sup>1</sup> PhD in Mathematics Education from the Universidade Estadual Paulista - Unesp. Post-doctoral student at Unesp, scholarship from Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq, Brazil. Email: carolina.batista@unesp.br. ORCID: <https://orcid.org/0000-0002-0923-647X>

<sup>2</sup> PhD in Mathematics Education from the Universidade Estadual Paulista - Unesp. Professor at the Mathematics Department of the School of Engineering and Sciences of Guaratinguetá, Unesp, Brazil. Email: rosa.paulo@unesp.br. ORCID: <https://orcid.org/0000-0001-9494-0359>

## Introduction

Classroom study<sup>3</sup> is a teacher professional development practice that originated in Japan under the name *jogyokenkyuu* (Baptista, Ponte, Velez & Costa, 2014) as a result of changes in the Japanese educational system during the early period known as the Meiji Revolution (Felix, 2010; Souza, Wrobel & Baldin, 2018).

After its inception in that country, it became popular in the United States in the late 1990s with the publication of the book *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom* (Stigler & Hiebert, 1999). It subsequently expanded around the world and is currently developed in several countries such as China (Han & Huang, 2019); South Africa (Adler & Alshwaikh, 2019); Denmark (Winsløw, Bahn & Rasmussen, 2018); Portugal (Ponte, Quaresma, Mata-Pereira & Baptista, 2018); Chile (Estrella, Mena & Olfos, 2018) and Brazil (Felix, 2010; Souza et al, 2018; Batista, 2017; Curi, 2018; Richit, Ponte & Tomkelski, 2019).

The proposal is for teachers to meet in small groups to discuss aspects of their own practices and advance in terms of content knowledge and teaching strategies (Fiorentini, Morelatti & Bezerra, 2019). The topic of the discussions can be elected by the group itself, according to the interest of the majority. A scenario of collaborative work is therefore established, in which the actions to be developed by the teachers focus on the student (Ponte et al., 2015; Curi, 2018; Richit & Ponte, 2020), that is, the type of reasoning they intend to develop, the previous knowledge, the strategies present in solving tasks, etc. For Ponte et al. (2018), having a focus on the learner rather than the methodology, for example, is the main feature that differentiates lesson study from other teacher education practices.

The lesson study is organized in "cycles" (Lewis & Perry, 2015), each consisting of four stages: the study of content and teaching materials to define a question or theme of common interest to the group; the planning of one or more lessons; the development of the lesson by a teacher of the group, with the participation of the other members as observers; and the discussion of the aspects of the lesson that prove relevant (Ponte, Lent, Mata-Pereira & Baptista, 2016; Curi, 2018). It should be noted that the formative work is not exhausted after the development of the last stage, because with each experience lived, new ideas with other work possibilities may arise, starting new cycles. In this way, authors such as Fiorentini et al. (2019, p. 2) consider the lesson study a "process [that] allows successive cycles of action and reflection to occur which can promote the teacher's professional development."

Understanding the lesson study as a formative space, we constituted a group with three mathematics teachers from a Full-Time School of the state education network in a municipality in the interior of São Paulo. We formed a partnership with the school for the formation of these teachers, and the class study scenario was the environment in which the

---

<sup>3</sup> We have chosen to use the term lesson study in Portuguese (Ponte, Quaresma, Mata-Pereira & Baptista, 2015). However, *jogyokenkyuu* also has translations in several languages, such as in English - lesson study (first translation) - and Spanish - estudio de clases.

data of a doctoral research (Batista, 2021) were constituted. Following the steps of the lesson study, we held biweekly meetings with the group during the period of one and a half years (from August 2018 to December 2019). In the meetings, we discussed the teaching of mathematics with technologies, more specifically, with the GeoGebra software. Moreover, the planning of lessons for six different themes was made and developed by the teachers and discussed in the group.

The objective in the formative actions of the class study was to give the teacher the possibility of understanding himself as a teacher with technology, experiencing situations in which mathematical content tasks were explored with software: GeoGebra. This formative action of the class study was conceived and conducted for the doctoral research in which we investigated: "how does the mathematics teacher perceive him/herself as a teacher with technology?"

As we analyzed the actions developed with the teachers, we saw that the "how" of the questioning, that is, the ways in which they were realizing the possibility of teaching with technology, evidenced different understandings that we are characterizing as learning. For this text, we focus specifically on the planning stage, which offers us conditions to expose: "what learnings of the teacher are shown in the planning of a lesson in which math tasks will be performed using the GeoGebra software?"

### **Planning: opening up possibilities for teaching and learning**

We understand that the four stages of the lesson study are intertwined and enhance different aspects of the teacher's teaching practice. Therefore, highlighting in the text one of these stages for the dialog does not mean to isolate it from the process, nor to emphasize it. We do this to talk about the experience lived with the teachers in our group and what was understood in the planning stage.

Lewis and Perry (2014) say that this stage of lesson study helps teachers to know how the content is present in the teaching materials and favors the development of the ability to negotiate points of view. We understand that this happens because planning is the stage in which teachers expose their practice, opening up to dialogue, discussing ideas and ways of conducting classroom actions, always considering student learning.

The way the lesson is planned is fundamental to the opening of possibilities for the constitution of teachers' knowledge and has an influence on the following stages, and can instigate the results and the very participation of the group members in the cycle. Thus, much attention is paid to this stage in the lesson study, which justifies the largest number of meetings with the group being devoted to it (Ponte et al., 2018; Fujii, 2016). In addition, research points out that teachers recognize that the appreciation given to planning causes it to be developed in a differentiated way, because, in addition to the development of tasks, the opportunity is given to explore the different ways of working with the same theme (Richit & Ponte, 2020). Thus, to turn to the planning stage is to seek to understand its particularities and see how it can contribute to the teacher's professional development.

It is in planning that the group organizes the tasks to work with the chosen theme, always taking into account that they will be developed by one of the teachers in one or more classes. Fujii (2018) says that, normally, the discussions begin by focusing on the tasks: which ones are more appropriate to the content; which skills can be developed; etc. However, in the course of the actions, the group directs the dialogue to other questions that go beyond "what to do" and move in the direction of "how to do" and "why to do".

Richit and Ponte (2020, p. 22) state that "planning consists in establishing a learning path of mathematics for students". Curi (2018, p. 21) emphasizes that by establishing this path, teachers begin to understand the importance of considering students' knowledge constitution, creating strategies for them to engage with the tasks, taking into account their prior knowledge, which becomes an important element in conducting the lesson. This organization and planning of future actions are guided by practices already developed, by the lived experience of these teachers in the classroom context.

Fujii (2016) reminds us that planning is also the moment of elaboration of a plan with description of details about what will be worked in the class, as well as indications of ways to expose the content, which reveals a concern with the record, which will be relevant for the teacher's work in the classroom.

Planning becomes an important space for dialogue and for preparing the teacher for his or her classroom practice, but, in the context of class study, this does not exhaust it. Considering planning beyond the concerns of "what to do" and "how to do it", one moves in the direction of "why to do it", highlighting the horizons of learning, not only of the student, but also of the teacher. In the class study, to plan is to learn, to explore the possibilities that are shown in the tasks developed by the group and that say something about the content to be learned and the way in which learning can take place.

The group discussions move in the direction of surveying the probable solutions that the students present for the proposed tasks and "predict" possible answers. This path leads teachers to look at the class as a whole, and at the individual student. Learning rhythms and characteristics that influence students' ways of understanding what is proposed to them are considered (Fujii, 2018).

For Richit et al. (2019, p. 58), this is "judicious collaborative work in which one seeks to predict students' ways of thinking, their strategies for solving proposed tasks, their difficulties, what they will say during class activities, etc." It is the moment when teachers collaboratively set out to solve the tasks, raising possible questions and trying to see what will lead students to make mistakes, identifying factors that generate difficulties in solving the task or favor the understanding of the content covered. In this respect, teaching experience and living in the classroom context are essential for planning, while the dialogue between teachers can bring very rich suggestions for strategies.

With regard to the content, Ponte et al. (2018) point out that the development of planning in the context of classroom study requires significant work in terms of mathematics,

which can be a challenge for the group, because in order to be able to predict possible problem-solving strategies, teachers need a deeper understanding of the mathematical content required to the subject and, also, discussions that, beyond teaching and student learning, involve the teacher's own learning. To subsidize the group's studies, several resources are considered, such as: the curriculum; the materials that teachers use to teach (textbooks, for example); research on the subject; and other means that favor "learning" mathematical content (Ponte et al., 2018). Such resources can also be significant for the group to design more challenging tasks, such as problem-solving or inquiry and exploration tasks (Takahashi & Mcdougal, 2016; Ponte et al., 2018; Richit & Ponte, 2020).

This openness so that the teacher can turn to the ways of thinking of students to learn new content and design more challenging tasks makes the planning of the lesson study different from other ways of planning a lesson. Even if this openness does not allow to foresee all the situations that may happen in the classroom, there is potential for understanding the ways in which the chosen tasks favor the constitution of knowledge in the students, since it gives the teacher the opportunity to turn to this constitutive process.

Thus considered, the planning sphere in the class study is a space for learning and organizing actions, which gives importance to the presence of a person from outside the group who can take charge of the "provocations", that is, someone to foster discussions, encourage dialogue, contribute with study materials, clarify doubts of the teachers, or propose ways to find solutions. This person can be in all cycles of the class study, however, it is fundamental that he or she be in the moments when dialogues are more evident, such as in the planning actions. This person may be someone who knows the steps of the lesson study by having lived them in previous experiences (Scheller, Ponte & Lent, 2019) or a researcher-trainer from a university (Curi; 2018; Batista, 2017) who is well acquainted with the proposal.

Scheller et al. (2019) state that this trainer can develop actions such as: inviting, encouraging teachers to participate in the discussion; informing/suggesting by complementing or extending the ideas expressed; supporting/guiding them to justify the suggestions given or the explanations made; and challenging teachers to advance in terms of proposals and their own knowledge of the content.

The presence of the trainer can contribute to the establishment, in the group, of a reflective attitude that fosters professional development. In this text, we focus on the context of the work with a group of teachers who, following the lesson study model, got involved with planning a lesson to teach relative position of lines.

## **Methodological procedures and the context of the training group**

As mentioned, the data discussed in this text are part of what was constituted for a doctoral research (Batista, 2021). In the research, a qualitative approach with a phenomenological stance was taken to conduct the actions and analyze the data. This perspective allows one to explore the "nuances of the ways in which quality is shown and

understandings and interpretations are made explicit so that it is not possible to generalize or transfer the constituted data to other contexts" (Bicudo, 2011, p. 21), but to clarify what is understood in the experience of the situations with the teachers of the group. What was understood in the research highlights the meaning that, for us, has the question: "how does the mathematics teacher perceive himself as a teacher with technology?"

To account for the questioning, that is, to make it possible to understand what we asked in the research, the class study showed itself as a way for teachers to develop and discuss the teaching of mathematics with digital technologies. In the sequence of this text, we will call the teachers who participated in our group as: Euclides, Luciana and Leonardo. They are mathematics teachers from a Basic Education school, linked to the Board of Education of a city in the interior of the state of São Paulo. The group meetings occurred weekly, on Thursdays, from the beginning of the second semester of 2018 to the end of the second semester of 2019. The participation of the researcher-trainer took place every other week. In the research, six lesson study cycles were carried out, each of which had about seven meetings of approximately one and a half hours duration, and four of them were dedicated to the lesson planning stage. We made video recordings of the meetings with the teachers and transcribed the recordings, which constituted the research data. For this text, we bring a cut of what was shown in the planning stage of the fourth cycle. We remind you that the intention is to explain: "what learning experiences do teachers show when planning a lesson in which math tasks will be performed using GeoGebra software? We understand that, in the planning stage, while the teacher is willing to teach, considering "what" he will develop in the classroom and "how" this will be done, he is equally willing to learn. In the group, with the class study, there is sharing, there is a certain way of being together - teacher-teacher, teacher-researcher, teacher-content - focused on understanding what is done and, in the planning, the dialog highlights the learning.

The GeoGebra software was the technology chosen by the group to develop the tasks, since the teachers were familiar with it, as they had participated in training courses offered by the partnership between the São Paulo State University (Unesp) and the city's Board of Education. The theme "relative position of lines" was suggested by teacher Euclides, who intended to perform the tasks with his 3rd year high school students in a recovery activity.

### **The group in action: lesson planning**

The topic "relative position of lines" had already been worked on with Professor Euclides' 3rd grade classes in the 1st school bimester of 2019. However, when analyzing the results of the bimonthly assessments, the teacher considered that the students had not understood the topic. Therefore, the suggestion to work in the class study group was with the intention of developing a plan for recovery in this class. The classmates agreed with Euclides' arguments, and the theme was defined. They organized to begin planning meetings in the month of May 2019.

The lesson planning was initiated by discussing the "type" of task that would be feasible to explore the topic with GeoGebra software. As a starting point, the group decided



to analyze the skills that were described in the curriculum and defined that the development of the skill of recognizing "conditions that ensure parallelism and perpendicularity between lines" (SEDUC-SP, 2011, p. 69) would be the goal to be achieved. The group also investigated the way the content was worked in the students' material - the Student's Notebook (SEDUC-SP, 2014) - and in the textbook, but chose not to use ready-made or equal tasks from these materials in the explorations they intended to propose. Also, before starting to prepare the tasks, the teachers considered it important to analyze the difficulties they identified in the evaluation the teacher had conducted with their students.

The three teachers actively participated in the discussions and assumed different postures. The researcher-trainer helped them with the functionality of the GeoGebra software tools and suggested different ways to explore the same content with the different tools. Sometimes she tried to challenge the teachers with questions that would lead them to analyze the viability of their suggestions for the task. The teachers organized the task with autonomy to choose the questions they wanted to propose and conjecture about the possible students' difficulties and solution strategies.

At first, the teachers were unsure about the way in which they should ask the students for their first explorations. Then they understood that this would be defined by their experience with the class and the knowledge of the students. Euclid suggested that they start by constructing parallel lines using the software tool "Parallel Line". Luciana argued that they could ask the students to construct any lines, leaving them free to move them in an attempt to make them parallel. Both teachers presented justifications for the suggested options, but could not define the possible explorations for each of the constructions, nor conclude which one would be more relevant for understanding the content. Seeing that the teachers could not move forward, the researcher-trainer intervened for the first time in the discussion, explaining her way of understanding each of the suggestions

Researcher Trainer: Suppose, if you construct a line parallel to another given line, with the tool "Parallel Line", they have the same slope. You could, for example, construct a line parallel to the x-axis, passing through a point A and another [line], parallel to the x-axis, passing through C [point], and then measure the inclination [of the lines] /.../ In this case, you will show that they have the same inclination and that they are parallel. Now, if you want a construction other than using the parallel [tool], you can show them lines with different slopes. For example, this one [constructed any two lines and measured the angle of inclination of the two lines]! You can see from the inclination that they are not parallel lines.

The intention was that teachers understood that the choice of construction should be guided by the content they wanted to explore with students. That is, if the objective was clear - recognizing conditions that ensure parallelism and perpendicularity between lines  $\neg$ , the choice of construction should be the one that best meets the objective. In the situation considered, the construction made using the "Parallel Line" tool would allow exploring the equality of the measure of the inclination angle. On the other hand, the choice of constructing any straight line would have equality between angles as a condition. That is, the characteristics of the constructed lines could be explored to define their relative position: parallel or competing (including the distinction for perpendicular).

In the discussion, the teachers considered that the construction using any straight lines would be more feasible for the objective, since, in addition to providing an opening to perform further explorations, it would also be an opportunity to see if students had difficulty identifying the characteristics that should be observed in the construction for the position of the lines to meet what was requested (in this case, to be parallel). Therefore, they chose to propose to the students the construction of a line  $r$  passing through two given points and, in the sequence, the construction of a line  $s$ , parallel to  $r$ , without specifying the mode of construction.

Once the starting point of the task was defined, the teachers made the construction in the software to organize the questions they would ask and conjecture ways for the student to solve the task. For example, Leonardo constructed any two lines, one passing through points A and B and the other passing through points C and D, and began to question what the students would do in such a situation.

Professor Leonardo: There, look! What he [student] is going to do. If he keeps doing this here [moving the two lines to a position where the lines appeared to be parallel].

Teacher Luciana: He will do just that.

Teacher Leonardo: Okay, but is it [a straight line] parallel? Then you...

Professor Euclid: [Moving ahead of him] Then you look [I mean, click] here on the "Angle" tool.

Professor Leonardo: Or you can do this here, look [zooming in]! You see? Start bringing it over here, look [dragged the lines to make a point of intersection visible on the screen]!

Researcher: We already saw that it is not parallel. This [zoom out] you [Euclid] can do.

Professor Leonardo: You keep doing it until you find this point of intersection. You find a way to make him see that it was not so parallel [laughs].

Leonardo believes that the students' first initiative would be to build any two lines and move them to a position where they "seem" parallel, suggesting that it is up to the teacher to create strategies that lead the student to verify if they are "really" parallel. For him, one possibility would be to use zoom, making more points of the line visible until it was possible to "see" a point of intersection. In the discussion that followed in the group, it was seen that this point could be found through the tool "Intersection of two objects", since by clicking on the two lines, if they were not parallel, the software would create a point of intersection.

Professor Leonardo: Hey! Another way to see if the two are parallel or not [...] we can say: what are two parallels? They don't intersect. So, I used this tool here, I asked for the intersection of this line [AB] with this line [CD] to see if a point appears, if a point appears it is because they are not parallel. So, they are not, do you agree? You can even tell them: do you agree with me? That if the two were parallel they wouldn't intersect. And there [in the algebra window] it shows the intersection, it means that at some point they intersect, so they are not parallel.

The teacher further complemented his explanation by considering that, in case the lines are parallel, when clicking on them asking for the intersection, GeoGebra would not be



able to find a point. In the dialog, Euclides reminded us that the tool "Angle" could also be a way to verify the parallelism between the lines. According to his argument, with this option, students could measure the inclination angles of the lines (in relation to the x-axis, for example) and leave the measurements displayed on the screen to move the lines and analyze the variation until they obtain lines whose inclination angles are the same. Leonardo, attentive to his colleague's words, showed interest in the suggested exploration and clicked on the "Angle" tool to measure the angle of inclination of the line AB (relative to the x-axis) and obtained the result shown in Figure 1.

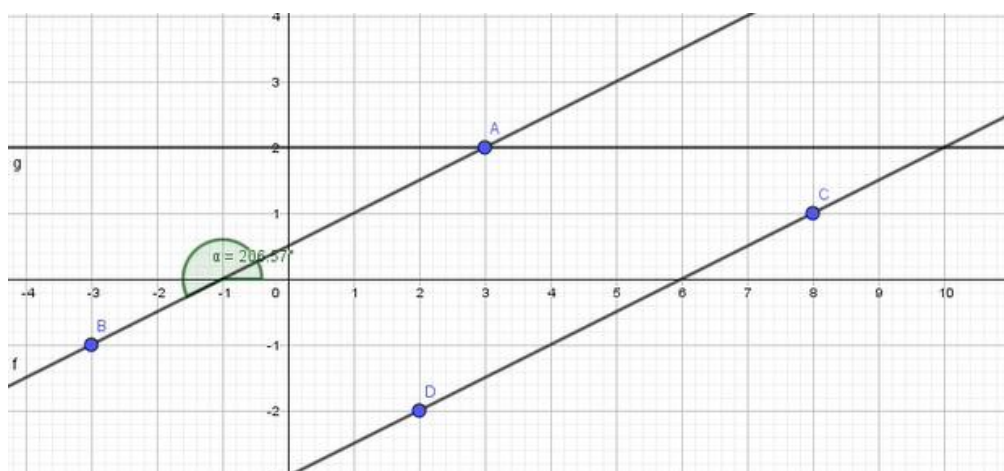


Figure 1: Straight lines AB and CD.

Source: Screen print of figure prepared by Professor Leonardo.

In the discussion, they noted that they had not correctly measured the desired angle, but that, by the measure presented, it would be possible to find the desired angle: "So [...] as he is measuring, just make the difference, right? [...] from the total measure of that angle take 180 [degrees] and take away" (Teacher Leonardo, 2019). That is, taking advantage of the mistake made when selecting the points to measure the desired angle, the teachers noted that if this was also a procedure done by the student, instead of asking him to erase and try again, it would be possible to challenge him to find the size of the desired angle from the measurement shown on the screen. Considering the example in Figure 1, the intention was to get students to observe that by subtracting from the total measure - 206.57 - the measure of the shallow angle (line AB), they would get the desired angle.

While analyzing Professor Leonardo's suggestion, Euclid and Luciana found the "Slope" tool, which indicates the tangent of the angle or the angular coefficient of the line. They clicked to measure the angle of inclination of the line AB (Figure 2) and were surprised by the value they found. Understanding what the tool was giving them, they began a dialog about possible explorations if the student used this option to solve the proposed task.

DOI: 10.20396/zet.v30i00.8663821

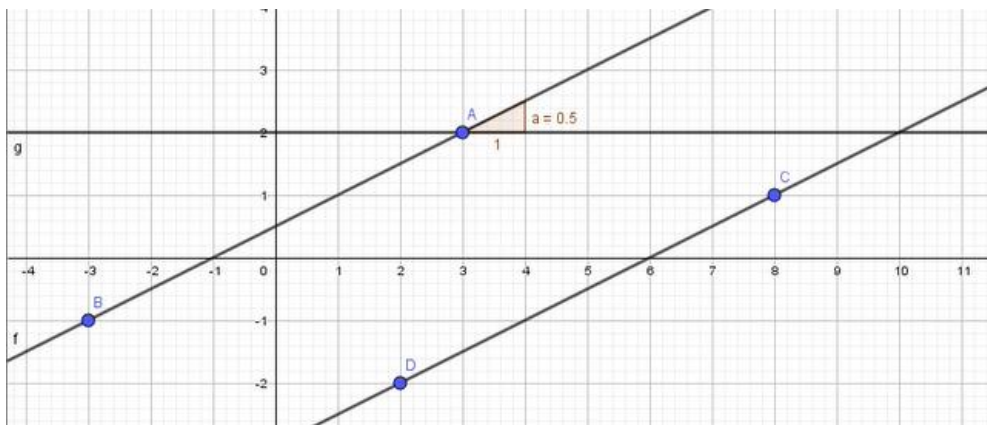


Figure 2: Measurement of the tangent of the angle.

Source: Screenshot of figure prepared by Professor Leonardo

Professor Euclid: There may be students who will find this tool [slope] [...] Right! I will ask them what is the relationship of the angular coefficient of this line [AB] to the other line here [CD] and they do an investigation. That would be a way too! Or not?

Teacher Luciana: Have they ever seen an angle coefficient? Do they know how to calculate?

Teacher Euclid: They can calculate by the two points and by the slope, when there is a measure of the angle. They can investigate, right?

Researcher: Yes [...] Let's think that we are the students doing it. What would be the possibilities for us to investigate?

Despite the initial surprise, Euclides considered that if the students found the value of the tangent instead of the measure of the angle, this would not be a problem, because he could still ask them to find the relationship between the angular coefficients of the two lines. In response to the colleague's speech, Luciana asks if the content "angular coefficient", which at first was not planned to be discussed in class, had already been worked on with the students. The teacher's question indicates a concern to identify if the students have the necessary prerequisites to find the relation suggested by the teacher, making the proposed task possible. When the teacher confirms that the students have the prerequisite and informs the ways in which they learned to calculate the angular coefficient of the line, the researcher-trainer intervenes suggesting that they analyze the possibilities of investigation based on the contents mentioned.

After thinking for a few moments, Luciana suggested a possibility of investigation: "if the coefficients are equal and the lines are parallel, [the students] can measure the angle of inclination and show that the tangent of those two angles is worth so much and that this value is the angular coefficient [of the line]. You can think about that too!" (Luciana, 2019). For her, this would be an opportunity for the student to see that the tangent is the angular coefficient of the line. She considered that students could compare the equations of two parallel lines, as they are displayed in the algebra window of the software, to find what these equations have in common. She states, "Then they will compare the two equations of the line and they will see what they have that is the same: the angular coefficient" (Luciana, 2019).

Moving on with the discussion, Leonardo considered that the tools explored make it possible to investigate other relative positions between straight lines and could create questions for students as they progressed through the task.

Teacher Leonardo: We can explore the positions. What straight line did you make? Ahh, those are crossing each other. What do you call them? They are competitors. Then you can get to this: find the coefficient of one and find the coefficient of the other [...] if everybody draws [lines] crossing, then you can show what happens to the angular coefficient of the two when they are competing [...] I think most people will do just that [...]

Professor Euclides: Then he can also use the angle tool, right? Ahh! 90 degrees, so it is perpendicular [...]

Researcher-Trainer: Yes, or you can ask him, but how can you guarantee that this line is perpendicular? He will have to think.

The dialogue in the group shows that the teachers consider that, even though it is not possible to predict exactly what the student will do during the task, if they are attentive, they can develop questions that allow them to advance with the exploration. It is noticeable that the teachers get involved in the explorations, advance with the survey of possibilities to treat the theme, but do not return to the most trivial construction mode. Again, there is intervention from the researcher-trainer.

Trainer: There can always be a [student] who goes for the "Parallel Line" or "Perpendicular Line" tool [...] if he wants to go straight to those tools he can say: Teacher, it's parallel because it's saying here [GeoGebra] that it's parallel [...] In that case, is the exploration over?

The teachers did not respond promptly to the questioning and remained a few minutes looking for an answer that would not end the possibilities of exploration planned for the lesson. Then, they considered that if this occurred, they should ask the students for some explorations that would lead them to highlight the characteristics of these lines, i.e., they thought about asking what allows classifying these lines as parallel or perpendicular. They also considered asking the students to find other ways of construction that did not use the "Parallel Line" or "Perpendicular Line" tool, instigating them to discuss what held in the different constructions.

In the course of the discussions, the concern with registration became explicit. Euclides wrote down in a notebook the "steps" of the explorations raised by the group and reported, "I'm making this script here to take to class" (Euclides, 2019). However, as they progressed with the elaboration of the questions, they understood that there was an opening to create new questions in the course of the student activity, and this allowed the researcher-trainer to ask them if a script would be useful to guide the work with technology.

The group argued that the development of a script was a way to make the teacher more confident in conducting the lesson with technology. However, they recognized that the preoccupation with following the script could take their attention away from the students' explorations, mischaracterizing the purpose of the work they were seeing in the class study, that is, the focus on the learner. Along the way, the idea of the script with the description of

the steps of the explorations was replaced by the elaboration of a document that the teachers named "Teaching action plan for intensive recovery - 2019".

They established that this document should contain: information about the skill to be developed - which they considered as the goal of the lesson -; the topic/object of knowledge; the date of the lesson; information about the gaps identified in the students' assessments; the difficulties that could arise due to the exploration - for example, when using the "Tilt" tool  $\rightarrow$  ; the goal of the recovery; and a brief description of the main explorations raised by the group. They also organized the class in two moments: the first with the explorations that would lead students to understand conditions of parallelism between lines; and in the second, the explorations to characterize the conditions of perpendicularity between lines would be systematized.

The teacher Euclides commented that he would try to fulfill the planned objective for the class, trying not to "stick" to what was written in the plan, but asked the other teachers to be attentive during the observation of the class, drawing his attention to exploits that were not perceived by him.

After finalizing the planning, the teachers decided that the lesson would be developed the next day, in teacher Euclides' 3rd grade B class, which, according to him, was the one showing the greatest difficulty in relation to the theme.

## Final Considerations

The experience lived with these teachers during the lesson planning showed that they were involved in the discussions and worked collaboratively to develop tasks and analyze possibilities of exploring mathematical content with GeoGebra software. In view of what has been discussed, to explain the question: "what teacher learning is shown in the planning of a lesson in which mathematical tasks will be performed using GeoGebra software?", it is evident the way in which teachers highlight the contents that draw their attention as a result of the option of building on the software. These contents are both mathematical ideas and aspects of the teaching of this subject. Choosing the "best way" to build in the software becomes a minor issue, since whatever it is, it opens up possibilities for exploration. This is a learning that is announced: the teacher understands that, by giving his class the freedom to build, several fronts open up and he must be attentive to be able to lead the discussions.

It is also shown that, when planning the lesson, it is necessary to have a defined objective, because this is what guides the actions. If we want the student to be able to characterize the parallelism between lines, it must be clear what the conditions are, because this will give subsidies to guide the students' explorations. Although prior knowledge is invoked as a requirement for a certain task, in the discussion it takes on a different perspective, since it can be "together", learned in the exploratory process.

In order to understand the possibilities of exploration, the teachers made the constructions in the software themselves. This process was important, leading them to

understand why, in class study, so much attention is drawn to the procedures adopted by the students (Ponte et al., 2015; Curi, 2018; Richit & Ponte, 2020). They saw that, as they explore the possibilities of constructions in the software, different ideas show themselves. These ideas, shared in the group, allow them to raise likely questions, analyze potential answers, and consider resolution strategies that learners might come to mobilize. Involved with the planning of a class in which the students' actions are encouraged to be considered, the teachers were able to negotiate routes that seemed relevant to discuss the mathematical content involved in the construction.

It was also possible to understand that in the exploration of the constructions made the knowledge is explicit, both in relation to the content and the software functionalities. The ways of teaching and learning in an environment in which one is with the software are experienced and analyzed by the teachers. It shows a way of conducting the class that is different from the actions already experienced, provoking them to new attitudes and new looks at the students' attitude and their involvement with the proposed task. This way of being a teacher makes it possible to understand that technology can be relevant to the construction of strategies, as well as to the search for mathematical solutions.

In addition, the presence of the researcher-trainer, who was together developing actions, supporting them in their decisions, showing solidarity with their difficulties and contributing with explanations and suggestions, made them feel more secure to work with the students. Participating in the explorations and facing the proposed challenges made the teachers think about the students' previous knowledge, the abilities enhanced by the task developed, and the possibility of arising unanticipated issues that could be dealt with during the lesson. Everyone's support strengthened the group and made them engaged in the discussions, contributing to advance in the elaboration of tasks and develop them with the students.

To conclude this text, we want to expose that, considering the dynamics of the student's activity when being with the software, the planning of a lesson with technology, however careful it may be, is not able to predict the "steps" to be followed by the teacher in the course of an entire class. This was understood by the group and, despite this or motivated by this, they saw the relevance of, in the lesson study, having the planning stage whose objective is not to predict exactly the students' actions, but to ensure the success of the teacher's practice, understood as the possibility of leading the student to think about what is being done, to question, to understand, and to be able to expose what is being done.

Thus, in line with what Richit and Ponte (2020) emphasize, the importance of planning in a formative process in which teachers realize that a lesson with technologies cannot be developed through instructions to be followed by students as a procedural script. The purpose of planning is the willingness of the teacher to learn with the student, to access new possibilities for investigating the task, allowing the production of knowledge of the contents that, in the process of exploration, will emerge. It is not a moment without the control of the teacher; it is a class in which the freedom of investigation gives the teacher the opportunity to explore mathematics with his or her students. Perhaps this is the learning that is desired for the entire formative process.



## References

- Adler, J., & Alshwaikh, A. (2019). Case of Lesson Study in South Africa. In R. Huang, A. Takahashi & J. P. Ponte (Eds.), *Theory and Practice of Lesson Study in Mathematics* (pp. 317-342). Suíça: Springer. Retirado em 10 de julho, 2020, de: [https://www.researchgate.net/profile/Jehad\\_Alshwaikh](https://www.researchgate.net/profile/Jehad_Alshwaikh).
- Baptista, M., Ponte, J. P., Velez, I., & Costa, E. (2014). Aprendizagens profissionais de professores dos primeiros anos participantes num estudo de aula. *Educação em Revista*, 30(4), 61-79. Disponível em: <http://www.scielo.br/pdf/edur/v30n4/04.pdf>.
- Batista, C. C. (2017). *O Estudo de Aula na Formação de Professores de Matemática para ensinar com tecnologia: a percepção dos professores sobre a produção de conhecimento dos alunos*. 109f. Dissertação de Mestrado em Educação Matemática. Rio Claro: Universidade Estadual Paulista “Julio de Mesquita Filho”. Retirado em 23 de janeiro, 2020, de: [https://repositorio.unesp.br/bitstream/handle/11449/152467/batista\\_cc\\_me\\_rcla.pdf?sequence=3](https://repositorio.unesp.br/bitstream/handle/11449/152467/batista_cc_me_rcla.pdf?sequence=3).
- Bicudo, M. A. V. (2011). A pesquisa qualitativa olhada para além dos seus procedimentos. In M. A. V. Bicudo (Org.), *Pesquisa qualitativa segundo uma visão fenomenológica* (pp. 11-28). São Paulo: Editora Cortez.
- Curi, E. (2018). Grupo de Pesquisa Colaborativo: espaço para promoção do desenvolvimento profissional docente. In E. Curi, J. C. P. Nascimento & J. P. Vece (Orgs.), *Grupos Colaborativos e Lesson Study: contribuições para a melhoria do ensino de matemática e desenvolvimento profissional de professores* (pp. 17-33). São Paulo: Alexa Cultural.
- Estrella, S., Mena, A., & Olfos, R. (2018). Lesson Study in Chile: A Very Promising but Still Uncertain Path. In M. Quaresma, C. Winslow, S. Clivaz, J. P. Ponte, A. N. Shúilleabháin & A. Takahashi (Eds.), *Mathematics Lesson Study Around the World: Theoretical and Methodological Issues* (pp. 105-122). Suíça: Springer. Retirado em 9 de julho, 2020, de: [https://www.researchgate.net/publication/323960799\\_Lesson\\_Study\\_in\\_Chile\\_A\\_Very\\_Promising\\_but\\_Still\\_Uncertain\\_Path](https://www.researchgate.net/publication/323960799_Lesson_Study_in_Chile_A_Very_Promising_but_Still_Uncertain_Path).
- Félix, T. F. (2010). Pesquisando a melhoria de aulas de Matemática seguindo a proposta curricular do Estado de São Paulo, com a metodologia da pesquisa de aulas (lesson study). Dissertação de Mestrado em Ensino de Ciências Exatas. São Carlos: Universidade Federal de São Carlos. Retirado em 23 de junho, 2020, de: <https://repositorio.ufscar.br/handle/ufscar/4412?show=full>.
- Fiorentini, D., Morelatti, M. R. M., & Bezerra, R. C. (2019). Apresentação Dossiê: Lesson Study em Matemática. *Revista Educere et Educare*, 14(32), 1-5. DOI: 10.17648/educare.v14i32.23708. Disponível em: <http://e-revista.unioeste.br/index.php/educereeteducare/article/view/23708>.
- Fujii, T. (2016). Designing and adapting tasks in lesson planning: A critical process of lesson study. *ZDM Mathematics Education*, 48(4), 411-423. Retirado em 23 março, 2020, de: [https://www.researchgate.net/publication/298213782\\_Designing\\_and\\_adapting\\_tasks\\_in\\_lesson\\_planning\\_a\\_critical\\_process\\_of\\_Lesson\\_Study](https://www.researchgate.net/publication/298213782_Designing_and_adapting_tasks_in_lesson_planning_a_critical_process_of_Lesson_Study).
- Fujii, T. (2018). Lesson Study and Teaching Mathematics Through Problem Solving: The Two Wheels of a Cart. In M. Quaresma, C. Winslow, S. Clivaz, J. P. Ponte, A. N.



- Shúilleabháin & A. Takahashi (Eds.), *Mathematics Lesson Study Around the World: Theoretical and Metodological Issues* (pp. 1-11). Suíça: Springer.
- Han, X., & Huang, R. (2019). Developing Teachers' Expertise in Mathematics Instruction as Deliberate Practice through Chinese Lesson Study. In R. Huang, A. Takahashi & J. P. Ponte (Eds.), *Theory and Practice of Lesson Study in Mathematics* (pp. 59-82). Suíça: Springer.
- Lewis, C., & Perry, R. R. (2014). Lesson Study with mathematical Resources: A Sustainable Model for Locally-led Teacher Professional Learning. *Mathematics Teacher Education and Development*, 16(1), 22-42. Retirado em 19 de dezembro, 2020, de: <https://files.eric.ed.gov/fulltext/EJ1046670.pdf>.
- Lewis, C., & Perry R. R. (2015). A Randomized trial of Lesson Study with Mathematical Resource Kits: Analysis of Impact on Teachers' Beliefs and learning Community. In J. Middleton, J. Cai & S. Hwang (Eds), *Large-Scale Studies in Research in Mathematics Education*, (pp. 133-158). Suíça: Springer. DOI: [https://doi.org/10.1007/978-3-319-07716-1\\_7](https://doi.org/10.1007/978-3-319-07716-1_7).
- Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2015). Exercícios, problemas e explorações: Perspectivas de professoras num estudo de aula. *Quadrante*, 24(2), 111-134. Disponível em: [http://repositorio.ul.pt/bitstream/10451/22628/1/Ponte,%20MQ,%20JMP,%20MB%20Quadrante%2024\(2\)%202015.pdf](http://repositorio.ul.pt/bitstream/10451/22628/1/Ponte,%20MQ,%20JMP,%20MB%20Quadrante%2024(2)%202015.pdf).
- Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2016). O estudo de aula como processo de desenvolvimento profissional de professores de matemática. *Bolema*, 30(56), 868-891. Retirado em 19 de dezembro, 2020, de: [https://www.scielo.br/scielo.php?pid=S0103-636X2016000300868&script=sci\\_abstract&tlng=pt](https://www.scielo.br/scielo.php?pid=S0103-636X2016000300868&script=sci_abstract&tlng=pt).
- Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2018). Fitting Lesson Study to the Portuguese Context. In M. Quaresma, C. Winslow, S. Clivaz, J. P. Ponte, A. N. Shúilleabháin & A. Takahashi (Eds.), *Mathematics Lesson Study Around the World: Theoretical and Metodological Issues* (pp. 87-104). Suíça: Springer. Retirado em 9 de julho, 2020, de: [https://www.researchgate.net/publication/323960988\\_Fitting\\_Lesson\\_Study\\_to\\_the\\_Portuguese\\_Context](https://www.researchgate.net/publication/323960988_Fitting_Lesson_Study_to_the_Portuguese_Context).
- Richit, A., Ponte, J. P., & Tomkelski, M. L. (2019). Estudos de aula na formação de professores de matemática do ensino médio. *Revista Brasileira de Estudos Pedagógicos*, 100(254), 54-81. Retirado em 10 de julho, 2020, de: [https://www.scielo.br/scielo.php?pid=S2176-66812019000100054&script=sci\\_arttext](https://www.scielo.br/scielo.php?pid=S2176-66812019000100054&script=sci_arttext).
- Richit, A., & Ponte, J. P. (2020). Conhecimentos Profissionais evidenciados em Estudos de Aula na perspectiva de Professores Participantes. *Educação em Revista*, 36, 1-29. Retirado em 10 de julho, 2020, de: [https://www.scielo.br/scielo.php?script=sci\\_arttext&pid=S0102-46982020000100201](https://www.scielo.br/scielo.php?script=sci_arttext&pid=S0102-46982020000100201).
- Scheller, M., Ponte, J. P., & Quaresma, M. (2019). O Formador na Condução de Sessões de um Estudo de Aula. *Revista Educere et Educare*, 14(32). Retirado em 10 de julho, 2020, de [https://www.researchgate.net/publication/337772913\\_O\\_FORMADOR\\_NA\\_CONDUC](https://www.researchgate.net/publication/337772913_O_FORMADOR_NA_CONDUC)

## AO\_DE\_SESSOES\_DE\_UM\_ESTUDO\_DE\_AULA.

- SEDUC-SP - Secretaria da Educação do Estado de São Paulo. (2011). *Currículo do Estado de São Paulo: Matemática e suas tecnologias*. Secretaria da Educação: São Paulo. Disponível em: <https://www.educacao.sp.gov.br/a2sitebox/arquivos/documentos/238.pdf>.
- SEDUC-SP - Secretaria da Educação do Estado de São Paulo. (2014). *Material de apoio ao Currículo do Estado de São Paulo: Caderno do Aluno – Matemática, Ensino Médio, 3ª série*. Secretaria da Educação: São Paulo.
- Souza, M. A. V. F., Wrobel, J. S., & Baldin, Y. Y. Lesson Study como Meio para a Formação Inicial e Continuada de Professores de Matemática – Entrevista com Yuriko Yamamoto Baldin. *Boletim Gepem*, (73), 115-130. Retirado em 5 de maio, 2020, de: <http://doi.editoracubo.com.br/10.4322/gepem.2018.020>.
- Stigler, J., & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. New York: The Free Press.
- Takahashi, A., & Mcdougal, T. (2016). Collaborative lesson research: maximizing the impact of lesson study. *ZDM - International Journal on Mathematics Education*, 48, 513-526. Disponível em: <https://link.springer.com/content/pdf/10.1007%2Fs11858-015-0752-x.pdf>.
- Winsløw, C., Bahn, J., & Rasmussen, K. (2018). Theorizing lesson study: two related frameworks and two Danish case-studies. In M. Quaresma, C. Winslow, S. Clivaz, J. P. Ponte, A. N. Shúilleabháin, & A. Takahashi (Eds.), *Mathematics Lesson Study Around the World: Theoretical and Metodological Issues* (pp. 123-142). Suíça: Springer. Retirado em 10 de julho, 2020, de: [https://www.researchgate.net/publication/323960989\\_Theorizing\\_Lesson\\_Study\\_Two\\_Related\\_Frameworks\\_and\\_Two\\_Danish\\_Case\\_Studies](https://www.researchgate.net/publication/323960989_Theorizing_Lesson_Study_Two_Related_Frameworks_and_Two_Danish_Case_Studies).