DOI: 10.20396/zet.v30i00.8665149

Abandonment of the teaching of geometry and Modern Mathematics: a historical review

Abandono do ensino de geometria e a Matemática Moderna: uma revisão histórica

Maria Célia Leme da Silva[1]

Abstract

This article aims to present the expansion of the HEM field and some research results developed during the 21st century on the teaching of geometry and the MMM, articulated, in order to problematize the representation built in the 20th century. The representation that the abandonment of geometry teaching is one of the consequences of the MMM can be revised, based on new theoretical-methodological contributions and research sources. A rereading indicates the need to review the abandonment, identifying a cooling of deductive geometry, questioned in the 1950s. The appropriations were multiple, with emphasis on proposals that sought a more didactic and comprehensive approach. We argue and defend with research results that such representation is no longer relevant considering the new HEM contributions. Understanding the complexity of the geometry teaching trajectory in the MMM period, through historiographical contributions, will certainly allow us to reconsider the representation of the abandonment of geometry.

Keywords: History of School Geometry; High school; Geometry of Transformations, Deductive Geometry.

Resumo

O presente artigo tem por objetivo apresentar a expansão do campo da HEM e alguns resultados de pesquisas desenvolvidas durante o século XXI sobre o ensino de geometria e o MMM, articuladamente, de modo a problematizar a representação construída no século XX. A representação de que o abandono do ensino de geometria seja uma das consequências do MMM pode ser revisada, a partir de novos aportes teóricos-metodológicos e fontes de pesquisa. Uma releitura indica a necessidade de rever o abandono, identificando um arrefecimento da geometria dedutiva, questionada nos anos 1950. As apropriações foram múltiplas, com destaque para propostas que buscaram uma abordagem mais didática e compreensiva. Argumentamos e defendemos com resultados de pesquisas que tal representação não é mais pertinente considerando os novos aportes da HEM. Entender a complexidade da trajetória do ensino da geometria no período do MMM, por aportes historiográficos, certamente nos permitirá uma reconsideração à representação de abandono da geometria.

Palavras-chave: História da Geometria Escolar; Ensino Secundário; Geometria das Transformações; Geometria Dedutiva

Initial Considerations

The first issue of Zetetiké, in 1993, published the article *The abandonment of geometry*

Submetido em: 31/03/2021 – **Aceito em**: 12/11/2022 – **Publicado em**: 30/12/2022

[1] PhD in Education (Curriculum) from the Pontifical Catholic University of São Paulo. Professor at Federal University of São Paulo (UNIFESP), Diadema Campus and of the Postgraduate Programs in Science Education at UNESP/Bauru and Postgraduate Program in Mathematics Education at UNESP/Rio Claro, Brazil. Email: celia.leme@unifesp.br

DOI: 10.20396/zet.v30i00.8665149

teaching in Brazil: causes and consequences, written by the mathematics educator Regina Pavanello. The article was the result of her Master's thesis defended at the University of Campinas, UNICAMP, a few years earlier, in 1989, entitled *The abandonment of geometry teaching: a historical overview.* The aim of the Master's thesis was to deal with the evolution of the teaching of geometry in different historical moments and to particularize it in relation to the Brazilian reality.

Another Master's thesis of a historiographical nature was defended in the same year, 1989, at the Federal University of Rio Grande do Sul, UFRGS, by the mathematics educator Elisabete Zardo Búrigo, entitled *Modern Mathematics Movement in Brazil:* A study of the action and thought of mathematics educators in the 1960s, whose aim was to study the movement for the renewal of mathematics education, known as the "*Modern Mathematics Movement" (MMM),* which emerged in Brazil in the early 1960s, through the study of the action, discourse and thought of the protagonists in relation to the historical context.

We also cite the doctoral thesis The dynamics and consequences of the modern mathematics reform movement for Brazilian mathematics education by the mathematics educator Beatriz Silva D'Ambrosio, defended at Indiana University, USA, in 1987, whose goal was to describe the dynamics of the MMM reform movement and its consequences in Brazilian education[2].

The three aforementioned studies were pioneering in a new field of research in Brazil, Mathematics Education (ME[3]). D'Ambrosio's dissertation, although developed in the United States, had as its object of study the MMM in Brazil. From the point of view of the field of History of Mathematics Education (HEM), the field of research would still take some years to begin its organization, as we will see below. In any case, the studies had a relevant contribution for the constitution of the first researches, because they allowed a first representation about the mathematics education in the past and provoked the desire for further studies.

All three studies addressed the MMM period, but Pavanello's dissertation (1989) focused specifically on the teaching of geometry and not on the MMM itself. The representation that this teaching was abandoned, and that this abandonment would have been an effect of the MMM was an assertion enunciated by Pavanello, both in the 1989 dissertation and in the article published by Zetetiké in 1993. Soon after, still in the 20th century, the National Curricular Parameters (PCN), published in 1998, reiterated the representation that the teaching proposals supported by the MMM would have been determinant in compromising the learning of Mathematics and consequently in the teaching of Geometry, as we observe:

> The teaching [influenced by the MMM] became overly concerned with formalizations, distancing itself from practical issues. The language of set theory, for example, emphasized the teaching of symbols and complex terminology, compromising the learning of calculus, geometry, and measures (Brazil, 1998, p. 19-20).

---

[2] The thesis was published with the title *The modern mathematics reform movement in Brazil and its consequences for Brazilian mathematics education* by Springer, in 1991.

[3] The first National Meeting on Mathematics Education (I ENEM) was held at PUC/SP in 1987. The following year, in 1988, the Brazilian Society of Mathematics Education (SBEM) was created. Pereira's (2015) thesis analyzes and reports on the creation process of SBEM.

DOI: 10.20396/zet.v30i00.8665149

Thus, we infer that, by the end of the twentieth century, the emerging community of mathematics educators understood that the MMM had contributed negatively to the teaching of geometry, leading to its abandonment. However, in the course of two decades of the twenty-first century, we observed a process of expansion and consolidation of the field of research on Brazilian Mathematics Education, as well as of the research area of the History of Mathematics Education, with new theoretical, methodological, and research sources, in order to allow the construction of other representations about the past of geometry education.

The present article aims to present the expansion of the field of HEM and some results of research developed during the 21st century about geometry teaching and the MMM, articulately, in order to problematize the representation built in the 20th century. Thus, we question: Was geometry "abandoned" in the MMM? What was this "abandoned" geometry? Do we have evidence, sources, interpretive analysis and studies that reiterate this representation built in the twentieth century? We argue and defend that such representation is no longer relevant considering the new sources and contributions of HEM.

The History of Mathematics Education research field

History of Mathematics Education in Brazil: Research issues, sources, theoretical-methodological references and elaborated histories was the title of the book, published in 2014, which in the words of its organizer Wagner Valente "reflects on all the scientific production disclosed in the I ENAPHEM - National Meeting of Research in History of Mathematics Education", held in Vitória da Conquista, Bahia, in 2012. The first national meeting of researchers on the theme of History of Mathematics Education (HEM) represented a significant milestone in the constitution of a national research field on the past of mathematics education.

Since then, every two years[4] , researchers on HEM have been meeting, discussing and reflecting about the scientific production of the new area. The proposal to collectively produce a book synthesizing the production and debates that took place in each of the events has been maintained so that we have a record about the consolidation process of these discussions organized in five books: Valente (2014); Garnica (2016); Dassie and Costa (2018), Leme da Silva and Pinto (2020); Cury, Morais and Garnica (2022).

Another relevant milestone was the creation of Working Group 15 (WG 15), dedicated to the History of Mathematics Education, within the Brazilian Society of Mathematics Education (SBEM), in the year 2016. Many other actions, such as the creation of the Journal of the History of Mathematics Education[5] (HISTEMAT) in 2015, numerous dossiers on HEM in national journals, such as BOLEMA (2010) and Zetetetiké (2019), as well as studies (Mendes and Gonçalves, 2020; Leme da Silva, 2022) have been reiterating the process of expansion and consolidation of the field of HEM research in Brazil, particularly during the last decade.

The brief presentation of the historical trajectory of a national movement of researchers

[4] In 2014, the II ENAPHEM took place in Bauru/SP; in 2016, the III ENAPHEM took place in São Mateus/ES; in 2018, the IV ENAPHEM took place in Campo Grande/MS; in 2020, the V ENAPHEM was scheduled to be held in Natal/RN, but, due to the COVID pandemic, it was held in virtual mode, and in 2022, the VI ENAPHEM took place in Florianópolis/SC.

[5] Between the years 2000 and 2002, volumes of the journal História & Educação Matemática were published.

DOI: 10.20396/zet.v30i00.8665149

who have worked over the last two decades to create and consolidate a new research field - HEM - indicates the expressive increase and advance of scientific knowledge in the field during the period. Thus, it seems pertinent to revisit and question pioneering representations constructed in the late twentieth century, such as "the teaching of geometry being abandoned by MMM", in face of a new moment of production in the scientific community. The article *Revisiting the past: contributions of Cultural History to critical research* reiterates the need to review and readjust previous studies in the face of the contribution of new theoretical contributions, such as Cultural History:

> A collective undertaking developed from 2005 by researchers from the Research Group on the History of Mathematics Education (GHEMAT) in dialogue with other researchers in the field, allowed to advance in the constitution of a wide repertoire of sources, and to build different looks at the protagonists of the movement and their practices of diffusion of proposals and curriculum materials inspired by modern mathematics (Búrigo, 2017, p. 56).

The large collective mentioned by Búrigo (2017) corresponds to the International Cooperation Project, *Modern Mathematics in the schools of Brazil and Portugal: comparative historical studies*[6] that produced, among numerous results on the understanding of MMM in Brazil and Portugal, a book synthesis whose title *The Modern Mathematics Movement: history of a curricular revolution* Oliveira, Leme da Silva and Valente, 2011) reinforced and highlighted the complexity of historical investigations about movements of international scope, such as the MMM.

On the other hand, in 2015, Caldatto and Pavanello resumed the theme in the article *A historical overview of geometry education in Brazil: from 1500 to the present day*[7] , whose goal was to present a historical analysis of the teaching of geometry in Brazil since 1500 until that year of publication. About the MMM period, the authors relativized and questioned the affirmative representation built in the twentieth century, by posing the question *The MMM and its influence on Brazilian education: an incentive to abandon geometry?* (p. 118). When analyzing the MMM period, the researchers used the same expression as in the book published in 2011, but they resumed Pavanello's (1989) conclusions:

> It was a "curricular revolution" that was still quite controversial in the academic community. The abrupt change in the approach of the contents in the mathematics textbook at that historical moment brought, above all, a great resistance from its main users, the teachers, who did not feel able to work mathematics and, especially, geometry in the way that MMM proposed. (Caldatto and Pavanello, 2015, p. 120).

It is important to note that the 2015 article, as well as Pavanello (1989, 1993), did not

[6] The Project *Modern Mathematics in Brazilian and Portuguese Schools:* comparative historical studies, developed in the cooperation CAPES/GRICES from 2006 to 2009, was coordinated on the Brazilian side by Dr. Wagner Rodrigues Valente and on the Portuguese side by Dr. José Manuel Matos. It brought together dozens of Brazilian researchers from several states, with numerous productions, such as dissertations, theses, books, and articles in qualified journals. One of the subprojects linked to the larger project was *The trajectory of school geometry in Brazil and Portugal and the Modern Mathematics Movement*, coordinated by Maria Célia Leme da Silva, from which it was possible to produce new readings and interpretations for the relationships between the teaching of geometry and the MMM.

[7] The article does not reference any of the numerous studies and results produced by the CAPES/GRICES project.

DOI: 10.20396/zet.v30i00.8665149

focus on the MMM but on a broad historical period, covering more than five hundred years of geometry education. However, there is a specific item to comment and analyze the MMM and again, the representations of the twentieth century are reiterated, without considering and dialoguing with specific research on the subject developed in more recent times. For example, the book chapter A modern geometry de Osvaldo Sangiorgi (Leme da Silva, 2008b), which analyzed the work of Sangiorgi - one of the main representatives of the MMM in Brazil and author of a textbook for high school considered a best seller of the period - and identified significant changes in the textbooks of the period.

It seems clear that the construction of representations about the MMM, and specifically about the teaching of geometry, requires an updated review based on specific studies, for the constitution and reconstitution of representations of the past. In this sense, this paper seeks to problematize the "abandonment" of the teaching of geometry by exposing research results, built in the twenty-first century, developed by the collective of researchers of GHEMAT and validated by the community of HEM in Brazil.

What does HEM research say about geometry teaching and the MMM?

After the initial phase contextualized in the introduction of the article, other studies on the MMM theme were produced (Vitti, 1998; Stephan, 2000; Soares, 2001, for example). In 2006, the Research Project developed by a group of researchers conducted a systematic study of bibliographic review on the theme; it leveraged a search for new sources in different Brazilian states and in Portugal and, over six years, produced scientific initiation, master's, doctoral and post-doctoral researches under the project. Moreover, it was during this project that the Group started to hold Thematic Seminars, as a dynamic of scientific production, involving the various participating researchers to exchange experiences, debates, and partial systematizations produced in the Annals and in books published at the end of the editions. Over six years, ten thematic seminars[8] were held, five of them in Brazil, at the different Universities participating in the Project, and five in Portugal.

The results we bring to the debate are related to the studies produced in the scope of this collective. As a HEM research group, the historical approach was supported by the theoretical-methodological tooling employed by historians of education for the production of historical facts. It can be said that historical facts are the constitutive elements of the writing of history, however, it is necessary to differentiate their role in teaching and in research: "We come here, no doubt, to the major difference between teaching and research, between the history that is didactically exposed and the one that is elaborated. In teaching, the facts are all ready-made. In research, it is necessary to construct them" (Prost, 1996, p. 55 apud Valente,

[8] The 1st Thematic Seminar took place at PUC/SP, in São Paulo (May/2006); the 2nd at UNL in Lisbon (September/2006); the 3rd in Curitiba, at PUC/PR (March/2007); at UNL in Lisbon (October/2007); o V at PUC/RS, in Porto Alegre (March/2008), at UNL in Lisbon (December/2008); o VI at USFC, in Florianópolis (July/2009), at UNL in Lisbon (December/2009); o IX at UFJF, in Juiz de Fora (July/2010) and o X at UNL in Lisbon (October/2011).

DOI: 10.20396/zet.v30i00.8665149

2007, p. 30).

We also consider that historical facts are always subject to criticism and revision, they can be confronted, refuted, questioned, either by the expansion of historiographical sources, always difficult to locate and store, or by the mobilization of new theoretical and methodological contributions. It is in this sense that the present article proposes new representations.

As mentioned before, the macro project brought together several subprojects, to deepen and investigate more specifically the MMM. Among them, one subproject aimed to understand how secondary school[9] geometry teaching took place at the time of the MMM, in other words, in what way the proposals that emerged from the Movement, with respect to geometry, were or were not incorporated into its teaching. What relevant characteristics in the Brazilian school culture[10] could be identified when studying the processes of appropriation of a new proposal for teaching? Finally, we seek to produce historical facts about the teaching of geometry in the period of the MMM, which means finding sustainable and appropriate justifications for the understanding and substantiation of a representation about the trajectory of this teaching.

The abandonment of geometry teaching

We resume the research of Regina Pavanello, whose objective was to approach the trajectory of geometry teaching in different historical moments and to particularize it in relation to the Brazilian reality. According to Pavanello (1989), the orientation resulting from the modernizing proposals for the teaching of mathematics presented in the textbooks published in Brazil from the 1960s on was centered on the use of the symbolic language of set theory:

> If this orientation, however, can easily be put into practice when it comes to algebra and arithmetic, the same is not true for geometry. Geometry can no longer be worked on in the traditional way. Thus, in a first moment, it is chosen to emphasize, in these books, the notions of geometric figure and intersection of figures as a set of points in the plane, by adopting, for geometry, the same symbology used for sets in general, and by working it according to an "intuitive" approach. This approach is concreted, in textbooks, by the use of theorems as postulates, by means of which we can solve some problems. There is no concern now with building a systematization from primitive and empirically elaborate notions (Pavanello, 1989, p. 163).

In relation to the proposals for the teaching of geometry in order to maintain coherence with the MMM, that is, to propose a work under the structures approach, the study pointed out

[9] It is important to point out that the researches mentioned above deal with secondary education in Brazil, which comprised the gymnasium course during the MMM period. Today, it would correspond to the final years of elementary school.

[10] The text by Faria Filho, Vidal and Paulilo (2004) presents an analysis of the different meanings for the term "school culture". In this study, we adopt the concept of Dominique Julia: "A set of norms that define knowledge to be taught and behaviors to be inculcated, and a set of practices that allow the transmission of this knowledge and the incorporation of these behaviors" (2001, p. 10).

DOI: 10.20396/zet.v30i00.8665149

two possibilities: geometry developed by vector planes or by transformations. In searching how these approaches arrived at the Brazilian legislation, Pavanello refers to the Mathematics Curriculum Guide, prepared by the Secretary of Education of the State of São Paulo and published in 1975, which highlighted the following recommendations for the teaching of geometry:

> - a course in intuitive geometry for the four first grades of primary school;
> - a study of measurements, done with much more propriety and greater possibility of assimilation in a science course;
> - the study, in the 5th grade, of geometry serving as a vehicle for the introduction of the language of Set Theory;
> - the introduction of the study of Geometry through Transformations starting in 7th grade (Pavanello, 1989, p. 164, our emphasis).

In this context, the researcher inferred that the recommendations proposed in the mentioned curricular standards did not gain space in the mathematics classrooms, because they did not contemplate the particularities of the teachers:

> The orientation to work geometry under the focus of transformations, a subject not mastered by most secondary teachers, ends up causing many of them to stop teaching geometry under any approach, and to work predominantly algebra - even because, as Modern Mathematics had been introduced through this subject, it had emphasized its importance. Law 5692/71, in turn, facilitates this procedure by allowing each teacher to adopt his or her own program according to the needs of the clientele (Pavanello, 1989, p. 164-165).

As already mentioned, Pavanello's (1989) research did not aim to focus on the MMM, but rather presented a broad retrospective of the different moments of mathematics education in Brazil in relation to the teaching of geometry. The norms analyzed by Pavanello were from the state of São Paulo and from the year 1975; however, the MMM ideology circulated in Brazil since the early 1960s and had different appropriations in different states and contexts of the country, as we will see below. In his analyses, Pavanello claims that the new approach proposed, geometry through transformations, which was not dominated by teachers, was a decisive element in the abandonment of geometry teaching in the MMM. However, we might ask, what was the "abandoned" geometry? Had the teacher's mastered geometry before the MMM?

Therefore, we now present the results of the above-mentioned project on the proposals for the teaching of geometry in secondary school and, after 1971[11], also in the later years of junior high school. We begin with the results of the teaching of geometry in the pre-modern period, in the 1950s, in order to better understand the changes resulting from the MMM. To this end, we have taken as sources of research the regulations, the debates in the National Congresses on Mathematics Education that began in 1955, articles in pedagogical journals, textbooks, among others. We emphasize that the sources analyzed were expanded, both in their nature and in their regional diversity, when documents from other states besides São Paulo were included. It was only through the collective effort of a group of researchers that the diverse

[11] Law 5.692/71 inaugurated a new organization in Brazilian education. Primary and secondary education was abolished and an eight-year junior high school was created.

DOI: 10.20396/zet.v30i00.8665149

set of sources could be inventoried.

Pre-modern times - the 1950s

Before addressing the arrival of modernizing ideas in the teaching of mathematics in Brazilian education, in particular the proposal for the teaching of geometry, in the early 1960s, we present a brief overview of the previous decade, in the 1950s, in order to contextualize the teaching of geometry before the MMM.

A milestone in Brazilian basic education in the 1950s was Ordinance No. 966[12] of 1951. Its purpose was to establish a minimum program to be developed in schools, given the expansion of basic education in Brazil and the impossibility of maintaining the control conducted by the Colégio Pedro II until then. Alexandre Marques, in analyzing the methodological instructions of the Ordinance, highlighted the following considerations:

> In the methodological instructions it is clear that the idea of rigor should not be exaggerated, that the teacher should avoid teaching by simple mechanization techniques and abusive use of definitions. In summary, the methodological instructions emphasize that
>
> - Each subject should be illustrated by applications and examples;
> - The unity of mathematics should be emphasized;
> - The teaching of mathematics in the early years should be practical and intuitive;
> - the student should be carefully and gradually introduced to the deductive method;
> - rigidity should be tempered (2005, pp. 60-61).

This was the orientation proposed in the legislation in force. However, in the classrooms, we know that the methodology adopted by teachers did not always correspond to the legal character[13] . According to Búrigo (1989):

> Math classes were expository, and not even the resolution of exercises by the students in class was a generalized practice. When it was done, what was presented to the students were standardized exercises, which had to be solved in the same way as a "model problem", with emphasis on bulky calculations. Demonstrations of theorems were presented by the teacher and memorized by the students, for presentation in the proofs. The resources used did not go beyond chalk, blackboard and textbook, if any (p. 40).

Another important source of research in the construction of a representation of the past of geometry teaching was the analysis of the Annals of the National Congresses that discussed the new trends, the problems diagnosed and presented possible solutions. It was precisely in the 1950s that the first National Congresses on Mathematics Teaching took place. The analyses

[12] Geometry is covered, according to the Ordinance, in the 3rd and 4th grades. The contents are: Plane geometric figures, line and circle; Proportional lines, similarity of polygons, Trigonometric relations in the right triangle, natural boards (3rd grade) and Metric relations in polygons and circle, calculation of π and Areas of plane figures (4th grade) (Marques, 2005, p. 56).

[13] The historian Chervel (1990) distinguishes a theoretical purpose from an actual purpose, warning that proposals contained in programs, official texts constitute only a theoretical purpose, and many of them do not become an actual purpose.

DOI: 10.20396/zet.v30i00.8665149

of the Proceedings of the three Congresses held in 1955, 1957 and 1959 were presented in the book chapter entitled Geometry in the National Congresses on Mathematics Teaching. The conclusion reached was that the debates revolved around the dualism between intuitive geometry and deductive geometry:

> The problems described, the suggestions pointed out refer to the following questions: Which should be taught, intuitive geometry or deductive geometry? At what point do we switch from one to the other? What does the presence of deductive geometry mean from the third grade on, for 13-year-old students? The rigorous demonstration of all the theorems that make up Euclidean geometry. (Leme da Silva, 2008a, p. 76)

The proposals to change the approach to geometry teaching were present in the Annals of the II Congress, held in Rio Grande do Sul, in 1957. The theses were defended by Professors Antonio Rodrigues[14] and Benedito Castrucci[15]. In both, we identified points of convergence in relation to the difficulties of the practice of geometry teaching and the solutions suggested. Briefly, the professors proposed to simplify the study of deductive geometry, reducing the number of theorems to be demonstrated and the inclusion of experimental geometry or intuitive demonstration (Leme da Silva, 2008a, p. 74).

The analysis of pedagogical journals that circulated in the period, another diverse research source, allowed researchers to understand the reports and discussions about teaching practices. Oliveira and Pietropaolo (2008) conducted an analysis about the articles published in *Revista Escola Secundária*[16] , highlighting those referring to geometry. According to these authors:

> It should also be noted that all these texts[17] basically discuss the issue of deductive geometry, leaving very little space - when they do - for the so-called intuitive geometry, which includes, fundamentally, nomenclatures and measures (areas and perimeters). (Oliveira and Pietropaolo, 2008, p. 98).

In summary, the articles in Revista Secundária agreed with the prescribed programs of 1951 in which deductive geometry should be developed in class, respecting the rhythm, needs and interests of students, provided that the practices of teachers were not too far from the maxim: the demonstrations should be rigorous, given that the proof is crucial in mathematical

---

[14] Full Professor of Geometry at the School of Philosophy of URGS. His thesis was presented in the Annals in the chapter entitled "The Teaching of Deductive Geometry".

[15] Full Professor and Professor of the School of Philosophy of the University of São Paulo. PhD in Mathematics. Author of specific didactic books on Geometry.

[16] Published by the Campanha de Aperfeiçoamento e Difusão do Ensino Secundário (Cades), linked to the Ministry of Education and Culture) from 1957 to 1963. Regarding mathematics and its teaching, in its 19 issues, Revista Escola Secundária published 34 texts, including articles, reports and teaching plans - all written by renowned teachers of the time (Oliveira and Pietropaolo, 2008, p. 96-98).

[17] The texts that discuss the teaching of geometry in the Revista Escola Secundária analyzed in this study are by: Antonio Rodrigues, professor of Geometry at the Faculty of Philosophy at URGS and who presented a thesis on this subject at the II National Congress on Teaching Mathematics in 1957; Eleonora Lôbo Ribeiro, who participated in the I National Congress on Teaching Mathematics in 1955 and whose thesis is covered in an article published in the Revista Escola Secundária; Thales Mello Carvalho, professor of Calculus Methodology at the Instituto de Educação (Rio de Janeiro) and Malba Tahan, pseudonym used by Júlio César de Mello e Sousa, professor at Colégio Pedro II (Oliveira and Pietropaolo, 2008).

DOI: 10.20396/zet.v30i00.8665149

culture (Oliveira and Pietropaolo, 2008, p. 105).

To complete the scenario of geometry teaching in the 1950s, we analyzed a textbook collection of wide circulation in the decade, the books published by Osvaldo Sangiorgi and that precisely in that decade gained recognition. In the collection *Matemática – curso ginasial*[18], in the preface to the textbook for the *3rd grade*[19], Sangiorgi emphasizes the role of geometry:

> In our opinion, this third volume has <u>a great responsibility in the deductive geometric initiation of secondary school students</u>. In fact, it is at this stage of the course that geometric knowledge must be deepened in order to allow secure assimilation by the students, within a demonstrative technique, accessible and uniform as much as possible. To this end, the demonstrative process that we employ, is composed of numbered parts, of which the first sees, almost always, the auxiliary constructions necessary to the demonstration, accompanied by evident properties; the second involves deduction, on the basis of successive reasonings, and the conclusion. Only exceptionally is there a third part for the purpose of dividing a very extensive reasoning from the second (Sangiorgi, 1964, p. 17, emphasis added).

Geometry was present in almost the entirety of the third volume for the junior high school course; of the four chapters, the last three dealt with the subject. It began in chapter II, which, on the first page, spelled out its objectives:

> Intuitive geometry. Objectives of deductive Geometry. Our first conscious contact with Geometry - called intuitive or experimental - was in primary school. Observation and experience were, at that time, the means employed to highlight the properties relative to the form and extension of bodies. Now, in a more advanced phase, in which Geometry begins to study these same properties of bodies, using only reason, it receives the name of deductive or rational and the objective of deductive Geometry is, precisely, to *study the geometrical properties of bodies by means of a logical chain of reasoning.* (Sangiorgi, 1964, p.89).

Sangiorgi accomplished what he set out to do, that is, he presented the axioms at the very beginning and from then on enunciated and demonstrated theorems and properties. There are no exercises of exploration, investigation, the so-called intuitive geometry was not addressed. In general, it followed the traditional didactic sequence: definitions, properties, theorems, and exercises, always at the end of each topic.

The 1960s and the arrival of MMM in Brazil

Only in the 1960s did the modernizing proposals for the teaching of mathematics gain

[18] The collection was launched in 1953. According to Valente (2008a), in the three years following the launch of the volume for the first series of the junior high school course, it had great acceptance. The print run did not stop rising, reaching, in 1957, for the first volume, the mark of 100 thousand copies. From then on, it remained at this level annually until 1963, the year in which, according to the archives of Cia. Editora Nacional, the 134th edition of the textbook was published! Considering that the school population of all secondary schools in the state of São Paulo, from the 1950s to the 1960s, as seen above, doubled, increasing to 360 thousand students, one can see how expressive were the numbers reached by Osvaldo Sangiorgi's collection "*Matemática - curso ginasial*".

[19] 3rd Gymnasial grade (13 year-old students) where geometry was taught (Portaria no.966 de 1951). The ginasial course in the 1950s consisted of four years, after the primary course. The third grade corresponds today to the eighth grade of elementary school.

DOI: 10.20396/zet.v30i00.8665149

space in mathematics classes in Brazilian basic education. More specifically, for the junior high school, it was triggered with the publication of the textbook *Matemática curso moderno*, volume 1, by Osvaldo Sangiorgi, in 1964[20]. The teaching of geometry was dealt with in volume 3, and was published in 1966, following the previous series.[21]

Many were the changes regarding the teaching of geometry in the textbook *Matemática curso moderno*. Osvaldo Sangiorgi drew the students' attention, right in the preface, to the new proposal for the teaching of geometry:

> My dear student, in this book - the third in the series of the modern teaching of mathematics in the Gymnasium - you will come into contact with a lot of new things (...). Finally, comes the "good stuff" of the book: the study of Geometry. Now, you will no longer need to "memorize" boring theorems and more theorems, against which, wrongly, some advanced students used to "warn" you. And he ends his message in the following way: "Be very happy on this trip to the wonderful country of Geometry and see you in the fourth grade. (Sangiorgi, 1969, p. XV)

We immediately see that the novelty announced by the author, in modern geometry, did not concern new content or a new geometry, but the emphasis on the demonstration of theorems that in times past, induced students to memorize. Leme da Silva[22] , in conducting a specific study comparing Sangiorgi's books of the 1950s with the modern book published in 1966, considered that:

> As for the methodological approach taken in the teaching of geometry, the book *"Matemática curso moderno*" brings, in addition to changes in language and postulates, a more careful development of geometric concepts and properties. In our view, this change, together with the attempt to recover exploratory aspects in the teaching of geometry, before the formalization of concepts and axiomatization, represents a significant change in the teaching of geometry. This aspect, hitherto little discussed in the MMM ideology, represents indications of a concern with the didactics of mathematics teaching, in an attempt to greater student participation in the learning process. It is not our goal to judge or evaluate the results of such methodological changes, but to recognize and identify that the two analyzed books clearly differ in the didactic proposal, in the activities, in the relationship that the author establishes with the students. In the 1964 textbook (before the MMM) there is a strong emphasis on the transmission of ready-made concepts, defined, without analysis. In the modern approach, from 1969, the author talks to the student, invites him to think, to seek his own ways, to explore situations before formalizing the concepts. (2008b, p. 91-92)

A pertinent reading of Sangiorgi's choice to propose a more comprehensive teaching, with the inclusion of experimental geometry, without so much emphasis on geometric demonstrations, can be made as a response to the discussions at the Congresses held in the 1950s and in the articles in the Revista Escola Secundária, which highlighted the difficulties of

[20] An in-depth analysis of the textbook collection that inaugurated the presence of modern mathematics in Brazil can be read in "Osvaldo Sangiorgi, um best seller" (Valente, 2008a).

[21] The modern textbooks for the junior high school were published, one each year: in 1965, volume 2; 1966, volume 3; 1967, volume 4, completing the collection.

[22] A deeper exploration of this study can be read in the chapter entitled "Osvaldo Sangiorgi's modern school geometry" in the textbook Osvaldo Sangiorgi: a modern teacher (2008b).

DOI: 10.20396/zet.v30i00.8665149

teachers and students facing deductive geometry.

However, the debate experimental geometry versus deductive geometry always permeated the discussions about the teaching of geometry at different historical moments, including the MMM, and was not characterized as the central element of the modernizing ideology. The focus of geometry education in the MMM was on the controversy about Euclidean geometry. We discuss the introduction of other approaches to the teaching of geometry, such as geometric transformations and with a theoretical basis supported by algebra, especially from vector spaces. By analyzing the modern collection of Sangiorgi, in relation to its appropriation[23] facing the international proposals, we conclude:

> Considering the two trends described by Fehr about the teaching of geometry, we can say that Sangiorgi, in the textbook *Matemática curso moderno*, came much closer to the trend used in the USA and based on Birkhoff, than to the proposal of teaching geometry through geometric transformations, linked to Klein. However, we do not find a clear positioning of Sangiorgi regarding the teaching of geometry. He includes measures in the postulates and develops deductive geometry along these lines but makes no reference to Birkhoff and does not highlight such a change as significant, either in the didactic work or in the "Guide for Teachers' Use". (Leme da Silva, 2008b, p. 90).

Certainly, there must have been many reasons that justified Sangiorgi's choice of the American proposal for the teaching of geometry, rather than geometry by transformations, which he included, but in a very timid way, in the appendix of the book. In any case, as this collection gained prominence with wide national circulation, the proposal for geometry teaching created a school.

Soon after the publication of Osvaldo Sangiorgi's textbook, many collections with the modern approach were launched. We bring for analysis the collection of Benedito Castrucci and Alcides Bóscolo[24] in 1968. Castrucci, besides being a full professor at the University of São Paulo, responsible for the chairs of geometry at the Faculty of Philosophy, Sciences and Languages, was, together with Sangiorgi, one of the founders of the Mathematics Teaching Study Group,[25] and a great promoter of the MMM. In the preface of the modern book, the authors explained their decisions regarding the teaching of geometry:

> As for Geometry, the topics, although often appealing to intuition, are presented in a

[23] We used Roger Chartier's (1991) concept of appropriation, understood as "a social history of uses and interpretations, referred to its fundamental determinations and inscribed in the specific practices that produce them. The essential thing is to understand how the same texts - in different printed forms - can be diversely learned, manipulated, understood" (p. 180-181).

[24] Graduated in Mathematics. Effective Professor by Contest of the Official Secondary and Normal Teaching of the State of São Paulo.

[25] In 1961, the GEEM - Grupo de Estudos do Ensino da Matemática (Mathematics Teaching Study Group) was created in the city of São Paulo, with Professor Osvaldo Sangiorgi at the head of the Group. Participating in the Group were university professors, secondary and primary school teachers, authors of textbooks, all with the objective of encouraging, coordinating, divulging and updating mathematics, as well as its teaching, in primary, secondary and normal courses. In other words, to propose actions that would subsidize the introduction of Modern Mathematics in basic education. Courses for teachers, lectures by foreigners in Brazil, publication of material, and translation of books were the main activities developed by GEEM (Leme da Silva, 2008b, p. 69).

DOI: 10.20396/zet.v30i00.8665149

> logical sequence that is essential to the good education of the students.
> As we understand that the Geometry part should not be sacrificed, as unfortunately often happens, we have tried to develop it within a minimum scheme that we think should be fully taught (Bóscolo and Castrucci, 1969, preface).

The teaching of geometry in the collection followed the same trend as Sangiorgi's, sticking to Euclidean geometry, using measures in the demonstrations but without emphasizing the measures as axioms, and presenting the geometric transformations in the appendix. Duarte (2007), comparing Sangiorgi's modern collection with that of Bóscolo and Castrucci, concluded that the latter followed in the footsteps of the former, sedimenting the new vulgate[26].

The Castrucci and Bóscolo collection did not last long, however, due to Bóscolo's death. Other collections followed Castrucci's didactic production for the junior high school, published by FTD, in the 1970s, together with other authors, especially Professor José Ruy Giovanni[27].

The textbooks analyzed constitute important evidence of the past for understanding how the teaching of geometry, intended for the junior high school course in the 1960s, were the first didactic productions that incorporated the MMM ideology. Osvaldo Sangiorgi's collection was a recognized editorial success, with national distribution, becoming a *bestseller*[28]. Besides, the figure of Sangiorgi is a symbol of the spread of the MMM in Brazil, either through his performance as president of GEEM, through the print and television media, through the organization and participation in national and international congresses, through his performance in educational commissions, among other relationships that legitimized him as leader of the movement. The Castrucci and Bóscolo collection, even without data on its sales, was justified by the fact that it came from authors who maintained close ties with the discussions about the MMM, especially Benedito Castrucci, who was responsible for the Chair of Geometry at the University of São Paulo.

The research of Elisabete Zardo Búrigo (1989), even without analyzing the teaching of geometry, interviewed several leaders of the MMM in Brazil. Among the mathematicians interviewed was Benedito Castrucci. When commenting on the reasons that might have led the movement to the so-called "failure", he said: "And the failure for me was in geometry" (Castrucci, 1988 *apud* Duarte, 2007, p. 277). In other words, for Professor Castrucci, geometry was the great villain of the MMM. However, in his textbook, the geometry of transformations was included as an appendix to the work.

The panorama of geometry teaching represented by the commented textbooks did not exclude the existence of other experimental modernizing proposals for geometry teaching in the 1960s. However, we believe that the experiments that included geometric transformations in the teaching of geometry in that decade could not be considered as representative. We can

---

[26] Chervel (1990, p. 203) states that "in every era, the teaching provided by teachers is, roughly speaking, identical, for the same subject and at the same level. All textbooks, or nearly all, then, say the same thing, or nearly so. The concepts taught, the terminology adopted, the collection of rubrics and chapters, the organization of the corpus of knowledge, even the examples used or the types of exercises practiced are identical, with approximate variations" calling this phenomenon the vulgate.

[27] He has a Bachelor's and a Bachelor's degree in Mathematics from Pontificia Universidade Católica de São Paulo (PUC-SP). He is the author of Math textbooks by Editora FTD until today.

[28] Best seller is the expression used by Valente (2008a) when describing in detail the trajectory of the first collection of textbooks for the gymnasium with the new modernizing proposal for the teaching of mathematics, elaborated by Osvaldo Sangiorgi and published by Companhia Editora Nacional.

Zetetiké
DOI: 10.20396/zet.v30i00.8665149

classify this moment as initial, the first appropriations of the MMM in the teaching of school geometry in Brazil.

The 1970s: a second moment of the MMM

The 1970s began with significant changes in Brazilian educational legislation. Law no. 5692/71 proposed a new structure for basic education, which became: primary education, with eight years of schooling, and secondary education, with three years.

As far as mathematics education was concerned, the decade was marked by evaluations and criticism of the MMM. The translation of Morris Kline's[29] book, *The Failure of Modern Mathematics,* published in 1973 in the United States and translated into Portuguese in 1976 under the title O Fracasso da Matemática Moderna, reinforced the cooling off of the Movement in Brazil. Osvaldo Sangiorgi himself, in an article in the Jornal do Estado de São Paulo, in 1975, pointed out what he considered to be the main effects of the MMM in mathematics teaching:

> 1. The gradual abandonment of the healthy habit of calculating (not knowing the multiplication tables in the middle of the 5th and 6th grade!), because operations on sets (especially with empty spaces!) are given priority over everything else; to this we can add the exclusive and premature use of calculators, which have become as popular as electronic toys,
> 2. Ordinary fractions and the decimal metric system - of great importance for life - are no longer taught, and set theory, which is extremely abstract for the age of the student, is usually taught incorrectly,
> 3. We no longer know how to calculate areas of plane geometric figures, much less of the solid bodies that surround us, in exchange for a rich vocabulary of external effects such as "geometric transformations",
> 4. Elementary problems - of daily life - are no longer solved because of the invasion of new symbols and abstractions completely removed from reality, such as "Is the set of parts of an empty set an empty set?", proposed in a 5th grade textbook. (Sangiorgi apud Soares, 2001, p. 116, our emphasis).

As we observed, Osvaldo Sangiorgi confirmed the assessment that in the teaching of geometry, old practices were replaced by new concepts, new vocabulary, including the emphasis on geometric transformations. But were geometric transformations included in the teaching of geometry, in the old high school course, which after 1971 is now called the 5th to 8th grade of the 1st year of high school? Did geometric transformations leave their position as an appendix to the textbooks and become part of the central proposal of the textbooks? And how were they included?

The Castrucci and Bóscolo collection, although short-lived, acquired a new configuration since Benedito Castrucci remained at the FTD and began to publish new collections of textbooks with other authors. In one of these collections, called *Matemática*, Castrucci, together with Ronaldo Peretti and José Ruy Giovanni, expressed their reflections on the study of geometry in the presentation of the 7th grade textbook:

[29] Professor of Mathematics at the Courant Institute for Mathematical Sciences at New York University. The book was a severe criticism of the MMM.

Zetetiké
DOI: 10.20396/zet.v30i00.8665149

> We allow ourselves here to draw attention to the study of geometry. Unfortunately, since the teaching of geometry, a source of inspiration and reasoning, has been abandoned, we have tried in this part, to give <u>a rather intuitive presentation, with few demonstrations, remembering that the important thing in this area, is the awakening to creativity and the development of fantasy.</u> In this phase, in second place, comes the knowledge of deductive reasoning, a basic tool in any intellectual activity. For this reason, there are short demonstrations arranged in two columns: assertion and reason. This is followed by other incomplete demonstrations for the student to finish (Castrucci, Peretti and Giovanni, 1976, emphasis added).

The textbook by Castrucci, Peretti and Giovanni was published in São Paulo, by FTD, after the publication of the Curricular Guides for Elementary School, elaborated by the Secretary of Education of the State of São Paulo in 1975 and analyzed by Pavanello (1989). The authors' emphasis, for the teaching of geometry, was, once again, to resume intuitive geometry, valuing creativity. Deduction was to take a back seat, and geometric transformations, which were among the content items listed in the Curriculum Guides of the State of São Paulo, did not appear either in the considerations or throughout the book. The geometry proposed in the book was Euclidean geometry, which the authors call elementary geometry, pointing out in a footnote the correspondence with Euclid's geometry (Castrucci, Peretti and Giovanni, 1976, p. 135). It was developed in the last eight units of the book and the appendix contained the geometric constructions.

There is also another textbook collection, also from FTD, written by Castrucci and Giovanni (s/d), entitled *Matemática (São Paulo)*. According to the Curricular Guides. It was not possible to determine the date of this publication, but it was certainly after 1975, due to the mention of the Curricular Guides. In the presentation of the 7th grade work, there is no specific consideration of the teaching of geometry. The index followed practically the same pattern as in the above-mentioned collection (Mathematics), and geometry was also developed in the last nine units. The significant difference was unit number 14, called "Transformation - Symmetry and Translation". In general, the authors present the concepts of the perpendicular bisector of a segment, axial symmetry, axis of symmetry of a figure, central symmetry, oriented segment, equipollent segments, and translation in the 11 pages devoted to this topic. In the 8th grade textbook, the development of geometric concepts followed the same order as in the Mathematics collection, in the last units of the textbook and without any reference to geometric transformations.

It seems obvious that the collection specifically intended for São Paulo had the inclusion of a unit with the theme of geometric transformations, in relation to the teaching of geometry, in order to meet the standards established by the norms of that state. However, the inclusion of a unit did not reveal a change in the proposal of geometry teaching, since it was not articulated with the other units, and did not change the structure of geometry teaching, neither in methodology nor in content. Everything indicates that for Castrucci and Giovanni (s/d) the publication of the Curricular Guides did not mean a change in the proposal, an innovation for the teaching of geometry.

A work that studied the teaching of geometry in Paraná was the master’s thesis of Ferreira (2006), whose goal was to analyze the pedagogical proposal of geometry developed

Zetetiké
DOI: 10.20396/zet.v30i00.8665149

by the Center for the Study and Dissemination of Mathematics Teaching (NEDEM). The research used as historical sources the archives of the Colégio Estadual do Paraná (Paraná State College), headquarters of the experience of the State of Paraná in the implementation of the MMM. The collection of textbooks *Ensino Moderno de Matemática* (Modern Mathematics Education for Grades 5 to 8), written by NEDEM and directed by Professor Osny Antonio Dacol, was analyzed.

The research showed that the main innovation of the proposal was the addition of a more advanced notion of homology, the language of set theory, demonstrations of theorems using logical propositions, and vector calculus. The introduction of vectors was a differential of the Paraná proposal with respect to other states. However, the proposal did not remain in the curriculum of the State School of Paraná for long, being replaced in 1974. The analysis of Ferreira (2006) showed that the proposal for the teaching of geometry in the book *Ensino Moderno de Matemática* (Modern Mathematics Teaching) was not included in the Paraná curriculum, that is, it was not accepted by the school culture and therefore it was put aside.

In analyzing the teaching of geometry and geometric transformations in the period in question, it is necessary to bring into debate the Bahian experience led by the teacher Martha de Souza Dantas[30]. Martha, together with Omar Catunda[31], Eliana Costa Nogueira, Eunice da Conceição Guimarães, Neide Clotilde de Pinho e Souza, Norma Coelho de Araújo and Maria Augusta Araújo Moreno, led the project "Development of a Curriculum for the Updated Teaching of Mathematics", with the aim of introducing modern mathematics in secondary education.

The team led by Dantas developed, among other things, didactic materials with the new proposal for the teaching of mathematics and used these materials on an experimental basis in the College of Application of the University of Bahia. According to Martha:

This experience, which began in the Science Center of Bahia (CECIBA), was an example of the incorporation of geometric transformations in the teaching of geometry. The materials produced went through several reformulations, different denominations, but unlike the case in Paraná, the project evolved and continued in the 1970s, 1980s and 1990s. An in-depth study of the teaching of geometry, present in the different editions of the teaching materials produced by this team, was developed in the master’s thesis of Camargo (2009).

Another collection of textbooks that was also used as a research source to study the teaching of geometry during the MMM was the collection *Matemática curso moderno* for Primary School, published since 1972 by the Mathematics Teaching Group (GRUEMA). The

---

[30] Martha Dantas (1923-2011). Professor at the University of Bahia School of Philosophy since 1945. In 1955, organized the I National Congress on Teaching Mathematics in the Secondary Course, in Salvador. She coordinated the Mathematics sector of CECIBA from 1965 to 1969 (Leme da Silva and Camargo, 2008).

[31] Omar Catunda (1906-1986). Professor at the School of Philosophy, Sciences and Languages at USP. When he retired in 1962, he went to Salvador and became Director of the Institute of Mathematics and Physics of the University of Bahia.

DOI: 10.20396/zet.v30i00.8665149

group was formed by Professors Lucília Bechara Sanchez[32], Manhúcia P. Liberman[33], Anna Averbuch[34] and Franca Cohen Gotlieb[35], with supervision and revision by L.H. Jacy Monteiro[36]. The authors played a prominent role during the MMM, especially in the teacher training courses organized by GEEM. The 7th and 8th grade textbooks in the collection worked with the concept of geometric transformations; Rios (2010) analyzed the proposal for teaching geometry in the collection in his master's thesis.

For the studies presented here, we believe that the 1970s represented a second moment of the MMM, in which the teaching of geometry was configured differently. Some isolated experiences that began in the 1960s were reformulated and, together with the already sedimented school culture, configured the teaching of geometry in a different context. It was a time of maturation, evaluation of pilot experiments, diffusion of modern textbooks, criticism and adaptation, and even resistance of the school culture to the new proposals.

Final considerations

As stated at the beginning, the article aimed to show the expansion and consolidation of the field of HEM and to synthesize studies developed after the end of the 20th century. Moreover, the synthesized results were produced in a collective project of international scope, in a moment of insertion of new theoretical contributions on how to produce the HEM and the gathering of a more representative set of historical sources. The intention was not to refute Pavanello's studies (1989, 1993), on the contrary, we wanted to value them in the sense of opening doors and bringing representations about the past at an early moment of Brazilian mathematics education.

In addition, the inclusion of the research in an international cooperation project, with Portugal, forced the research group to limit itself to certain contexts, in this case the Brazilian point of view, and to a certain educational segment, in this case the junior high school, today the last years of elementary school. The experience of moving through different cultures, whether in Portugal or in the different Brazilian states, brought gains in the process of history production:

> Making himself a foreigner in the midst of another culture that he wants to know, the researcher relativizes, reconstructs the knowledge that he was so familiar with, of

[32] Lucília Bechara. Master's and PhD in Education from USP, founding member of GEEM.

[33] Manhúcia Liberman (1927-2017). Bachelor and graduate from UFRJ, founding member of GEEM.

[34] Anna Averbuch (1928-2004). Graduated and Bachelor in Mathematics from UFRJ, professor at Universidade Santa Ursula (RJ), founding member of Gupo de Estudos e Pesquisa em Educação Matemática -GEPEM.

[35] Franca Cohen Gotlieb has a graduate and bachelors degree in mathematics from UFRJ, is a professor at the Universidade Santa Ursula (RJ), and a founding member of the Grupo de Estudos e Pesquisa em Educação Matemática GEPEM.

[36] Jacy Monteiro (1921-1975). Professor at the University of São Paulo, member of GEEM.

DOI: 10.20396/zet.v30i00.8665149

> elements of the school culture that was so close to him in his own country, in his own culture. This is where the idea of experimentation seems to take root. When testing, in the field of the other, elements present in the culture that is one's own, one gets to know more about one's own culture, as the encounter of strangeness, differences and asymmetries that, at first, make inexplicable what was familiar. (Valente, ICME 11, 2008b)

Perhaps it is still early to produce new historical facts about the teaching of geometry and the MMM in Brazil, but everything indicates that there is sufficient evidence to support that the teaching of geometry in the MMM should not be tied only to the presence of geometric transformations in the list of contents of a curriculum proposal. The concept of geometric transformations has been treated in isolated experiences, and everything leads us to believe that it has not penetrated the mathematics classroom, except in special contexts, as was the case in Bahia. Focusing discussions of geometry education and the MMM on geometric transformations, without looking at how this proposal was inserted into school culture, often prevents us from really understanding the trajectory of geometry education at the time of the MMM. The arguments presented here, in light of the new contributions of the HEM, show that the narrative of the MMM's abandonment of geometry is not relevant.

The results of the studies of the collective project showed that the geometry proposal of the 1950s emphasized deductive geometry, theorems and demonstrations, and were the subject of debate and concern in the national congresses. Thus, from the 1960s, during the MMM in Brazil, we observe a cooling of deductive geometry, which was characterized in the pre-modern period. Another representation that could be defended is that the modernizing proposals tried to revive the teaching of geometry by proposing a more experimental and intuitive approach, in accordance with the desire expressed in the debates present at the National Congresses. However, this attempt, at a time of emphasis on algebraization, of valorization of set theory, may have caused the effective proposal for the teaching of geometry to go unnoticed by the community.

In fact, the terms abandonment or failure in the historical production can be interpreted as a value judgment. Educational historians warn us that the role of historical production is to compare sources in search of a plausible explanation that is as close to the truth as possible, not to judge, as Marc Bloch points out:

> There are two ways of being impartial: that of the scientist and that of the judge. They have a common root, which is honest submission to the truth. There comes a time, however, when the paths separate. When the scientist has observed and explained, his task is finished. It remains for the judge to declare his sentence. (Bloch [1994], 2001, p. 125)

Perhaps we need to recognize and understand why the path of geometry teaching seems to have been different from other mathematical concepts, such as algebraic structures, which even without the mastery of teachers on the subject, were inserted directly into textbooks in teacher training courses. Understanding the complexity of the trajectory of geometry teaching, associated with the MMM period and the appropriations that textbook authors, teachers and students have made, will certainly allow us to take a more critical look at the current teaching.

Zetetiké
DOI: 10.20396/zet.v30i00.8665149

References

Bloch, M. (2001). *Apologia da História ou O ofício de historiador.* Rio de Janeiro: Jorge Zahar Ed.

Bóscolo, A & Castrucci, B. (1969). *Matemática: curso moderno*. v.3. São Paulo: FTD.

Brasil (1998). Secretaria de Educação Fundamental. Parâmetros Curriculares Nacionais (5ª a 8ª Séries): Matemática. Brasília: MEC/SEF.

Búrigo, E. Z. (1989). *Movimento da matemática moderna no Brasil: estudo da ação e do pensamento de educadores matemáticos nos anos 60*. [Dissertação de Mestrado em Educação], Universidade Federal do Rio Grande do Sul, Porto Alegre.

Búrigo, E. Z. (2017). Revisitações do passado: contribuições da História Cultural à crítica da pesquisa. *Revista de História da Educação Matemática*, 3 (2), 56-76.

Caldatto, M. E. & Pavanello, R. M. (2015). Um panorama histórico do ensino de geometria no Brasil: de 1500 até os dias atuais. *Quadrante*, XXIV (1), 103-128.

Camargo, K. C. (2009). O ensino da geometria nas coleções didáticas em tempos do Movimento da Matemática Moderna na capital da Bahia. [Dissertação de Mestrado]. Universidade Bandeirantes, São Paulo.

Castrucci, B.; Peretti, R. G. & Giovanni, J. R. (1976). *Matemática: 7ª série*. São Paulo: FTD.

____ (1976). *Matemática: 8ª série*. São Paulo: FTD.

Castrucci, B. & Giovanni, J. R. (s/d). *Matemática (São Paulo).* De acordo com os Guias Curriculares. 7ª série. São Paulo: FTD

_____________________________ (s/d). *Matemática (São Paulo).* De acordo com os Guias Curriculares. 8ª série. São Paulo: FTD.

Chartier, R. (1991). O mundo como representação. Tradução: Andréa Daher e Zenir Campos Reis. *Estudos Avançados*, 11 (5), 173-191.

Chervel, A. (1990). História das disciplinas escolares: reflexões sobre um campo de pesquisa. *Teoria & Educação*, 2, 177-229.

Cury, F. G.; Morais, M. B. & Garnica, A. V. M. (orgs.) (2022). *História da educação matemática: desenvolvimento e consolidação de um campo de pesquisa*. São Paulo: Editora da Física.

D'Ambrosio, B. S. (1987). *The dynamics and consequences of the modern mathematics reform movement for Brazilian mathematics education.* [Tese de Doutorado]. Indiana University, EUA.

Dassie, B. A. & Costa, D. A. (orgs.) (2018). *História da educação matemática e formação de professores.* São Paulo: Editora da Física.

Duarte, A. R. S. (2007). *Matemática e Educação Matemática: a dinâmica de suas relações ao tempo do Movimento da Matemática Moderna no Brasil*. **[**Tese de Doutorado]. Pontifícia Universidade Católica de São Paulo, São Paulo.

Faria Filho, L. M.; Vidal, D. & Paulilo, A. L. (2004, jan/abr.). A cultura escolar como categoria de análise e como campo de investigação na história da educação brasileira. *Educação e Pesquisa*, 30 (1), 139-159.

DOI: 10.20396/zet.v30i00.8665149

Ferreira, A. C. C. (2006). *Propostas Pedagógicas de Geometria no Movimento Paranaense de Matemática Moderna*. [Dissertação de Mestrado]. Pontifícia Universidade Católica do Paraná, Curitiba.

Garnica, A. V. M. (org.) (2016). *Pesquisa em História da Educação Matemática no Brasil: sob o signo da pluralidade.* São Paulo: Livraria da Física.

Julia, D. (2001, jan/jun.). A cultura escolar como objeto histórico. *Revista Brasileira de História da Educação*, 1, 9-43.

Kline, M. (1976). *O fracasso da matemática Moderna.* Trad. Leonidas Gontijo de Carvalho. São Paulo: IBRASA.

Leme da Silva, M. C. (2008a). A geometria nos congressos nacionais de ensino de matemática. In: Búrigo, E. Z.; Fischer, M. C. B. & Santos, M. B. (orgs.). *A Matemática Moderna nas escolas do Brasil e de Portugal: novos estudos.* Porto Alegre: Redes Editora, 69-80.

Leme da Silva, M. C. (2008b). A geometria escolar moderna de Osvaldo Sangiorgi. In: Valente, W. R. (org.). *Osvaldo Sangiorgi – um professor moderno.* São Paulo: Editora Annablume, 69-93.

Leme da Silva, M. C. & Camargo, K. C. (2008). Martha Dantas: o ensino da geometria na Bahia. *Revista Diálogo Educacional.* 8 (25), 701-714.

Leme da Silva, M. C. & Pinto, T. P. (orgs.) (2020). *História da Educação Matemática e Formação de Professores: aproximações possíveis.* São Paulo: Editora Livraria da Física, 2020.

Leme da Silva, M. C. (2022). Um breve panorama sobre a História da educação matemática no Brasil. *Revista de História da Educação Matemática*, 8, 1-17.

Marques, A. S. (2005). Tempos pré-modernos: a matemática escolar dos anos 1950. [Dissertação de Mestrado]. Pontifícia Universidade Católica de São Paulo, São Paulo.

Mendes, I. A. & Gonçalves, F. D. S. (2020). Caracterização da pesquisa brasileira em Dissertações e Teses sobre História da Educação Matemática (1990-2010). *Revista Colombiana de Matemática Educativa,* 5, 55-69.

Oliveira, M. C. A.; Leme da Silva, M. C. & Valente, W. R. (org.) (2011). *O Movimento da Matemática Moderna: história de uma revolução curricular.* Editora UFJF, Juiz de Fora.

Pavanello, R. M. (1989). *O abandono do ensino de geometria: uma visão histórica*. Dissertação de Mestrado em Educação. Universidade Estadual de Campinas, Campinas.

Pavanello, R. M. (1993). O abandono do ensino de geometria no Brasil: causas e consequências. *Zetetiké*, 1, 7-17.

Pereira, D. J. R. (2005). *História do Movimento Democrático que criou a Sociedade Brasileira de Educação Matemática – SBEM*. [Tese de Doutorado]. Universidade Estadual de Campinas, Campinas.

Rios, M. S. B. (2010). A proposta de ensino da geometria nos livros do GRUEMA. [Dissertação de Mestrado]. Universidade Bandeirantes, São Paulo.

Sangiorgi, O. (1964). *Matemática, para a terceira série ginasial.* Companhia Editoria Nacional, São Paulo, 78ª Edição.

____________(1969). *Matemática curso moderno – 3º volume para os ginásios.* Companhia

DOI: 10.20396/zet.v30i00.8665149

Editora Nacional, 6ª Edição.

São Paulo (Estado). (1975). Secretaria de Educação. *Guias Curriculares para o ensino de 1º grau*. São Paulo: CERHUPE

Soares, F. S. (2001). *Movimento da Matemática Moderna no Brasil: Avanço ou Retrocesso?* [Dissertação de Mestrado]. Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro.

Stephan, A. M. (2000). *Reflexão histórica sobre o Movimento da Matemática Moderna em Juiz de Fora.* [Dissertação de Mestrado]. Universidade Federal de Juiz de Fora, Juiz de Fora.

Valente, W. R. (2007). História da Educação Matemática: interrogações metodológicas. *REVEMAT – Revista Eletrônica de Educação Matemática*, 22, 28-49.

_______________ (2008a). Osvaldo Sangiorgi, um *best seller*. In: Valente, W. R. (org.). *Osvaldo Sangiorgi – um professor moderno.* São Paulo: Editora Annablume, 13-41.

_______________ (2008b). *For a comparative history of mathematics teaching.* Palestra proferida no ICME 11. México.

Valente, W. R. (org.) (2014). *História da Educação matemática no Brasil: Problemáticas de pesquisa, fontes, referências teórico-metodológicas e histórias elaboradas*. São Paulo: Livraria da Física.

Vitti, C. M. (1998). Movimento da matemática Moderna: Memória, Vaias e Aplausos. Tese de Doutorado. Universidade Metodista de Piracicaba.