



Significance indications from different semiotic systems in the algebraic thinking

Indícios de significação a partir de diferentes sistemas semióticos no pensamento algébrico

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Abstract

This article is an excerpt from an investigation carried out in a classroom in the 7th year of teaching elementary school in a state public school in the countryside of São Paulo. It is an investigation of the own practice, with a qualitative approach, which is supported by the cultural historical psychology and studies of Luis Radford for the discussion of the elaboration of algebraic thinking. The purpose of this text is to analyze the evidence of the process of producing meanings with the use of different semiotic systems for the representation of algebraic thinking. Lessons were videotaped based on the resolution of problems, focusing on the construction of algebraic generalizations, from the observation of symbolic-numeric sequences. The analysis of an interactive episode that occurred during the synthesis of the students' productions involving one of the proposed problems revealed signs of significance for the use of formal algebraic language.

Keywords: elementary school; algebraic thinking; algebraic language; cultural historical psychology.

Resumo

O presente artigo é recorte de uma investigação realizada em uma sala de aula do 7.º ano do ensino fundamental em uma escola pública estadual do interior de São Paulo. Trata-se de uma investigação da própria prática, de abordagem qualitativa, que se apoia na perspectiva histórico-cultural e nos estudos de Luis Radford para a discussão da elaboração do pensamento algébrico. O objetivo deste texto é analisar os indícios do processo de produção de significados com a utilização de diferentes sistemas semióticos para a representação do pensamento algébrico. Foram videogravadas aulas pautadas na resolução de problemas, tendo como foco a construção de generalizações algébricas, a partir da observação de sequências simbólico-numéricas. A análise de um episódio interativo ocorrido durante o processo de síntese das produções dos alunos envolvendo um dos problemas propostos revelou indícios de significação para a utilização da linguagem algébrica formal.

Palavras-chave: Ensino Fundamental; Pensamento algébrico; Linguagem algébrica; Perspectiva Histórico-cultural.

Introduction

The teaching of algebra, historically, has been marked by the formalism of symbolic representations, often leaving aside the possibility of using other systems of representation to express this form of thought. A large literature addresses the issues related to algebraic

Submetido em: 12/04/2021 – **Aceito em:** 17/11/2022 – **Publicado em:** 14/12/2022

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thinking and has enabled pertinent discussions about the subject; however, many of these discussions do not reach the classrooms. There are good teaching materials to be used in the classroom, but they are not always understood by teachers, since this requires a break with formalistic conceptions of teaching. However, the investigation of one's own practice has the potential to encourage a break with these models, since it promotes the teacher's study and reflection on the theme, the ways of conducting the class, and the interactive movement with the students.

In this perspective, the sample presented here is from a master's research of the first author³, who investigated his own practice, analyzing the processes of meaning in the development of algebraic thought in a 7th grade classroom of a school belonging to the São Paulo state system.

Among the possible approaches to the theme, we chose to analyze the signs of meaning production of the different semiotic systems for the representation of algebraic thought.

The text is organized in three sections: initially we will present the theoretical reflections based on the cultural-historical theory and on the studies of Luis Radford about the semiotic systems of representation; next we will describe the context where the research was conducted, highlighting the methodological procedures for the organization of the classroom and data production; in the third section we will report the interactive episode and its analysis; and we will finish with some considerations.

Our pillars

Starting from our objective for this text, we present here some fundamental concepts, from the cultural-historical theory, to analyze the processes that occur during the elaboration of algebraic thought, when using the language systems in its rhetorical and symbolic representation.

We argue that all knowledge is built historically, from the social relations that take place between subjects, and is (re)elaborated by each one of them, based on the meanings produced. In this process, signs play a central role. Fontana and Cruz (1997, p. 67) help us understand this basic concept:

The sign is compared by Vygotsky to the instrument⁴ and is called by him a "psychological instrument". Everything that is used by man to represent, evoke, or make present what is absent constitutes a sign: the word, the drawing, the symbols (such as the flag or the emblem of a soccer team), etc.

Therefore, the understanding of how we elaborate concepts inevitably goes through the mediation of signs that are mobilized for the construction of meanings. Mediation, therefore, assumes a role of utmost importance for the individual to produce his meanings,

³ Guided by the second author.

⁴ Fontana and Cruz (1997, p. 66) define instrument as everything that interposes itself between man and the environment, extending and modifying its forms of action.

because knowledge is (re)constructed from social relations with the other (social subject). According to Fontana and Cruz (1997, p. 73), based on the Vygotskian theory

to consolidate and master autonomously the cultural activities and operations, the child [adolescent] needs the mediation of the other. The child's mere contact with the objects of knowledge or even immersion in informative and stimulating environments does not guarantee learning or necessarily promotes development, since it does not have, as an individual, instrumental to reorganize or recreate the cultural process alone.

So, there is no way to understand the processes aimed at the (re)elaboration of knowledge without mediation, that is, contact with the other. We, as historically constituted beings, integrate a chain of links, which give subsidies to all the knowledge generated by us, as a society.

Mediation is a process that allows semiotic elements (symbols) to develop and mobilize in the relationship with the other, enabling a favorable scenario for the production of meanings. The sign, as a human construction, acts as a mediating element, which refers to something; as an operator, which makes it create; or even as a transformer, which modifies an existing meaning. Smolka (2004, p. 56) collaborates with us to understand the concept of signification:

[...] signification implies but is not restricted to representation. Representation, as the possibility of forming images, ideas, thoughts, has a character, or functions, on an individual level. But these images, ideas, thoughts are not formed, are not composed independently of the relations between persons, outside of the web of significations, that is, without the mediation, the operation with signs. The sign, as that which is produced and stabilized in interpersonal relationships, acts, resonates, reverberates in the subjects. Its characteristic is impregnation and reversibility, that is, it affects the subjects in (and in the history of) relations.

The process of construction of meanings can be understood as an action of (re)construction of knowledge by the individual, having as a conductive factor the mediation performed by the other, from the operation of semiotic elements (signs) necessary to build a favorable environment for this whole process.

Returning to our research object, which is based on the observation of the use of languages during the process of elaboration of algebraic thought, we need to understand how thought and language interrelate. For this, we resort to Vygotsky (2001, p. 396), who tells us that "thought and language are not linked to each other by a primary bond; what happens is that thought arises, modifies and expands in the process of the very development of thought and word".

Although we understand that thought and the use of language are consolidated as two distinct processes, these activities do not occur independently.

The constitution and appropriation of the word, as a sign that is being mobilized, are not simply external representations of thought, because the word, as a unit of analysis, joins thought and the production of language. That is, the word as a semiotic instrument, through

mediation processes, is endowed with meaning by the subject, so as to structure the dynamics of concept (re)construction, as Vygotsky (2001, p. 170) shows us:

Concept is impossible without words, thinking in concepts is impossible outside of verbal thinking; in this whole process, the central moment, which we all have the grounds to be considered a cause arising from the maturation of concepts, is the specific employment of the word, the functional employment of the sign as a means of concept formation.

Therefore, the word, semiotic instrument, acts in a way to build the external representation of thought, goes through the mediation process, which enables the use of the word in a specific way, endowed with the meaning built by the subject, to elaborate and externalize the concept.

In order to deal with an investigation that will be conducted around algebraic thinking, it is essential that we explain our understanding of the theme. In line with the cultural-historical theory, we rely on Radford (2006a, p. 2, our translation⁵), who lists three constituent elements of algebraic thinking:

The first deals with the sense of indeterminacy that is proper to basic algebraic objects such as unknowns, variables, and parameters. It is indeterminacy (as opposed to numerical determinacy) that makes it possible, for example, to replace a variable or an unknown object with another; it does not make sense to replace "3" with "3", but it may make sense to replace one unknown with another under certain conditions. Second, indeterminate objects are treated analytically (...). Third, what makes algebraic thinking also is the peculiar symbolic mode it has for designating its objects. Indeed, as the German philosopher Immanuel Kant suggested in the 18th century, while the objects of geometry can be represented ostensibly, unknowns, variables, and other algebraic objects can only be represented indirectly, by means of constructions based on signs (see Kant, 1929, p. 579). These signs can be letters, but not necessarily. Using letters is not equivalent to doing algebra.

Therefore, algebraic thinking emerges from situations in which certain mathematical objects are treated from an "indeterminacy", whose representations are constructions based on signs. In particular, in this article, we will take situations linked to the analysis of numerical or symbolic patterns, seeking to build an algebraic generalization for the observed regularity.

Radford (2006b) argues that the process of constructing an algebraic generalization is based on the ability to perceive communalities in particular cases, belonging to the pattern studied (called $p_1, p_2, p_3, \dots, p_k$), and allows extending them to subsequent terms

⁵ Original text: "The first deals with a sense of indeterminacy proper to basic algebraic objects, such as unknowns, variables, and parameters. It is indeterminacy (as opposed to numerical determinacy) that makes it possible, for example, to replace one unknown variable or object with another; it does not make sense to replace "3" with "3", but it may make sense to replace one unknown with another under certain conditions. Second, indeterminate objects are treated analytically. (...) Third, what makes algebraic thinking algebraic is also the peculiar symbolic way it has of designating its objects. Indeed, as the German philosopher Immanuel Kant suggested in the 18th century, while objects of geometry can be represented ostensibly, unknowns, variables, and other algebraic objects can only be represented indirectly, through constructions based on signs (see Kant, 1929, p. 579). These signs can be letters, but not necessarily. Using letters is not equivalent to doing algebra."

($p_{(k+1)}, p_{(k+2)}, p_{(k+3)}, \dots$). In this way, the observed regularity is generalized, generating an expression through which one can have direct access to any term of the sequence. We can better observe this process in the flowchart below:

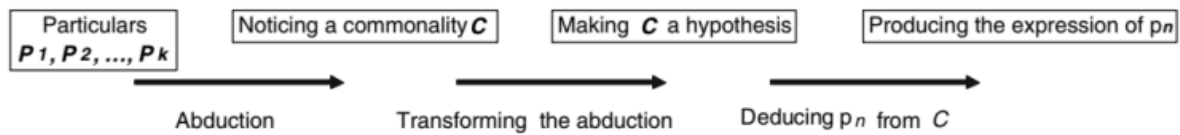


Figure 1: Architecture of algebraic generalization of patterns

Source: Radford (2007, p. 3)

We call attention to the risk of building a generalization that is not based on the conditions established for the constitution of algebraic thought. This type of generalization stems from the observation of a recursive pattern, i.e., makes use of a previous term to get the next term of the sequence, but does not make the relationship between the position of the term in the sequence and the number that makes up the term. Radford (2007, p. 2, our translation⁶) helps us understand the difference between what we call algebraic generalization and arithmetic generalization:

The dividing line between arithmetic and algebraic generalization of patterns must therefore be located in differences in what is computable within one domain as opposed to the other. Although in both domains [in arithmetic and algebraic] some generalization will certainly occur, in algebra, a generalization will lead to results that cannot be achieved within the arithmetic domain.

To externalize the constructions produced from situations involving these patterns, we can use several systems of representation. By common sense, it may seem that this representation should be given only by an alphanumeric semiotic system, based on the algebraic writing of the pattern. Although the development of skills aimed at understanding this semiotic system is clearly empowering for further studies in algebra, Ribeiro, and Cury (2015, p. 14) say that

Effectively, at the beginning of the work with algebra, we can express a problem in everyday language, we think about it, we try to express it with the help of symbols - which, depending on the age group of the students, can be figures or letters - and we arrive at the algebraic language that, in turn, through generalization, allows us to use the same thought in other problem situations.

It is also important to point out that the transition from a system of representation that makes use of native language to a system based on alphanumeric representation, constituted by formal algebraic language, does not happen by simple transposition. As Radford (2007, p. 02) states

⁶ Original text: “The dividing line between the arithmetic and algebraic generalization of patterns should hence be located in differences in what is calculable within one domain as opposed to the other. While in both domains some generalizations do certainly occur, in algebra, a generalization will lead to results that cannot be reached within the arithmetic domain.”

[...] algebraic language emerged as a technical tool and later evolved socio-culturally to a level of being considered as a mathematical object. Usually, in the modern curriculum, algebraic language appears from the beginning as a mathematical object itself. Taking this aspect into account, is any change possible regarding the introduction of algebraic language in the classroom

That is, the introduction of formal algebraic language in teaching practices should consider that it is a historically constructed object and, therefore, its use should be gradual, aiming at the production of meanings related to this new semiotic system.

Moreover, using different semiotic systems to represent mathematical situations contributes to building a scenario rich in details, thus facilitating the understanding of the context that is being analyzed. This is what we observe in the statements of Brizuela and Earnest (2008, p. 275, our translation⁷):

A solitary representation brings clarity to some part of mathematics, but that clarity hides an indistinct treatment of other mathematical attributes. The ambiguity inherent in any representation needs the incorporation and support of additional representations and their underlying mathematical concepts or structures to fully appreciate the nuances of a mathematical situation and thus resolve some ambiguity in any system.

That is, the use of different semiotic systems, considering verbal and nonverbal representations, helps to eliminate possible gaps or even to realize the communalities needed to build a generalization from numerical or symbolic pattern.

As a way to integrate the concepts arising from the cultural-historical theory to the assumptions adopted here about algebraic thinking, we rely on the cultural theory of objectification, developed by Radford (2000), which considers, from Bakhtinian and Marxist philosophical conjectures, that the way we know the objects of knowledge and think about them is framed by historically constructed meanings, which go beyond the content of the activity itself in which the act of thinking occurs.

We see algebra learning as the appropriation of a new and specific mathematical way of behaving and thinking, dialectically intertwined with a new use and production of signs whose meanings are acquired by students as a result of their social immersion in mathematical activities. (Radford, 2000, p. 241, our translation⁸)

The semiotic triangle presented by the cultural theory of objectification (Figure 2) synthesizes the relationships that are established between the sign, its object (object of knowledge) and its meaning. For its analysis and understanding, we highlight the interactions that occur with the subjects and influence the signs and the production of meanings.

⁷ Original text: *A lone representation brings clarity to some part of the mathematics, but this clarity hides an indistinct treatment of other mathematical attributes. The inherent ambiguity of any one representation necessitates the embodiment and support of additional representations and their underlying mathematical constructs or concepts to fully appreciate the nuances of a mathematical situation, and thereby resolve some ambiguity in any one system.*

⁸ Original text: *We consider the learning of algebra as the appropriation of a new and specific mathematical way of acting and thinking which is dialectically interwoven with a novel use and production of signs whose meanings are acquired by the students as a result of their social immersion into mathematical activities.*

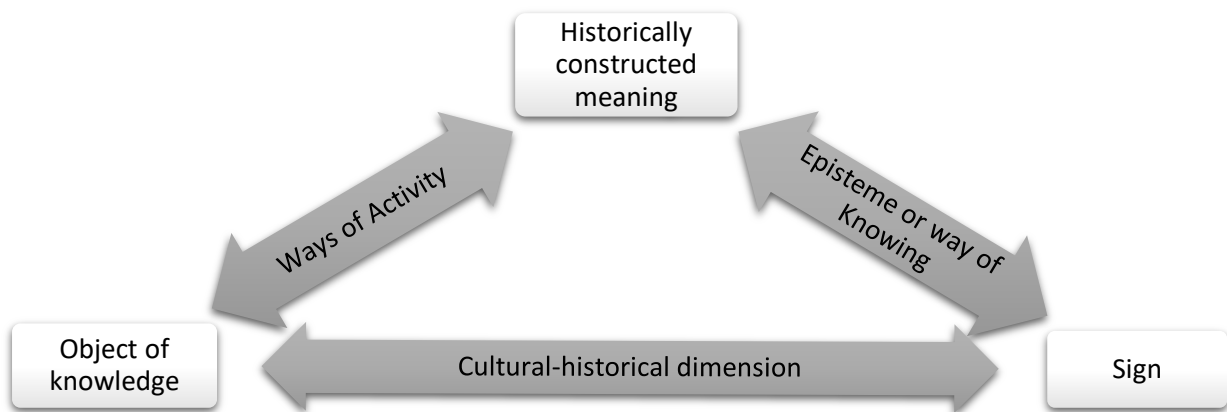


Figure 2: Semiotic Triangle - relation meaning, object of knowledge and sign

Source: Radford (2006b, p. 109, our translation and adaptation)

Therefore, we understand that the mathematical objects addressed here are historically generated. "More precisely, mathematical objects are fixed patterns of reflexive activity embedded in the world in the constant exchange of social practice mediated by instruments" (Radford, 2006b, p. 111, our translation⁹).

The entire production of meaning and, consequently, the appropriation of concepts occurs through the tension between students' subjectivity and the means of semiotic objectification.

These are the assumptions that guide our study. Next, we will present our methodological choices for data production and analysis.

Organizing our ways

The cut presented here refers to the master's research of the first author (Pereira, 2019), and the data were produced in his classroom, characterized as an investigation of his own practice, focusing on the production of meanings that emerge during the process of development of algebraic thinking. As occurs in a master's research, student and supervisor interacted throughout the process, both in the theoretical and methodological construction of the work, and in the shared analysis of the data.

The research, of qualitative approach, was conducted in a school unit of the state public network, located in the outskirts, in a small town in the state of São Paulo. The school's student body, at the time, was composed of about 510 students, with 260 enrolled in elementary school, distributed in the morning and afternoon periods, and 250 in high school, in the morning and evening periods.

⁹ Original text: *De manera más precisa, los objetos matemáticos son patrones fijos de actividad reflexiva incrustados en el mundo en cambio constante de la práctica social mediatizada por los artefactos.*

The episode highlighted here occurred in a 7th grade classroom, formed by 32 students, aged between 12 and 13 years old, the vast majority living in the same neighborhood where the school is located.

The data analyzed here were taken from the production conducted for a larger study, and the selection of the interactive episode, which is not included in the text of the dissertation, is justified by the emergence of questions related to the research objective chosen for this article.

The classroom was organized in groups of 4 to 5 students, thus prioritizing the interaction among students to discuss the proposed tasks, since the genesis of meaning production emerges from social relations, which presupposes the interactions among students and between them and the teacher. The classes were videotaped, in order to capture the students' voices and gestures, enabling a micro genetic analysis, consistent with the cultural-historical theory. Góes (2000) presents videotaping as a fundamental procedure for research that seeks to apply micro genetic analysis, since it allows the observation of processes that emerge in the minutiae of social relations that are presented in the object of analysis.

The videotapes were transcribed and used together with the research diary produced by the teacher-researcher. In the transcription the episodes were organized in sequential shifts, designated by the initial T, followed by numbering (for example: T01, T02, T03...). The students are identified by fictitious names¹⁰, and the teacher's lines are indicated by the letter P. All direct lines are written in italics. The data in brackets refer to the descriptions of the gestures and other details important for the characterization of the data.

In the search for a teaching and learning situation that could be characterized by its potential for dynamic interactions and discussions, we defined problem solving as the methodological option for the class used as the basis for building the research scenario. This choice is justified because

when students engage with well-chosen problem-solving-based [problem] tasks and focus on the methods of solving them, what results are new understandings of the mathematics embedded in the task. While students are actively looking for relationships, analyzing patterns, figuring out which methods work and which don't, and justifying results or evaluating and challenging the reasoning of others, they are necessarily and favorably engaging in reflective thinking about the ideas involved. (Van de Walle, 2009, p. 57)

Thus, as problems to be investigated in our research scenario, we chose to use a sequence of tasks consisting of nine problems, taken from the curriculum material of the public network of the State of São Paulo (Programa São Paulo Faz Escola, 2017)¹¹. The choice of this material is justified by the fact that the school where the investigations were developed is part of the state public school network and of use of the teacher-researcher and

¹⁰ The present research was approved by the Ethics and Research Committee (CEP), under process CAAE 69062517.0.0000.5514.

¹¹ Program created by the São Paulo State Education Department (2008), which consists of an official curriculum as well as teaching materials standardized in all schools belonging to this sphere.

also because the tasks showed promising to discuss the development of algebraic thinking, in particular the processes of elaboration of generalization. The material underwent minor changes, in a shared action between the student and the researcher, in order to align it to the class methodology proposed by the researcher-teacher. The lesson planning was based on the model suggested by Smith and Stein (2012), which defines three fundamental phases for the teacher to orchestrate the discussions in a productive way, regarding the production of meanings: launching; exploration; discussion and synthesis.

The "launching" phase is the moment when the teacher presents the problem to be solved by the students. This is when he defines what kind of production, he expects his students to conduct, as well as the tools available for this. In the "exploration" phase, the students should observe the proposed problem, raise possibilities for solving it, and discuss them in groups in order to draw a possible resolution strategy. And finally, in the "discussion and synthesis" phase, the students present to the class as a whole their elaborate solution strategy. Here everyone has the opportunity to give their opinion on the socialized constructions.

From the organization of the lesson in these three phases, Smith and Stein (2012) suggest that the teacher apply five practices, so that he or she can orchestrate these discussions, always keeping the focus on the object of knowledge that he or she intends to work on in the lesson in question - these are the practices of "anticipating", "monitoring", "selecting", "sequencing" and "connecting".

The authors describe the practice of "anticipating" this way:

Anticipating student responses involves setting thoughtful expectations about how students might mathematically interpret a problem, the set of strategies—correct and incorrect—that might be used to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn. (Smith & Stein, 2012, p. 8, our translation¹²)

The practice of "anticipating" takes place before the beginning of the three phases described here, since it presents itself as a moment prior to those to be developed in the classroom, and shows itself as an action of great complexity, because here the teacher will have to think about how he expects his students to think, simulating different possibilities. However, although of considerable difficulty, it will be fundamental for the next practices. The teacher prepares for possible doubts or questions posed by the students, even though unpredictability in the classroom cannot be ruled out; many times, the resolutions presented by the students are not expected by the teacher but may bring other perspectives to the discussion.

¹² Original text: *Anticipating students' responses involves developing considered expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—that they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn.*"

The practice of "monitoring" takes place during the "exploration" phase, when the teacher must observe the progress of the discussions underway in the groups. Here it is important to highlight that the proposal brought by Smith and Stein (2012) is not based on a purely passive monitoring process by the teacher, because it is essential the interaction between students and teacher, and the latter should perform mediations through the conscious use of the word, seeking to mobilize the concepts necessary for the production of meaning.

Then we move on to the practices of "selecting" and "sequencing", which will also be developed during the "exploration" phase, because it is when the teacher selects and sequences the strategies that are being developed by the students, which should occur during socialization, to favor the construction of a logical sequence in the presentations, having in mind the objective that is sought with the proposed situation.

The last practice, the "connect", will be developed during the last phase of the lesson. At this moment, it is the teacher's task to point out the points of convergence and divergence between the different resolution strategies presented by the students, trying to create the proper connections between these productions. Since our proposal is based on the construction of meanings through interactions between subjects, we consider what Smith and Stein (2012, p. 11) define:

the teacher can help students make judgments about the consequences of different approaches to the range of problems that can be solved, about the accuracy and efficiency in solving them, and about the kinds of mathematical patterns that can be most easily discerned. Rather than having mathematical discussions that consist of picking apart the different ways of solving a particular problem, the goal is to have students' socializations build on each other to develop powerful mathematical ideas.

Therefore, from these methodological perspectives, the classes were planned and constituted the research scenario. The episode, object of analysis in this article, occurred during the "discussion and synthesis" phase, during the socialization of the productions related to the seventh problem applied. It is important to consider that the students had already been working for six classes with the perception of regularities in different types of sequences, but still without using the symbolic language; the generalizations were produced with the native language.

We used the micro genetic analysis (Góes, 2000), which considers that the emergence of meaning and conceptual elaboration processes takes place in the minutiae of social interactions. These interactive moments were videotaped and transcribed. In the analysis we chose interactive moments among students and between them and the teacher, in which concepts related to algebraic thinking emerged. Each episode presents dialogues that are organized in turns, which are numbered (T01, T02,) and, in the analytic process, the signs of meaning production and conceptual elaboration are highlighted.

We will present, next, the analysis of the selected episode.

Our production


The episode described here occurred during the discussion and synthesis phase of the class held on 11/08/2017, from the situation proposed by problem 7 (P7). Therefore, this was a moment of socialization of the productions made by each group. In Figure 3 is the proposal of P7.

Sequência extraída do material curricular oficial da Secretaria da Educação do Estado de São Paulo.

Objetivo: realizar generalizações com linguagem escrita e com expressões matemáticas que envolvem o uso de letras.

A) Os grupos discutirão e proporão uma solução para cada um dos itens descritos abaixo, registrando suas conclusões no material curricular utilizado. (15 min.)

11. Observe a sequência de bolinhas e responda às perguntas.



a) Desenhe as bolinhas que devem ocupar as posições 5 e 6.

b) Preencha a tabela, associando o número de bolinhas com a posição da figura.

Posição	1	2	3	4	5	6
Número de bolinhas						

c) Quantas bolinhas terá a figura que ocupa a 10ª posição?

d) E a figura que ocupa a 45ª posição?

e) Descreva, em palavras, o padrão de formação dessa sequência.

Figure 3 - Problem 7 (P7)

Source: São Paulo (State) (2014, pp. 55-56)

Here is the transcript of the episode¹³ :

¹³ In order to preserve the identity of the participating subjects, as already mentioned in the methodological section of this article, fictitious names were used to identify them.

DOI: 10.20396/zet.v30i00.8665274

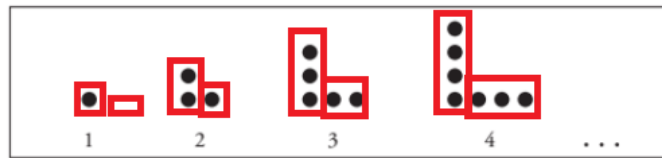


Figure 5 - Representation of student Ivo's speech, in T10

Source: own author

T 01 Q: Tell me how you did to find position 5 and position 6. [the teacher had asked the groups to socialize the strategies created to solve item "c" of the task in question. He refers to position 5 and 6, trying to help in the process of socialization of the item in question]

T 02 Gisele: Since we were already in 6th position [because we had already socialized item "a" of the task], we went counting up to 10th position, then we gave 19. We went counting 2 by 2.

T 03 Q: Until you got to 10th [place]?

T4 Gisele: Yes, 13, 15, 17, and 19.

T 05 Q: Okay, guys, is there a different way?

T 06 Ivo: Here, professor, we did like this: we took the 10, because it was the 10th position, plus the back number, which is 9, so it became $10 + 9 = 19$.

T 07 Q: Okay. But why did you do it like this?

W 08 Ivo: It's like this. Since I want the 10th position, so I took the 10, then I added it to 9, which is the number that comes before.

T 09 Paula: I don't understand!

T 10 Ivo: It's like this, in the picture, there's a little ball standing up and a little ball lying down [refers to the balls in the horizontal and vertical lines that form the figures of each position of the sequence]. There, standing, you always have the same position, and lying down is the number before. [presents his vision as illustrated by figure 5].

T 11 Ivo: It's the position and what's behind it.

T 12 Q: Another different way to solve that one, then. Do you understand, Paula?

T 13 Paula: I get it now.

T 14 Q: Did anyone do it another way?

T 15 Several students [after a silence]: No, teacher.

T 16 Q: Let's go to the next one, then. Item "d" asked us to find the 45th position. So, how did it turn out, guys?

T 17 Kelly: We did it twice...

T 18 Paula: Two times 45, which is 90. Then Kelly said we had to take 1, because...

T 19 Kelly: Like in 3 [refers to the picture in the 3rd position of the sequence], it would be 2 times 3, which is six, but it's actually minus 1. There have to be 5 balls.

T20 Paula: So, we did the same thing. 45 times 2, which is 90, then take away 1. That's 89.

T 21 Q: Does that apply to all the pictures? Did you test it?

T 22 Kelly: It does, teacher, if it's 4 [refers to the figure that occupies the 4th position in the sequence], it gets 4 times 2, which gives 8. Take away 1 and it gets 7. It works out.

T 23 Q: Okay, I think that's a good way, very cool. I think you have one group that thought a little differently. Which one was it?

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W 24 John: Here, teacher. We saw that they are all odd and that it is increasing every 2 by 2 [refers to the number of balls in each figure in the sequence].

T 25 Q: But I think there was something else you had seen, what was it again?

W 26 John: Eduardo who saw it, do you want to talk [Eduardo]?

W 27 Edward: We took the 45, added it to 45, gave 90. Then we take 1, that's 89.

W 28 Q: Okay, one more way then. Did anyone else do it differently? [silence]. Well, I don't think so. Let's go to the next one, then. Item "e." It asked you to write in words the formation pattern of that sequence. Cool. So, how did you do it? The girls' group over there.

T 29 Eloá: It was like this, the position 45 times two, gives 90.

T 30 Q: But why did you multiply 45 by two?

T 31 Eliana: Because it's the position.

T 32 Q: So, we took the position and...

T 33 Eloá: We multiply by 2.

T 34 Q: But why multiply by 2?

T 35 Paula: I think it's because it's increasing by 2 by 2.

W 36 P: That's right!

T 37 John: That's it, teacher, if you multiply by 2, it's just like the multiplication table, then it's going to increase by 2 by 2.

W 38 Eloah: Then, from the result, take 1.

T 39 Q: So, how can I write it on the blackboard?

T 40 Eloá: Multiply the position by 2, then take a 1. [Teacher writes this on the blackboard]

T 41 Q: So, guys, that's cool. Do you have any other ways to explain it?

T 42 John: You can write its position plus position, then subtract 1.

T 43 Q: I'll write here, the position plus the position, then subtract 1 [Teacher writes this on the blackboard]. It can also be. Both groups, John's, and Eloah's, said you need to subtract 1, I wonder why that's happening in the sequence?

T 44 Ivo: I think it's because of the lying line [refers to the horizontal line of each figure in the sequence, according to Figure 5]. It's always 1 less than the position. That's why you have to take 1.

T 45 Q: What do you think, guys?

W 46 John: I think Ivo is right, professor. It makes sense.

T 47 Eloah: The line lying down is always 1 less.

T 48 Q: That's right, and, as Ivo said, there we find the back number, the number that precedes the position. Very good, guys. To finish, I have a question. Can we write these explanations using mathematical symbols instead of words?

W 49 John: I think so if you're asking! [laughter from the room]

W 50 Q: Very good point, John. But what would it look like here? [points to the writing brought in by Eloa's group "multiply the position by 2, then take a 1"].

T 51 Ivo: You can write two times [makes the "×" sign] something, minus [makes the "-" sign in the air].

T 52 Q: That's a good way. But that something, what is it?

T 53 Eloah: It's the position.

T 54 Q: So, let's use a symbol to represent this position. Let's use a letter.

DOI: 10.20396/zet.v30i00.8665274

W 55 John: Can it be the letter "P"?

T 56 P: That's fine. What does it look like?

T 57 John: It'll be $2 \times P - 1$ [draws this in the air].

W 58 Q: Like this [writes expression on blackboard]?

W 59 John: That's right, teacher. And if it's our way [refers to the other way of expressing the sequence pattern brought up by the group "the position plus the position, then subtract 1"], it gets $P + P - 1$.

T 60 P: Okay, guys, so we were able to write the sequence pattern using mathematical language. Very good...

At the very beginning of this episode (T01), the teacher begins the discussion by asking the students to explain the strategy they had used to answer a previous item (item a), in order to recover the constructions performed during the socialization of this item. Here we highlight the intentionality of the teacher's action regarding the establishment of connections between the socialized items, thus seeking to gradually build the generalization of the observed pattern. The teacher considers the relationship between thought and language (Vigostki, 2001), as fundamental to this process.

In response to this request, Gisele (T02) uses the observation of a recursive pattern, one that makes use of the previous term of the sequence for the elaboration of the next one. Here we emphasize that there is the observation of the communality present in the sequence as to its linear growth "*We were counting from 2 to 2*". And we have, therefore, a process of elaboration of an arithmetic generalization (Radford, 2007).

Continuing the discussion, the teacher asks the students if anyone has constructed a different strategy (T05). Promptly, Ivo (T06) presents the strategy created by his group. In this contribution, we see that Ivo's group observed other communalities found in the sequence, establishing a relationship between the geometric pattern of organization of the balls in each figure, with the number of balls found.

The observation of communalities present in different fields leads to the production of various signs, which can function as important factors to develop the generalization intended as a goal, constituting semiotic instruments. It is through these signs that the subjects will have access to the object of knowledge, and through a dialectical process that is established between subjects, from the modes of activity (observation) the meaning is constituted, as depicted by the cultural theory of objectification (Radford, 2000).

In T07, the teacher asks the students to explain the strategy adopted. Then Ivo continues his explanation, highlighting the relationship that his group established from the observation of the geometric pattern represented in Figure 5 - horizontal line and vertical line of balls. Here we have evidence of the elaboration process of an algebraic generalization (Radford, 2007), because, through the strategy presented by Ivo's group, it is possible to obtain any term of the sequence in a direct way (Radford, 2006a) and thus establish a functional relationship between the position of the term and its number of balls.

When, in T16, the teacher asks about the strategies developed by the class for item d

of the proposed problem, in response, Kelly (T17) presents the solution found by her group. The strategy presented here also establishes a relationship between the position of the term in the sequence and the number of balls in the term. When we compare the strategies presented by Ivo's group and Kelly's group, we notice that the latter did not make use of the geometric pattern identified by Ivo's group.

After Kelly's group exposes the strategy created the teacher questions about the hypothesis in question, whether its validity applies to all the terms of the sequence. In T22, Kelly states that it does and presents tests of the hypothesis, related to the accessible terms of the observed sequence.

The observations made by the teacher during the exploration phase allowed him to know that there was another strategy constructed, which would bring contributions to the process of elaboration of the generalization. In view of this, the teacher (T23) requests that this strategy be socialized.

In T24, John begins his presentation, bringing communalities of the arithmetic scope, and then, after the teacher's questioning, in T27, Eduardo complements, presenting a strategy based on the relationship between the position of the term in the sequence and the number of balls that compose it. The generalization presented is based on the following thought: position + position - 1. We observe that they are using an additive principle, differently from the multiplicative principle presented by Kelly's group.

The strategies of João and Kelly's group start from the transposition of a figurative representation - geometric and numerical pattern - to a purely numerical representation, using the table proposed by the problem itself, in item b.

In the dynamic discussion that takes place during the socialization of the strategies developed, we can find evidence that these different representations end up contributing to the process of elaboration of algebraic generalization (Radford, 2007). Through them, we can obtain a clearer and more comprehensive representation of the mathematical situation observed, thus allowing access to a better detailing of the sequence (Brizuela & Earnest, 2008).

From John and Edward's statement (multiply the position by 2 and then subtract 1) in T34, the teacher asks, "*But why multiply by 2?*" In response, Paula states, "*I think it is because it is increasing by 2 by 2.*" In addition, John says: "Yes, teacher, if you multiply by 2, it's the same as the multiplication table, so it will increase by 2 in 2". In this turn, we notice signs of observation of the regularity that is established from a multiplication by a certain factor (factor 2).

Forwarding the discussion to the last item, the teacher asks how the generalization developed by the groups could be represented (written). Right away, Eloá - she is in the same group as Kelly - states (T40): "*Multiply the position by 2, then take away 1*".

In T42, John presents another possibility: "*You can write that it is the position plus the position, then subtract 1*". After that, in T43, the teacher questions the occurrence of

subtracting one unit in both writings. Answering the questioning, Ivo (T44) states that this is due to what occurs in the "*lying*" (horizontal) line of little balls, referring to the geometric pattern observed; and in T47 Eloá concludes that this "*line*" always has one less little ball than the vertical "*line*".

In T47 Eloá concludes that this "*line*" always has one less ball than the vertical "*line*". Noting that the representation written in the native language has already been materialized and aiming to propose a new form of representation, based on an alphanumeric system, directed to the formal algebraic language, the teacher asks the question contained in T48.

We highlight the answer given by John (T49), which brings up the legitimation of the speech made by the teacher, thus presenting his possible understanding of the intentionality of the teacher's speech. We can also interpret how much the students understand the relations present in a classroom didactic contract, because if the teacher asks, it is because he has an answer; John quickly manifests himself and his speech is validated by his classmates, through laughter. The teacher respects John's and the class's position and reformulates his question. The dialogue in the classroom is fundamental because the students can manifest their ways of thinking, and the teacher can rework his ways of questioning.

During the attempts to create this new system, based on the use of mathematical symbols to represent the generalizations elaborated, the teacher proposes (T54) the use of a letter to represent the position of the figure (variable), a situation of indeterminacy (Radford, 2006a). From this proposal, John suggests writing $2 \times P - 1$, to represent the generalization, which starts from the multiplicative principle, and $P + P - 1$ for the additive principle.

At this moment it is possible to find indications of the beginning of the process of meaning construction, with the use of this new semiotic representation system - the formal algebraic language. This transition does not happen directly, it is not a simple transposition. This process takes place through a process of meaning production of this new system.

We close P7 with the representation of the law of formation (the generalization) of the sequence in algebraic language.

To conclude...

By assuming that all knowledge is historically constructed from the relationship established among social subjects, we are defining that the teaching-learning process takes place through mediations performed with semiotic instruments by the participants of the social dynamic in which we are inserted. It was based on this principle that this article was built. Our goal was to analyze the signs of the process of meaning production regarding the use of different semiotic systems for the representation of algebraic thought. To this end, during the analysis we seek to present evidence of this elaboration process. The students' statements complement each other, and meanings are being produced, as John (T46) said: "*It makes sense*".

The episode highlights the importance of the organization of the class, in the practices

suggested by Smith and Stein (2012), because the teacher, in the exploration phase, already observes which groups have built strategies that can expand the discussion at the moment of synthesis. This occurs in T23, when the teacher says, "*I think you have one group that thought a little differently. What was it?*". In this way, the teacher mobilizes the students to communicate their strategies as they are validated.

The students participated in the discussion, and in their speech, it is possible to perceive marks of school mathematics, such as the use of the multiplication table: "*if you multiply by 2, it equals the multiplication table*" (T37). The students, in a 7th grade, have already studied the multiples; however, the discourse of the multiplication table is stronger than the use of the concept of 'is a multiple of'; to be in the multiplication table of 2 is to be a multiple of 2.

We present here the paths taken by 7th grade students in their initiation in the use of a new semiotic system. It is important to highlight that this process does not happen in a linear way. The constructs presented here are part of a larger research scenario, which was full of comings and goings, and the problem presented here was the seventh of the developed sequence. Therefore, the students were already immersed in a context in which they observed the sequences and looked for commonalities. Some generalizations in their native language had already been made, but in this problem, they needed to reach the symbolic representation, which was possible with the teacher's mediation.

Although the teaching of algebra is historically marked by the use of great formalism in its representation, what we argue is that algebraic thinking is not directly related to the use of this representational system.

The use of formal algebraic language allows for the continued development of mathematical thinking and expands the universe of possibilities, but the teacher must consider that its use must occur through the transformation of these symbols into real signs, historically constructed, which must be appropriated by the subjects that use them, with meanings. This is a process. It is important to pay attention to the fact that this process, as Radford (2007) states, is not consolidated by a simple transposition. By introducing the algebraic language, we are proposing the use of a new semiotic system.

The episode presented here is part of a larger context, in which we can identify several possibilities of constructions not addressed by the selected cut.

This article presents a possible interpretation to the data produced, based on the theoretical assumptions made here. Other interpretations may occur in the analysis of the same episode.

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