



School geometry in the early years: a history of movements in partnership with drawing

Geometria escolar nos anos iniciais: uma história de movimentos em parceria com o desenho

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Abstract

The present article takes categories built by the historical studies of the relationship between geometry and drawing that marked the different movements of the mathematics curriculum for the teaching of geometry, proposed to the primary school, from the 19th century to the mid-20th century. Regulations and school manuals of each period, which were intended to meet the norms, are used as a source. The results of current research dialogue with the past, which highlight perspectives for the teaching of geometry in the 21st century and its insertion in the BNCC. None of the three movements has been maintained in the way it was prescribed, however, there are traces of them in the 21st century scenario. The history of knowledge construction movements for the teaching of geometry can bring contributions and fruitful reflections to think, analyze and trigger proposals and relevant actions for pedagogical practices of geometry teaching.

Keywords: Free hand drawing; Geometric drawing; Natural drawing.

Resumo

O presente artigo toma categorias construídas pelos estudos históricos de relações entre geometria e desenho que marcaram os diferentes movimentos do currículo de matemática para o ensino de geometria, proposto ao curso primário, do século XIX até meados do século XX. Utilizam-se como fonte regulamentações e manuais escolares de cada período que tiveram como intenção atender às normativas. Dialogam com o passado os resultados de pesquisas atuais, que destacam perspectivas para o ensino de geometria no século XXI e sua inserção na BNCC. Nenhum dos três movimentos foi mantido da maneira como foi prescrito, entretanto, há vestígios deles no cenário do século XXI. A história dos movimentos de construção de saberes para o ensino de geometria pode trazer contribuições e frutíferas reflexões para pensar, analisar e desencadear proposta e ações pertinentes para práticas pedagógicas de um ensino de geometria.

Palavras-chave: Desenho à mão livre; Desenho geométrico; Desenho Natural

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Initial Considerations

The field of investigation on the history of geometry teaching is quite recent, in the process of being constituted. However, the few results point to the importance of the passage from the 19th to the 20th century, as a period marked by the circulation of new methodologies and educational reforms for the teaching of geometry in the international scenario (Barbin & Menghini, 2014). The period also corresponds to the arrival of pedagogical theories in the educational field, especially in the circulation and incorporation into teaching proposals, of the so-called intuitive method, which has in Pestalozzi² one of its main references.

In Brazil, studies on the history of geometry teaching have had a significant growth in the last two decades, with several research projects³, which took as their object of investigation the proposals for the teaching of geometry from the early years between the 19th century and the mid-20th century. Such projects produced numerous results, including the construction of a systematized representation of the past of school geometry in the early years of schooling, which pointed to three relevant partners in the process of construction and consolidation of a school geometry since the 19th century: drawing, manual work and measurements (Leme da Silva, 2021).

The projects that investigated the teaching of geometry in the first years of school in Brazil followed a methodological trajectory in accordance with umbrella projects, developed by the Research Group on the History of Mathematics Education - GHEMAT, based on the following research sources: normative legislation, pedagogical journal, school manuals, notebooks, tests and exams and finally, teaching materials. The initial project entitled Geometry and drawing in São Paulo primary education, 1890-1930, developed between 2010 and 2012, gathered, organized and examined the set of teaching programs of school groups in the state of São Paulo in the period. Therefore, the first results on the history of geometry teaching pointed to the analysis of regulations and, since then, we have been in contact with the challenges of research to be developed in the first years of schooling: the school culture of the primary course, which, as Julia (2001) already warned us, differs from the school culture of secondary courses:

² Johann Heinrich Pestalozzi (1746-1826), Swiss educator. Known for his action as a teacher, director and founder of schools, his ideas demarcate intuitive pedagogy, whose basic characteristic is to offer, as far as possible, sensitive data to the perception and observation of students. (Zanatta, 2012).

³ Projects developed under the coordination of Maria Célia Leme da Silva, since 2009, within the scope of the Research Group on the History of Mathematics Education – GHEMAT. The last two are cited: “Transformations of geometric knowledge in Brazilian primary education”, financed by FAPESP, between 2018 and 2020 and “A history of the professional knowledge of geometry teachers”, funded by the Universal Public Notice – CNPq, in progress. since 2019.

Norms and practices cannot be analyzed without taking into account the professional body of agents who are called upon to obey these orders and, therefore, to use pedagogical devices in charge of facilitating their application, namely, primary teachers and other teachers. ... by school culture it is also convenient to understand, when possible, the children's cultures that develop in schoolyards. (pp. 10-11)

In addition to children's culture, among other differences is the training of primary teachers, different from the training of secondary teachers. While the former takes different school knowledge as the object of their work, the latter is a specialist in an area of knowledge.

Later, in the project 'The practical dimension and the schooling of elementary geometric knowledge, between 2015 and 2016, of international cooperation with France, we had the opportunity to consult French drawing, geometry and primary school textbooks, which were translated, adapted to Portuguese in the 19th century. The comparison of the original French manuals with the Brazilian production allowed a more significant understanding of the teaching of geometry during the Brazilian Empire, given that the normative prescriptions of the first Brazilian courses, called "school of first letters", were succinct. At that time, there was not even the organization, so natural today, of ranking primary courses. The recommendations of content to be taught were general and broad, without delimitation by school year or subjects. Only after the Republic did we get closer to what we now call curriculum for the early years, with the creation of school subjects (in some states of the Federation, the expression school discipline was used). We also highlight the diversity of regulations and curricula, considering that each state had the autonomy to develop and change its own school program, with different school subjects in each one of them.

Faced with this particular reality, the historian Julia (2001) was again the one who warned us that "the historian knows how to make an arrow with any wood" (p. 17). In other words, the relevance of producing a history based on a normative diversity that was not well organized was established. Thus, understanding and interpreting the teaching of geometry, having as an object of investigation only the legislative norms, was an arduous task and, certainly, produced results that were often partial and incomplete.

From the inventory of other documents and the inclusion of researchers from other states in the GHEMAT, in other words, with the expansion and diversification of sources, the possibility of comparisons and the maturing of the historian's craft, the studies, after ten years of research, tend to make narratives about the past of mathematics teaching in the early years more consistent. According to Valente (2021), the book '*Histórias do ensino de geometria nos anos iniciais e seus parceiros: desenho, trabalhos manuais e medidas*' (Leme da Silva, 2021)⁴ resulted in the characterization of a mathematics of teaching. In summary, the book

⁴ Translator's Note (TN): Histories of teaching geometry in the early years and its partners: drawing, manual work and measurements.

did not intend to deepen theoretical studies on the subject, but rather to offer subsidies for the process of theorizing about school knowledge, given the difficulty and, perhaps, the impossibility of particularizing a school geometry for all of Brazil. Thus, the call for this thematic issue with the term “movements for the construction of knowledge” seems, in our view, very opportune, because, even without specifying, we are fully able to identify remarkable and long-lasting movements in the history of geometry teaching.

From the above considerations, this article starts from the results of historical research developed and takes as a methodological path the comparison of regulations, norms of different Brazilian states with school manuals, more specifically, with the analysis of questions destined to students, contained in these manuals to meet legal requirements, in different historical movements: 19th century (freehand drawing), late 19th century (geometric drawing) and first decades of the 20th century (natural drawing). It also adds an inventory of 21st century research, in dialogue with the past, and seeks to emphasize the drawing movements and permanences of as a methodological contribution to the teaching of geometry over time.

As main conclusions, pointed out in Leme da Silva (2021), studies on teaching proposals for geometry in Brazil indicate the close connection between geometry teaching and drawing throughout the 19th and early 20th centuries. It is not possible, in our view, to analyze and understand the teaching of geometry of the period, without considering the drawing, since the two school knowledges were constantly articulated. The article, entitled “Desenho e Geometria na escola primária: um casamento duradouro que termina com separação litigiosa” (Leme da Silva, 2014a)⁵, which examined the regulations of the state of São Paulo, deserves new considerations and understandings about the connections between drawing and geometry. Most likely, the expression 'litigious separation' indicated, as early as 2014, the presence of inheritances that certainly left traces to the present day. Finally, we highlight the relevance, for the field of investigation, of subsidized research in long-term projects and on the same theme, so that the analyses, interpretations and constructions of representations about the past can be enriched, better clarified and, mainly, strengthened from the methodological point of view.

In this way, we selected three categories - freehand drawing, geometric drawing and natural drawing -, built by historical studies, of relations between geometry and drawing proposals that marked the different movements of the mathematics curriculum for the teaching of geometry proposed to the first years of schooling, from the 19th century to the mid-20th century. We used as a central source for the investigation the regulations and school manuals of each period, with the intention of adapting and responding to the recommendations made by the school programs, which corresponded to what we currently call mathematics curriculum for the first years of schooling. In dialogue with the past, we

⁵ TN: Drawing and Geometry in primary school: a lasting marriage that ends with a litigious separation.

present the results of current research, which highlight the relationships and interactions between drawing and geometry as perspectives for the teaching of geometry in the 21st century, and we discuss their insertion in the most recent Brazilian normative prescription, the Base Nacional Comum Curricular (BNCC, or Common National Curriculum Base, in English).

Freehand drawing

The first Brazilian public instruction law, after Independence, dates from 1827 and establishes that:

Teachers will teach reading, writing, the four operations of arithmetic, practice of broken numbers, decimals and proportions, the general notions of practical geometry, the grammar of the national language, the principles of morals and doctrine of the Roman Catholic and Apostolic Religion [emphasis added]. (Moacyr, 1936, p. 189)

One of the first manuals that circulated in Brazil with the aim of subsidizing such regulation for the teaching of geometry, which could be identified as a guide to public instruction, had as its title *Princípios do Desenho Linear compreendendo os de Geometria Pratica, pelo método do ensino mutuo*⁶. It was published in 1829 and represents a translation/adaptation of the French work, carried out by Albuquerque. The original book was considered a method of teaching linear drawing, an innovation for the time and had the adhesion of other European countries, such as Belgium, Denmark, Sweden, Switzerland. Despite dating from the beginning of the 19th century, it presents pedagogical proposals, ideas, theories that circulated internationally, through different means, designated by cultural agents, such as manuals, teachers, pedagogical materials, among others. Contacts and interactions were intensified with the Universal Exhibitions (the first held in 1851, in London), with the pedagogical missions of educators, among other political actions (Matasci, 2015).

The French book proposes tasks to be performed by students and was intended to exercise the vision and hands, since students should make freehand tracing of geometric figures, starting from straight lines, before introducing curved lines. D'Enfert (2007) analyzed the proposal and considered it as a simplified formula of Pestalozzi's intuitive method, in which knowledge was based on the sensitive perception of nature, and, more particularly, on sensations and visual observations.

The translation of the work into Portuguese constituted a compilation of tasks, in which students should reproduce drawings of geometric figures, organized in classes, due to the difficulty in freehand tracing of drawings. In 1st grade, it started with straight lines, flat geometric figures. In the 2nd grade, we moved on to spatial geometric figures, but still drawn

⁶ TN: Principles of Linear Drawing comprising those of Practical Geometry, by the method of mutual teaching.

in straight lines. The circle and arcs were introduced in the 3rd grade, as they required a greater mastery of the hands, in order to make a curvilinear tracing, as we can see in Figure 1:

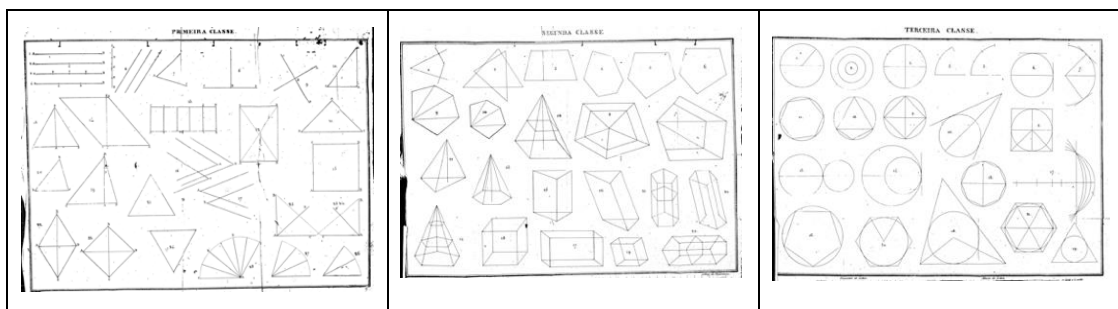


Figure 1 – Drawings from the Albuquerque's book (1829)

Source: Albuquerque (1829, s./p.)

In the guidelines for teachers, it was indicated that the tasks should be performed by the students, as a perfect copy of the model presented, without any guidance, only by imitation. The measuring instruments (ruler, protractor, etc.) were for the exclusive use of the teachers, for verification, and if the drawing was not perfect, the students had to redo it until the teacher validated it. The orientation was to measure with the eye, according to the intuitive method. The central aspect of the teaching method was the use of the senses as fundamental in the learning process and, in the case of drawings, the vision would allow the detailed observation of the model to be copied, together with the ability of the hands to execute the tracings, developed during the performance of the tasks. It was a purely practical activity, in order to respond to the normative indication “general notions of practical geometry”, without conceptualization, reasoning, but which, for its full execution, required an acute observation of the particularities of the different geometric figures drawn (Leme da Silva, 2021).

The role of drawing, in addition to the relationship with geometry, was also intended to contribute to writing, in the sense of developing manual skill in tracing. However, performing numerous and successive tasks of drawing geometric figures freehand, even without highlighting the geometric properties in question, can be interpreted as ways to better understand, in the future, the geometric concepts studied. In any case, the practice of drawing geometric figures represented, in Brazil, a response to the normative “general notions of practical geometry”, as well as the first contacts of children with the teaching of geometry (Leme da Silva, 2021).

Linear or geometric drawing

At the end of the 19th century, new proposals for the teaching of geometry gained space in Brazil, no longer exactly as translations of foreign works. The organization and consolidation of the first Normal Schools to train primary teachers took place in this period and emerged, in many regions of the country, as institutions linked to secondary schools, in which the teaching of geometry had a theoretical and deductive approach. This bond and

approximation, most likely, ended up influencing the production of new school textbooks for the teaching of geometry, produced in Brazil. The moment was also marked by the creation of the school groups model, a new public teaching organization, which began in the state of São Paulo in 1893 and gradually expanded to the other states of the federation.

Thus, the state of São Paulo was a pioneer in the restructuring of Public Instruction, soon after the Proclamation of the Republic, and underwent several reforms in the late 19th century and early decades of the 20th century, especially in the years 1894, 1905, 1918, 1921 and 1925. The first teaching program of São Paulo school groups was made official by Decree 248, of June 26, 1894 (São Paulo, 1894). The rubrics linear drawing or practical geometry were no longer used, however the subjects of geometry and drawing remained strongly intertwined. The drawing, present since the 1st year, begins with the tracing of horizontal and vertical lines, angles. Geometry begins in the following year, 2nd year, and deals with geometric figures, such as triangles and quadrilaterals, with great similarity of content present in drawing, but construction is explicitly proposed in the matter of drawing, which reads: “construction right triangle, isosceles triangle, equilateral triangle. Squares: diagonals and diameters: their construction. Different construction methods – by the sides, by the diameters, by the diagonals” (Drawing Program, 1894). There are no indications in the normative guidelines on how such constructions should be carried out.

The 1905 reform, regulated by Decree 1281 (São Paulo, 1905) broke with the articulation between the subjects of Drawing and Geometry; drawing became committed to the representation of the natural, of concrete objects and, on the other hand, geometry, a science that has always been committed to abstract geometric entities and to rigor. The following reforms maintained the division between drawing and geometry, but highlighted the geometric instruments in the practical prescriptions of the subject of geometry. The 1918 Reform, by Decree 2944 (São Paulo, 1918), brought for the first time, in the 3rd primary year, the orientation “lines drawn using the compass” [emphasis added]. The same indication would be present in the 1921 Reform, in Decree 3356 (São Paulo, 1921), for the 2nd year, since the aforementioned reform reduced primary education to two years. Finally, in 1925, the presence of construction tools in the field of geometry is explained and detailed in the 3rd year syllabus, in which it is observed “Circumference traced by freehand and with a compass, Make an angle equal to the other with a compass and protractor. Perpendiculars traced with the help of a ruler, square and compass. Triangles traced with instruments” [emphasis added] (Program of 1925, São Paulo, 1925).

The study on the regulations referring to primary education in the states of São Paulo, Minas Gerais, Sergipe, Goiás and Rio Grande do Sul, during the late 19th century to the first decades of the 20th century, found that operations using ruler and compass were present in subjects such as Drawing, Linear Drawing, Geometry, Practical Geometry in all these states (Leme da Silva, 2014b). Subsequent research (Camara, 2019; Guimarães, 2017; Kuhn, 2015), in regulations from other states (Santa Catarina, Paraná), reiterates the presence of geometric

drawing as prescriptive in school programs.

The book *Primeiras Noções de Geometria Prática*⁷, by Olavo Freire da Silva, published in 1894, was a representative manual of the late 19th century. Silva's book, as a reference work in several Brazilian states, such as São Paulo, Paraná and Rio de Janeiro, met the demand for the indication of geometric constructions with ruler and compass in different programs of school groups (Leme da Silva et al., 2017).

The book *Primeiras Noções de Geometria Prática* (Silva, 1907) is structured by the tasks in which students should perform geometric drawings using instruments, such as a ruler and compass, through a sequence of instructions. The teaching proposal is configured by a traditional approach, which began with definitions, followed by properties and ended with synopses. At the end of each chapter, the book presented a list of many tasks, in general for the student to reproduce the definitions and properties already discussed, but also proposing that he draw constructions with the instruments, similar to those previously solved. (Figura 2)

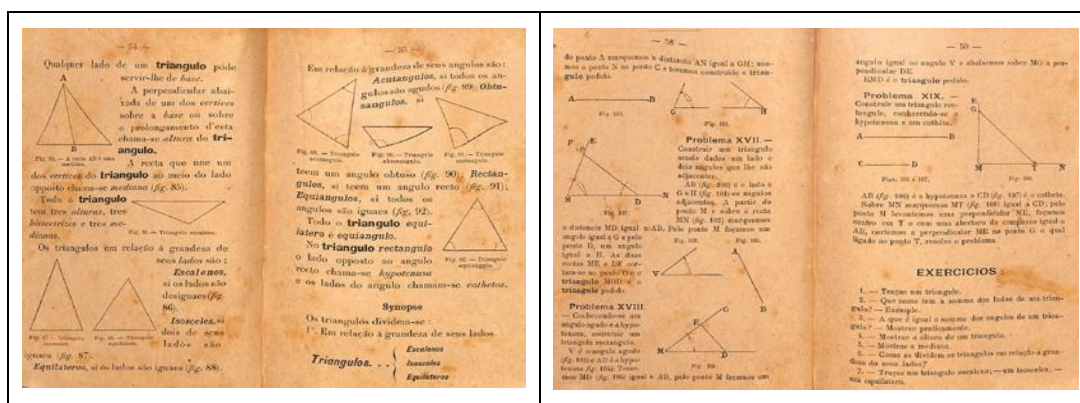


Figure 2 – Drawings from Silva's book (1907)

Source: Silva (1907, pp. 54-55 and pp. 58-59)

The visual aspect was left out, as the students had to memorize the sequence of steps to produce the final figure. The geometric figures remained as the final object to be drawn, however, the aspect of skill with manual tracing changes to the operational skill in the use of instruments. In the tasks proposed in Silva's manual (1907), students should know the definitions and properties of the constructed geometric figures, even without highlighting the reasoning involved in the use of instruments and in the conceptualization of the figures. The geometric concepts were evidenced, however, without the concern of articulating them with the traces produced through geometric drawings (Leme da Silva, 2021).

Natural drawing

Still in the late 19th and early 20th centuries, a new proposal, now focusing on the

⁷ TN: First Notions of Practical Geometry.

teaching of drawing, and not exactly on geometry, gained space in Brazilian pedagogical journals, an important vehicle for the circulation of international ideas at that time. The method, called natural drawing, gained visibility in articles in journals that bring debates, such as the International Congress of Drawing, held in 1904, in Berne, Switzerland.

The purpose of natural drawing in elementary school was seen as a practice of educating the eye and the hand, developed in the exercise of visual observation and tracing, through the reproduction of models. However, one should abandon the drawing of geometric figures and, in its place, insert the drawing of natural objects, familiar to the student. In the same way, there was no longer the objective of perfection, but rather, to develop taste and motor and operational skills in the practice of drawing, to educate the eye.

Pedagogical journals point to two contradictory points of view: on the one hand, the perspective of the artistic community defending natural drawing as a strategy for the formation of a craft, not restricted to artists, but to the future citizen. On the other hand, the vision of science betting on geometric drawing for future studies of Geometry, whose purpose is instruction and preparation for a propaedeutic study.

It is necessary to reiterate that, throughout the first half of the 20th century, each state in Brazil adopted its own regulations in the organization of subjects, with their respective teaching programs. For example:

The Sergipe program does not cover the subject of Geometry from 1911 to 1931, but indicates the construction of geometric figures. Santa Catarina follows the same nomenclature as São Paulo, Drawing and Geometry as two different subjects in all programs, but the geometric constructions remain together with the natural drawing in the subject of Drawing, while in São Paulo, they migrate to Geometry, a fact similar to the programs from 1917 to 1921 in Paraná. (Leme da Silva, 2021, pp. 53-54)

Thus, in Brazil, despite the opposing points of view between natural drawing and geometric drawing, both coexist as pedagogical proposals in school programs in different subjects in the various states (Leme da Silva, 2021).

School textbooks that presented questions using the natural drawing methodology were rare in Brazil. However, we can take as an example the work 'O Discípulo Parisiense'⁸, without authorship, published in Paris, France, found in a second-hand bookshop in Curitiba, with the student's name and the date of 1926 (Camara, 2019). It is a collection, composed of 12 notebooks, in which the first 2 are considered as preparatory for the others. From Notebook 3 onwards, the drawings are themed, for example, Notebook 4 presents figures of common objects to be reproduced; and Notebook 7, figures of landscapes, as we can see in Figure 3.

⁸ TN: The Parisian Disciple.

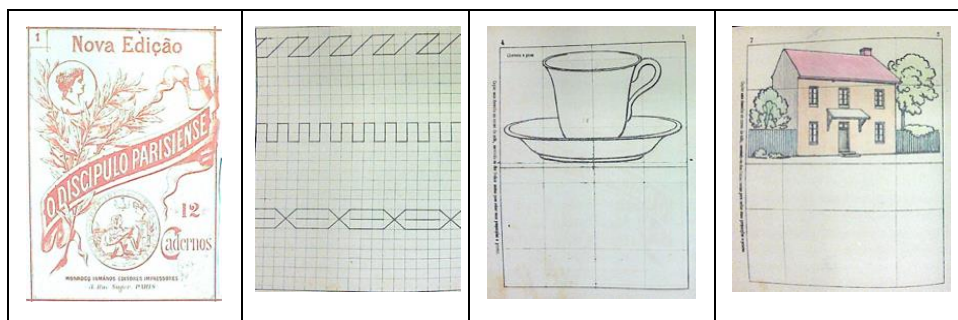


Figure 3 – Drawings from O Discípulo Parisiense (s./d.)

Source: *O Discípulo Parisiense* notebook (Capa, Cadernos 1, 4 and 7)

In the natural drawing method, the importance of the visual aspect is resumed, with the observation of figures to be reproduced. However, unlike the freehand drawing proposal of geometric figures, the natural drawing mobilizes geometric traces, but without the prerogative of limiting itself to figures and geometric properties. The objective in this approach was to quickly start tracing three-dimensional objects, from the students' daily lives, as an attraction to arouse students' interest in counterpoint to the aridity of geometric figures. The pedagogical method of Escola Nova, which has John Dewey⁹ as one of its defenders, supported the need to make the interest of the child the force around which all school tasks must be organized and developed. (Leme da Silva, 2021).

Drawing and geometry in the past

The three different historical movements indicate different approaches to introduce geometry teaching in the early years. The presence of geometric shapes as content to be developed was a constant, as well as their visual representation and, because of that, the permanence of drawings as an integral part of the teaching of geometric shapes. What problematizes and differentiates the three proposals concerns the different ways of drawing: freehand and with perfection; freehand, but without perfection and, therefore, far from geometric shapes; and finally with geometric instruments, based on properties and perfect.

The changes in approaches, over time, in the tasks proposed to the students to start the study of geometric shapes followed pedagogical movements of international scope and, not, exactly, the concepts of the science in question, in this case, Euclidean geometry. In the characterized period (19th century to mid-20th century) we did not find, in Brazil, problematizations in the geometric concepts, what the studies indicate to prevail are changes of pedagogical nature, of how the Educational Sciences understood the children's learning process. In this sense, we emphasize the specificity of the history of geometry teaching as

⁹ In 1896, John Dewey (1859-1952) established the Laboratory School at the University of Chicago and directed an experience that served as a practical reference base for theories, which came to be designated, in Brazil, as Escola Nova.

distinct from the history of geometry, given that disturbances in school knowledge are distinguished from changes in scientific knowledge (Karp & Schubring, 2014).

Another component to be observed is the alternation between drawings of flat figures (2D) or of spatial figures (3D) as a starting point for teaching geometry and drawing traces. In the demand for perfection, the most suitable teaching sequence for students seems to be from flat to space; on the other hand, if the everyday object is privileged, the first figures must be 3D objects and then draw flat figures (2D). Such aspects produce different modes of normative organization and learning and involve visual, reasoning and operational skills, which are equally different.

Drawing and geometry in current studies

The article by Sinclair and Bruce (2015) opens the issue of Geometry in Elementary School, published in June 2015, by ZDM Magazine – Mathematics Education, describing the new opportunities that can change the landscape of geometry education in primary school: (1) role of spatial reasoning and its connections with school mathematics, (2) role of drawing in the construction of geometric sense, (3) resources of digital technologies in geometry and spatial reasoning, (4) importance of geometric transformation in the curriculum and (5) passive emphasis on vocabulary-based geometry for a more active stance towards geometry creation.

One of the highlighted opportunities is the role of drawing in the construction of geometric sense. And to discuss the function, or role, of drawing in the teaching of geometry, we present the study by Thom and McGarvey (2015), which questions: “can the act of drawing serve as a means by which children become aware of geometric concepts and relations, rather than being seen as a product of that knowledge?” (p. 465). In other words, the question is whether drawing can be a methodology, or even a tool or instrument to support the development of geometric thinking in the early years of schooling.

Thom and McGarvey (2015) analyzed, in their research, several vignettes of drawings made by children at the beginning of schooling. Two cases are presented, the reproduction drawing, made by a 5-year-old child, and the wire and pole drawing, made by a 2nd grade student, as shown in Figure 4:

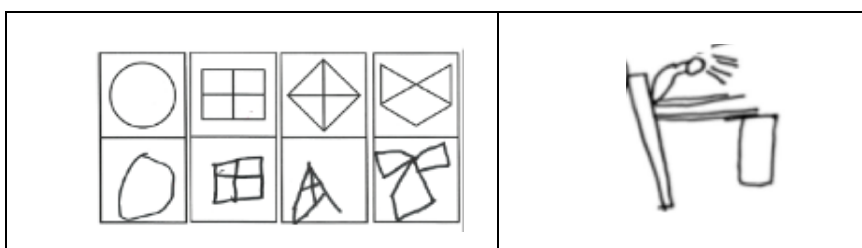


Figure 4 – Drawings reviewed by Thom and McGarvey (2015)

Source: Thom and McGarvey (2015, pp. 466 and 470)

The drawing on the left, of reproduction, can be identified as an inheritance of proposals from the 19th century intuitive method with the guise of interpretation proper to the 21st century, and in line with current pedagogical theories. The authors highlight the differentiation between straight and curved lines, measurements of angles and lengths in freehand tracings. It is not asked to remake it to perfection, but it is committed to valuing the visual observation of characteristic properties of the figures. The figures mix straight and curved lines and then the students' answers are analyzed. The drawing is copied from visual measurements.

In the drawing on the right, the trace was again freehand, but the copy is of an object observed in the schoolyard, which can be read as incorporated into the interest in the student's daily space. The highlight of the drawing is for the teacher's dialogue with the student, asking: "And in what way are they, the wires and the pole?". The student responds that "they are small straight lines, but this pole has a shape similar to a paper towel roll". The researchers analyze aspects of the student's geometric thinking, which articulates the wires as straight lines (1D) and the pole, which appears as an oblong (2D) and as a roll of paper towels (3D). We can observe that the student decomposes the pole, from a 3D object to a 2D figure. The dialogue enabled the emergence of certain geometric ideas and understandings that define how the student understands the observed shapes. The researchers suggest, in the end, that visual, physical, verbal aspects and temporal ways of thinking about the wires and the pole allow the student and the teacher to change their thinking from everyday conceptions to more geometric ones, that is, the drawing and the reflection on it, mediated by the teacher, create possibilities to understand geometric concepts, such as the development of geometric thinking (Thom & McGarvey, 2015).

The dialogic interaction between teacher and student, as well as their reflection on drawing, seems to bring new possibilities for learning, without, however, failing to preserve heritage from the past. Everything indicates that the visual aspect and the intuitive method, expressed by freehand drawing without the need for perfection, together with the practices of reproduction of figures, whether 2D or 3D, related to the students' daily situations, create rich opportunities to develop students' geometric thinking, particularly in their first contacts with geometric aspects. Highlighting and valuing dialogue and the use of natural language seem to be more recent aspects, as Ponte (2007) points out, translated into collaborative and interactive learning, which enriches understandings, reflections on practices and operational skills, no longer mobilized as mechanics.

Past and present: some reflections

Studies taken as a current example consider that the drawing tasks performed are means to assess the understanding of spatial geometric knowledge, in addition to the drawings generated by the students expressing representations, which, in turn, are at the heart of mathematical understanding. Then, they comment that the view of children's drawings as

“external visual representations” of the internal cognitive schema has a long history in the psychological literature, becoming a focus of study in the early 20th century along with the field of child development (Thom & McGarvey, 2015).

In addition to the psychological approach to the subject, we bring to the dialogue results of studies on the history of geometry teaching in the first years of schooling, developed in Brazil, which reiterate this assertion. The aforementioned studies, now systematized in this text, also highlight the relevance of children's drawings in primary school, with multiple functions and, in particular, in their articulation with the teaching of geometry, in different historical movements, taking into account the prerogatives of methods of teaching that circulated internationally.

The permanence of freehand drawing, derived from the intuitive method of the 19th century, can be highlighted as an important element of the 21st century. The insertion of the drawing to the natural, whose defense meant the incorporation of everyday objects of the student, together with the respective associations of geometric figures, is another mark of permanence incorporated by the school culture. However, despite the long period of the linear or geometric drawing approach, in Brazil, as a prescriptive indicative of school regulations, this proposal was not established as an adequate recommendation for children's first contacts with geometric figures. The school refused or expelled the traditional and conceptual character of this approach for the early years.

The novelty of the dialogic interaction between teacher and student, as well as the reflection of both on drawing, pointed out in current studies, may also have been the subject of debate in proposals from the past, since studies of the history of geometry teaching, as already said, are recent and need to be expanded, especially for the second half of the 20th century. On the other hand, we can question to what extent the current curriculum for teaching geometry in the early years, indicated in the BNCC, meets the indicated international studies, or approaches it, in particular, considering the role of drawing in the construction of the geometric sense. The thematic unit Geometry, proposed for all five initial years of Elementary School, is presented at the BNCC in a table with the knowledge objectives associated with the respective skills in the different years.

Chart 1, below, took the theme of flat figures as an example to analyze the role of drawing as a skill to be developed to achieve the knowledge in question.

Chart 1 – Drawings of flat figures in Geometry [added emphases]

Year	Knowledge objects	Skills
1	Flat geometric figures: recognition of the shape of the faces of spatial figures	Identify and name flat figures (circle, square, rectangle and triangle) in drawings presented in different layouts or in contours of geometric solids' sides..
2	Flat geometric figures (circle, square, rectangle and triangle): recognition	Recognize, compare and name flat figures (circle, square, rectangle and triangle) through common

	and characteristics	features, in drawings presented in different layouts or in geometric solids.
3	Congruence of flat figures	Recognize congruent figures, using overlay and drawing on square or triangular meshes , including the use of digital technologies.
4	Straight and non-straight angles: use of foldings, brackets and software	There is no drawing
5	Flat geometric figures: characteristics, representations and angles	Recognize, name and compare polygons, considering sides, vertices and angles, and draw them, using drawing material or digital technologies.

Source: BNCC (2018)

The analysis of Table 1 allows us to observe the presence of the drawing in practically every year, but its function is as drawings presented and not to be executed. The verb to draw is present only in the 5th grade, in which the practice must be done with drawing material or technology, that is, the practice of drawing is not configured as a participant in the construction of the geometric thinking proposed in the first years. The drawing is mobilized with the function of being observed, so that the students can recognize the properties there, compare measures, but not produce it. The production moment is at the end of the cycle, in which the set of characteristics was studied and then the student draws the geometric figures with the respective properties, supposedly already acquired in previous years. Drawing is the point of arrival, but not a pedagogical support tool to identify, recognize and compare. The practice of observing, of visualizing drawings of geometric figures, seen in the 19th century, remains, but not associated with the skill of the hands in tracing. Vision remains a determining factor in identifying regularities in flat figures, but there is no proposal to draw them freehand.

For the study of spatial geometric figures proposed in every year at BNCC along with flat figures, it is suggested to relate such figures with familiar objects of the physical world since the first year, to establish associations, compare attributes, name. However, it is also not recommended to draw spatial geometric figures, either freehand, nor with drawing material or digital technologies. It is noteworthy that from the 3rd year onwards, it is recommended to associate spatial figures (such as prisms, pyramids, etc.) to their plans, in order to establish relationships between flat and spatial representations.

Everything indicates that the act of drawing, as a methodological instrument for teaching the first notions of geometry, in particular, for the study of flat and spatial geometric figures, is not considered in the BNCC guidelines, and does not incorporate the results of the study by Thom and McGarvey (2015) who highlight the importance of drawing tasks as a means to assess understanding of spatial geometric knowledge, and that student-generated drawings express representations, which are at the heart of mathematical understanding.

Final considerations

The study in question listed three categories of drawing as an example of knowledge construction movements for the teaching of geometry in a historical period of relevance and pedagogical changes for primary education. To better understand how the normative prescriptions for the teaching of geometry were appropriate, an analysis of school manuals considered as a reference to subsidize the respective prescribed programs was taken. In addition to prescriptions placed in the different mathematics curricula, a lot of evidence (Leme da Silva, 2011) indicates that the three identified movements – freehand drawing, geometric drawing and natural drawing – effectively gained emphasis in the first year classes of past times, participating in the primary school culture. And so, they established strong interactions with the proposals for the teaching of geometry over more than a hundred years, from the 20s of the 19th century to the middle of the 20th century.

The production of a representation of the past of geometry teaching, in particular, in a long-term study, allowed us to identify remarkable movements in the investigated period. It highlighted the dynamic character of school culture in response to normative prescriptions, which, despite the legal and determinant aspect of the curriculum, underwent changes at the time of appropriation by cultural agents, such as authors of textbooks, responsible for translating curriculum regulations into pedagogical practices. The understanding of the movements identified in the past, in the same way, allowed us to dialogue, question and reflect with the current moment, analyze how such movements remained or, better still, left legacies that we can signal in the research debate on the teaching of geometry in the 21st century, as well as ending with a reflection on the most recent Brazilian regulation, the BNCC.

Freehand drawing, the first movement in the construction of knowledge for the teaching of geometry, was not incorporated as a relevant knowledge of methodological support after the 19th century, at the time of the organization of primary public education in Brazil, after the Republic. The geometric drawing, the most expressive and longest-lasting in the normative prescriptions, in different states of the Federation, was also not incorporated by the primary school culture, given that, from the mid-twentieth century, the indication of the use of construction instruments in geometry teaching programs for the primary school practically disappeared. The natural drawing was born with obstacles and debates with the geometric drawing and, later, gained its space detached from the prescriptions for the teaching of geometry.

Certainly none of the three movements was maintained in the way it was prescribed, however, it is possible to identify traces of such movements in the 21st century scenario. In the study, taken as an example, we identified characteristics of freehand drawing and natural drawing; and in the prescription of the BNCC, for the last year, the indication of making drawings with materials and digital technology. Freehand drawing is linked to the intuitive

method and the importance of observation, a visual method for identifying regularities in figures. The geometric drawing and the use of instruments correspond to a stage to be reached, however, it seems to be a consensus today that such an approach should not be the starting point for the child's first contacts with the teaching of geometry. In the same way, it presents itself with other available supports that add possibilities in the methodology of building the geometric figures. Finally, the natural drawing can be perfectly combined with the proposals for teaching geometry, in particular, in the identification of the geometric thinking of students at different stages of development, with the aid of the dialogic interaction between student and teacher to better understand the geometric thinking of both.

Finally, we believe that the history of geometry teaching, the history of the movements of knowledge construction for the teaching of geometry over time, can bring contributions and fruitful reflections to think, analyze and put into perspective relevant proposals and actions to think about pedagogical practices of a teaching of geometry nowadays. It is necessary to make the knowledge of this new field of investigation reach the working teachers and the teaching formation, in order to add its results to the other fields of Mathematics Education.

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