Zetetiké

DOI: 10.20396/zet.v30i00.8667600

Characterization of a “tables for teaching”

Caracterização de uma “tabuada para ensinar”

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Abstract

This paper aims to characterize the knowledge of the professional field of the teacher who teaches mathematics. It will seek to answer the following question: What “table for teaching” can be characterized based on the analysis of Albuquerque’s guidelines in her manual *Mathematics Methodology* (1951)? The research, qualitative nature; bibliographic and documentary is based on authors who defend two types of knowledge in the constitution of teaching and training professions, “knowledge to teach” and “knowledge for teaching” and their interpretations by authors of Mathematics Education in “mathematics to teach” and “mathematics for teaching”. For theorical-methodological analyses, it will be supported by authors who defend that the interpretation of information in knowledge occurs in stages. As a result, two groups of knowledge were characterized. Un group of the knowledge about the tables as object of teaching and the knowledge about the teaching practices for teaching tables.

Keywords: knowledge for teaching; mathematics for teaching; fundamental facts.

Resumo

Este texto tem por objetivo caracterizar saberes do campo profissional do professor que ensina matemática. Buscar-se-á responder à seguinte questão: Que “tabuada para ensinar” pode ser caracterizada, a partir da análise das orientações de Irene de Albuquerque em seu manual *Metodologia da Matemática* (1951)? A pesquisa, de cunho qualitativo, bibliográfica e documental, embasa-se em autores que defendem dois tipos de saberes na constituição das profissões do ensino e da formação, saberes *a* e *para* ensinar, e volvendo para a matemática, de autores que definem matemática *a* e *para* ensinar a partir dos primeiros referenciais. Para as análises teórico-metodológicas, apoiar-se-á em autores que consideram que a interpretação de uma informação em saber ocorre por etapas. Como resultados foram caracterizados elementos de uma tabuada para ensinar em dois grupos, dos saberes sobre a tabuada a ensinar e dos saberes sobre as práticas de ensino da tabuada.

Palavras-chave: saberes para ensinar; matemática para ensinar; fatos fundamentais.

"Is it still necessary to teach multiplication tables in the early years?"

[1] **Submitted**: 19/11/2021 - **Accepted**: 15/02/2022 - **Published**: 22/04/2022.

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DOI: 10.20396/zet.v30i00.8667600

Recently, a student of the Pedagogy course, while developing her end-of-course work about guidelines for teaching multiplication tables in a pedagogical manual, namely, *Mathematics Methodology*, by Irene de Albuquerque, published in 1951, asked her supervisor the following question, "Teacher, is it still necessary to teach multiplication tables in the early years? Her advisor asked her what had been the motivation for such a questioning. She replied that, when searching for the term "multiplication table" in the current curriculum guideline document, the Common National Curriculum Base (CNCB) (MEC, 2018), she had not found even one occurrence, concluding that multiplication tables had disappeared from the curriculum proposals. Not satisfied, she conducted the same search in the National Curricular Parameters (NCP) (MEC, 1998), the previous curricular guiding document, finding only two occurrences containing the term *tabuada* (times tables) (MEC, 1998, pp. 74-75).

Early in the development of her research, the student came across the title of one of the chapters of the textbook she was analyzing, "Learning the fundamental facts of the four integer operations - the multiplication table" (Albuquerque, 1951, p. 80). She thus realized that the multiplication table would be related to the term "facts". She returned to the search in the official curriculum documents, with the new expression. Then she verified that the "fundamental facts" were relatively explored in the NCP and that in the CNCB the "facts" appeared only in the 3rd year of Elementary I, in the "thematic unit" - "numbers" -, according to the "object of knowledge" - "Construction of fundamental facts of addition, subtraction and multiplication" (MEC, 2018, p.286). Two "skills" were related to the object,

> (EF03MA03) Construct and use basic addition and multiplication facts for mental or written calculation.
>
> (EF03MA04) Establish the relationship between natural numbers and points on the number line to use it in the ordering of natural numbers and also in the construction of addition and subtraction facts, relating them to shifts to the right or left (MEC, 2018, p. 287).

The first skill concerns the fundamental (or basic) facts of addition and multiplication; and the second relates the facts of addition and subtraction to shifts on the number line. There is no guidance on division facts. It should be noted that the multiplication table appears late (only in the 3rd year of elementary school) and ephemerally in the CNCB, but this is another discussion.

Despite the learner's concern about the whereabouts of the multiplication table in current curriculum proposals, this subject had a marked presence in teaching programs and textbooks aimed at elementary school in the 20th century. Almeida and Pinto (2017) state that this theme has been the object of recent studies, which sought to investigate the relationship between multiplication tables and memorization practices in primary education in Brazil. The authors also highlighted that, in times of intuitive teaching, predominant in the first decades of the 20th century, the multiplication tables were called to new uses, opposing to the practices of learning by heart, according to the analysis that the authors performed in

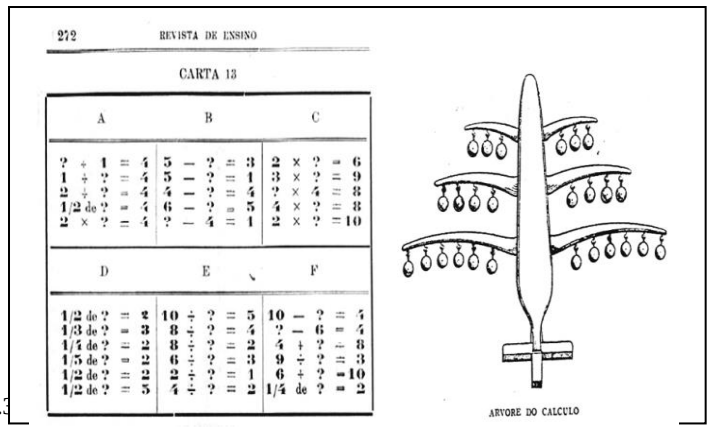
DOI: 10.20396/zet.v30i00.8667600

the textbooks prescribed to the teaching of arithmetic such as Büchler (1919) and Trajano (1922), for example.

One of the studies that identified changes in the purposes and uses of the multiplication table was that of Valente and Pinheiro (2015). In the article published by these authors, they aimed to identify trajectories of the multiplication tables, in Brazilian schools, that countered the culture of memorization. They showed that Parker's Cards, represented by a "set of pictures/charts/tables" with the purpose of "assisting the teacher to methodically conduct the teaching, especially, of the four fundamental operations", was one of the pedagogical devices used for purposes of facilitating the teaching and learning of multiplication tables. However, with continued use in school, these Letters became so popular that students began to memorize them, which, according to the authors, became an "alternative to memorization" (Valente & Pinheiro, 2015, p. 27) and with that, the Letters began to receive criticism.

One of the critics of Parker's Letters for the teaching of multiplication tables, according to Valente and Pinheiro (2015), was Professor Ferraz de Campos, a São Paulo native, textbook author, occupant of important positions in public education, who proposed a new model of multiplication tables, the Calculation Tree. According to the authors, Campos intended a more efficient passage from the concrete (use of objects) to the abstract (numbers and operations), so he suggested the use in class of the didactic-pedagogical device Calculation Tree "as a pleasant way to objectify all questions that about numbers [could] be proposed to beginners" (Campos, 1931, p. 9, cited by Valente & Pinheiro, 2015, p. 24). Objectify in the sense of representing the numerical quantity by objects. For Campos, Parker's Letters did not achieve such an objective.

According to Valente and Pinheiro (2015), Parker's Letters and the Calculation Tree (Figure 1) "proposed a simultaneous teaching of the different operations in the presentation of each number to be studied and its possible "formations" (p. 35).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CARTA 13

A				B				C			
?	+	1	= 4	5	−	?	= 3	2	×	?	= 6
1	+	?	= 4	5	−	?	= 1	3	×	?	= 9
2	+	?	= 4	4	−	?	= 4	?	×	4	= 8
1/2 de	?		= 4	6	−	?	= 5	4	×	?	= 8
2	×	?	= 4	?	−	4	= 1	2	×	?	=10

D				E				F			
1/2 de	?		= 2	10	+	?	= 5	10	−	?	= 4
1/3 de	?		= 3	8	+	?	= 4	?	−	6	= 4
1/4 de	?		= 2	8	+	?	= 2	4	+	?	= 8
1/5 de	?		= 2	6	+	?	= 3	9	+	?	= 3
1/2 de	?		= 2	2	+	?	= 1	6	+	?	=10
1/2 de	?		= 5	4	+	?	= 2	1/4	de	?	= 2

ARVORE DO CALCULO

 ISSN 2176-1744

DOI: 10.20396/zet.v30i00.8667600

Figure 1: Parker's Chart (left) and Calculation Tree (right).

Source: (Valente & Pinheiro, 2015, p. 28 and 32)

The authors state that the tradition of memorizing was supported by the contribution of other devices, such as the Pythagorean Table (Figure 2), which helped the student to obtain and memorize the results of multiplication operations from one (1) to ten (10).

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Figure 2: Pythagorean table

Source: (Valente & Pinheiro, 2015, p. 24)

Note that in the Pythagorean table, multiplying a number at the end of a row by another number at the end of a column or vice versa, the product, the result of the multiplication of these two numbers, will be represented by the numeral at the intersection of the row and column, respectively, representing the first and second number that were multiplied. The board, according to Valente and Pinheiro (2015), was a pedagogical device that facilitated the memorization exercise of the child.

Both the presence of the multiplication table in teaching programs, as Almeida and Pinto (2017) point out, and the transformations in the purposes and uses that were made of it in the first decades of the 20th century, according to Valente and Pinheiro (2015), instigated further research on the presence and orientation of this didactic-pedagogical device in the teaching of the four operations in later times.

This article presents the results and advances of a research, developed as a final work for the graduation course in Pedagogy, of the student mentioned in the beginning of this text (Morais, 2021), supervised by the author of this text. The research sought to identify pedagogical consensuses for teaching multiplication tables, based on the analysis of the guidelines given to teachers in the pedagogical manual, *Mathematics Methodology*, published in 1951, by Irene de Albuquerque. Thus, which "multiplication table to teach" can be characterized, according to Irene Albuquerque's orientations to teachers in elementary school,

DOI: 10.20396/zet.v30i00.8667600

in her pedagogical manual? To answer this question, it will be referenced authors who define the general knowledge to teaching and teaching in two types: knowledge "to" and "to" teach (Hofstetter&Schneuwly, 2017), and mathematics educators, who appropriate this theory and define the professional knowledge of the teacher who teaches mathematics, in mathematics "to" and "to" teach (Bertini et al., 2017), according to the next subtopic.

Steps for the decantation of a "table to teach".

Thinking about the question just enunciated, this topic has a double objective, to outline the steps to take in interpreting the guidelines for the teaching of multiplication tables (in the *Mathematics Methodology* manual) in knowledge and, also, to explain how the process of decanting a "multiplication table to teach" will occur, from the analysis of these guidelines.

The researcher - the one who will analyze and interpret the guidelines - will be considered as a social, historical and cultural being. Thus, the way of analyzing and reading the orientations, that is, forms of appropriation (Chartier, 1990), is specific to each researcher and will be supported by his or her research question and theoretical and methodological references, in order to achieve his or her goals.

The guidelines present in a pedagogical manual is a product, a result, of "recompilations of teaching experiences" (Valente, 2018, p. 380). Such recompilations can be organized and interpreted as teachers' knowledge, which can be analyzed and compared, thus enabling their interpretation and systematization as knowledge (stages of interpretation of information into knowledge, according to Valente, 2018). In this text, knowledge is understood as objectified, depersonalized knowledge; and knowledge, properly speaking, as something intrinsic to the subject (Burke, 2016; Pais, 2001; Valente, 2018).

However, there is no point in understanding that the passage from information to knowledge takes place in stages, if there is no understanding of the object of study, which one wishes to analyze, interpret, systematize. The guidelines for teaching multiplication tables, scattered in Albuquerque's manual, should be detailed, so that they can be compared and categorized into knowledge to teach. By analyzing the knowledge, one can verify pedagogical consensuses that can be characterized as knowledge to teach.

The expression " times tables to teach" is linked to "mathematics to teach", defined by Bertini et al. (2017), according to these authors' appropriation of Hofstetter and Schneuwly's (2017) "knowledges to teach". Knowledge to teach is not a common sense expression, it would be easy to deduce that every teacher needs to know the multiplication table to then teach it; neither does it refer to how the teacher should or can teach it. According to

DOI: 10.20396/zet.v30i00.8667600

Hofstetter and Schneuwly (2017), the "knowledge to teach" are the "teacher's working tools," those didactic and pedagogical devices and methodologies to teach, the knowledge to be taught, the subjects to be taught, the institution where this teaching takes place with its norms and teaching programs, the educational purposes, and other specificities. This knowledge, from the professional field, is linked to the "knowledge to teach", linked to the disciplinary field, which, according to these authors, comprises the "object of the teacher's work".

The "mathematics to teach", defined by Bertini et al. (2017), similarly, encompasses all the aforementioned examples of tools, but specific to the teaching of mathematics. Recent works have sought to characterize mathematics to teach (Basei, 2020, Fortaleza, 2019; Maciel, 2019, among others) and have shown that they are articulated to mathematics to teach, according to interpretations of the research by Oliveira (2017) and Pinheiro (2017), for example.

Hypothetically, the " times tables to teach" links to the mathematics to teach, thus comprising the teacher's working tools specific to the teaching of times tables. In general, the multiplication table to teach, associated with the multiplication table to teach, is linked to the mathematics to teach, which, articulated to the mathematics to teach, is linked to the knowledge to teach, also articulated to the knowledge to teach, respectively. Thus, "what ' times tables to teach' can be characterized, from the analysis of Irene de Albuquerque's guidance to elementary school teachers, in the textbook Methodology of Mathematics (1951)?

Santos and França (2019) summarize the trajectory of Irene de Albuquerque, who held several prominent positions in the educational field.

> Born on October 30, 1915, she attended the Secondary School of the Escola Normal do Distrito Federal from 1930 to 1933, joining the Regular Primary Teacher Training Course (or School of Education) of this Institute. She taught Teaching Practice of the Normal Course of the Institute of Education of Rio de Janeiro, and Methodology of the Improvement Courses of INEP, being the author of didactic books and articles in pedagogical magazines. (p. 188)

As we can see, Albuquerque, besides being a teacher trainer to work in primary education, was the author of textbooks and articles in pedagogical magazines. Borges et al. (2020) present an inventory of works in which Albuquerque's works were the object of several researches. Among them, the authors highlight "Working with measures: guidelines for primary education by the hands of Albuquerque" (Zuin, 2016); "The games for teaching mathematics: the proposal of Irene de Albuquerque in her book Mathematical games and recreations (1958)" (Schneider & Costa (2016); " "Knowledge to teach in the work Methodology of Mathematics by Irene de Albuquerque" (Carneiro & Pinto, 2019); "The teaching of arithmetic in pedagogical manuals: principles vulgarized in the new school"

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(Felisberto, 2019); "Mathematics to teach the notion of number and counting seen in the manual *Methodology of Mathematics* (1964)" (Gregório & Costa, 2019), among others.

Briefly, in the manual *Methodology of Mathematics*, published in 1951, Albuquerque sought to provide guidance and practical suggestions regarding the points of the elementary school programs. The manual was aimed at primary school teachers, guidance counselors and teachers in training at the Escola Normal. The manual, divided in two parts, in the first part brought general guidelines for teaching mathematics; in the second, specific guidelines for the teaching of certain topics. The second part contains Chapter III, in which the author deals with "learning the fundamental facts of the four integer operations - the multiplication table and practical suggestions" for its teaching. In this chapter, she presents 16 guidelines for the teacher. Each one of them is well detailed and exemplified. Following the guidelines, she offers practical suggestions, with examples of activities and exercises. The guidelines for teaching multiplication tables, together with the practical suggestions, are spread over 20 pages of her manual.

Some of these guidelines and suggestions, as well as the movement of analysis of knowledge from them, will be detailed in the following subtopic.

From knowledge analysis to the characterization of a multiplication table to teach

For the limited number of pages of this text, eight of the sixteen guidelines for teaching the fundamental facts will be taken for detail. In the operations of addition, subtraction and multiplication there would total 100 facts in all, for example, from addition, there would be 81 facts without zeros, starting with, 1 + 1, 1+ 2, [...], 1+ 9; then, 2 + 1, 2 + 2, [...], 2 + 9, this would be repeated until 9 + 1, 9 + 2, [...], 9 + 9; and 19 facts with zeros, 0 + 1, 0 + 2, [...], 0 + 9 and then, 1 + 0, 2 + 0, [...], 9 + 0, and finally, 0 + 0. In division, since there is no zero divisor, there would be a total of 90 suits. The 390 facts, according to Albuquerque, would constitute the "famous multiplication table" (Albuquerque, 1951, p. 81).

Among the guidelines that will be analyzed, at first we have the fourth (n.4) and the fifth (n.5), as they are presented in the manual. In the fourth (n.4), Albuquerque asks the teacher to start from a "real situation, arising in class, and quite objective. One suggestion of the author was that the facts should be presented with the use of stories to make them meaningful to the children, which was fully recommended, since the lack of meaning would lead to difficulties in learning and memorization by the child. The fifth (n.5) guideline complements the fourth, in it the author recommends that the teacher make use of objectification for calculation, which meant using objects to teach calculation. Objects in the

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classroom, equal groups of objects, are examples of objectification material, according to the author.

In the eighth (n.8) guideline, Albuquerque advises that facts should not be taught in isolation, but each fact together with its inverse, which would facilitate learning and memorization. The author proposes that for each operation the teacher should present its inverse in parallel, indicating for each one objects to represent these quantities. For example, teaching 3 + 4 = 7 next to 4 + 3 = 7, in addition; 2 x 3 = 6 next to 3 x 2 = 6, in multiplication; 7 - 3 = 4 next to 7 - 4 = 3, in subtraction, and, finally, 6: 2 = 3 next to 6: 3 = 2, in division. She said that this way would help learning by "transference" (Albuquerque, 1951, p. 83).

The author explains in the tenth guideline (n.10), that each 4 facts, related from addition with subtraction, or from multiplication together with division, would form a "didactic unit for teaching the fundamental facts" (p. 83). As an example, he cites as a didactic unit of addition together with subtraction: 2 + 1 = 3; 1 + 2 = 3; 3 - 1 = 2 and 3 - 2 = 1; of multiplication together with division: 2 x 4 = 8; 4 x 2 = 8; 8 :2 = 4 and 8: 4 = 2 (Albuquerque, 1951, p. 85).

Albuquerque, in the twelfth (n.12), orientates that, in the presentation of the didactic units, it would be necessary to obey "the order of difficulty of the fundamental facts". One can see, in this guideline, that Albuquerque intended to innovate something that was set in the logical order of multiplication tables taught until then. She states that memorizing the facts, according to the logical order, would make learning difficult and would slow down the answers to everyday situations that might arise. She explains that, for the child to answer 7 x 9, he would have to go back through the whole previous sequence (7 x 1; 7 x 2; 7 x 3; [...]) until he reached the desired operation (Albuquerque, 1951, p. 86).

The author says that "the order of difficulty of addition and subtraction facts, according to Clapp's research in the United States and Alfredina Paiva e Souza's in Brazil[2]" (Albuquerque, 1951, p. 87) helped her to draw some conclusions about this. The easiest combinations would be those below 10. Combinations with the number 1 and with equal numbers, like 3 + 1; 1 + 3 and 3 + 3, for example, were still classified as easy. The combinations with zero presented average difficulty. In subtractions, the easiest ones were those with minuend up to nine (9), in the same way with subtraction one (1 or zero), for

[2] The author provides the references at the end of her textbook. From Clapp, F. L., the work The *NumberCombinations, TheirRelativeDifficultyandtheFrequencyoftheirAppearance in Text-books, in "Bureau ofEducationalReasearch* Bulletin", n. º 1 and 2, 1924. From Alfredina Paiva e Souza, she mentions two works: "O Cálculo na Escola Primária" (Calculus in Primary School), published in 1942, in Rio de Janeiro; and "Metodologia do Cálculo", published in Revista de Educação pública, volume I, n. 4, by the Federal District Municipality (RJ) (the year was not informed by the author).

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example 7 - 1; or 3 - 0, with remainder (1), as in 4 - 3 or numbers subtracted from itself, for example 3 - 3. Regarding multiplication, facts with zero would be the most difficult (and with one (1) the easiest). Facts with 6, 7, 8 and 9 would be the most difficult. Finally, for division, the author states that facts with a zero dividend would be of average difficulty, with a dividend equal to the divisor, they would be the most [easy] difficult[3] . She also highlights that, equally, divisions with quotient or divisor 6, 7, 8 and 9 would be the most difficult (Albuquerque,1951, pp. 86-87).

In guideline number 13 (n.13), Albuquerque organizes teaching in groups of teaching units for addition, subtraction, multiplication, and division, in the order they should be presented to students. As an example, he mentions the 1st group, which would be formed by 6 (six) teaching units, dealing with "combinations with 1 or with equal numbers (in addition); up to 5 (in total or minuend)" (Albuquerque, 1951, p.88). The following facts would belong to this group:

1 + 1	2 + 2	3 + 1	2 + 2	4 + 1	2 + 3
2 – 1	1 + 2	1 + 3	4 – 2	1 + 4	3 + 2
	3 – 1	4 – 1		5 – 1	5 – 2
	3 – 2	4 – 3		5 – 4	5 – 3

(Albuquerque, 1951, p.88)

Also noteworthy are the last two guidelines (n.15 and n.16) which revolved around how the teacher would exercise the fundamental facts with the students. She indicates exercises, games, problems, and contests, especially, in "training" the more difficult facts. The author points out that there would be many other suggestions in her manual *Games and Mathematical Recreations*[4] (Albuquerque, 1958, p. 93).

The practical suggestions, which came after the guidelines, dealt with examples of activities for the teacher to perform with the students. They were divided into four groups. In the first, called "Exercises that serve to learn and fix the fundamental facts and the meaning of the operations", there were suggestions such as making a parallel between drawings and icons and the indication of the addition facts. This would then be done for subtraction. There was even a proposal that the student should invent a problem of one of the operations from a

[3] In the Manual the term "difficult" is found, when the term "easy" should have been used. In multiplication Albuquerque (1951) cites examples of easy and difficult levels of difficulty. In division this was possibly an error in this edition and continued in the following edition of 1954.

[4] The third edition of the manual Mathematical Games and Recreation, by Irene de Albuquerque, was published in 1958 and is available at https://repositorio.ufsc.br/handle/123456789/161042.

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given drawing and solve it. The second group, "Preparatory exercises for multiplication", included exercises that would lead the student to understand multiplication as an addition of equal parts. In the third group, the author presented the "Organization of the teacher's notebook, for noting the frequency with which the various fundamental facts were exercised. In it she presented a table containing the operations and a list of the facts taught and how many times each one was taught. There was also a chart for monthly control of the facts taught. The last group was Games, composed of examples of "outdoor games", "in the classroom", "fishing", etc.

The last group was the Games, composed of examples of "outdoor games", "in the classroom", "fishing", etc. Based on the teacher's guidelines, we will analyze and categorize the knowledge needed by teachers, as presented in the pedagogical *manual Mathematics Methodology*, *by Albuquerque* (1951). To this end, Table 1 was prepared to facilitate the visualization of the relationship between guidelines and knowledge, in bold, the eight guidelines that were highlighted at the beginning of this subtopic.

Table 1 - Knowledge from the guidelines for teachers on teaching the fundamental facts

Guidance	From the orientation, the teacher should be able to:
01	Get students to give quick and correct answers.
02	Introduce students to the fundamental facts.
03	Use methods that would facilitate the memorization of the fundamental facts.
04	**Introduce the fundamental facts to the students by means of a real situation.**
05	**Introduce the fundamental facts to students with the help of objects.**
06	Get students to understand numbers before the fundamental facts.
07	To lead students to understand the meaning of the fundamental operations.
08	**Present each fundamental fact in parallel to its inverse.**
09	Teach addition and then subtraction to students, relating fundamental facts.
10	**Teach the fundamental facts to the students by making use of the teaching units.**
11	Teach the teaching units respecting order to be taught, with the exception of zero in the unit.
12	**Present the teaching units following the order of difficulty of the fundamental facts.**
13	**Distribute the fundamental facts into groups of teaching units, in the order they are to be taught.**
14	To get the student to fix the fundamental facts with practice and exercises such as games and problems.
15	**Teach counting, review it orally, and practice repeating the combinations of the fundamental facts through exercises, games, and problems to avoid mistakes.**
16	**Practice the most difficult facts with the students by means of exercises games and problems.**
Practical suggestions	Apply training and fixation activities such as exercises that relate drawings and icons to fundamental facts. Ask the students to develop problems based on a figure and solve them.

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	Perform multiplication exercises so that the student perceives it as an addition of equal parts. To train and exercise the fundamental facts with the students, especially the more difficult ones, through varied games.

Source: Created by the researcher.

In Table 1, we tried to present an organized systematization of the knowledge necessary for teachers to comply with each of the guidelines and practical suggestions.

Based on this knowledge, we sought, in this analysis, to interpret it taking into account the major concept of Hofstetter and Schneuwly's (2017) knowledge to teach. The authors state that the knowledge to teach deals with the

> knowledge about "the object" of the work of teaching and training (about the knowledge to be taught and its knowledge, its development, ways of learning, etc.), about teaching practices (methods, procedures, devices, choices of the knowledge to be taught, organizational and management modalities), and about the institution that defines their field of professional activity (syllabuses, instructions, purposes, administrative and political structures, etc.) [emphasis in original]. (p. 134)

Thus, it was intended to relate each of the knowledges to the first two groups of knowledges, according to Hofstetter and Schneuwly (2017), to facilitate interpretation. The first two groups of the knowledges about "the object" of teaching and training work and the knowledges about teaching practices. The third group (knowledge about the institution that defines the field of professional activity) is implied to be partially contemplated, since the manual attended to the program of the primary course and the program of the primary teacher training course, according to information found, respectively, on the front and back cover of the Manual.

To better organize the interpretations of the knowledge as objectified knowledge and its relation to the knowledge to teach is that Table 2 was elaborated. According to Valente (2019), the objectification of knowledge is directly related to the pedagogical consensuses interpreted in an objective, depersonalized way. Objectified knowledges "show themselves as systematized discourses, ready to be mobilized, with the capacity to circulate" (p. 10).

Thus, one can interpret each of these knowledges, taking into account the working tools of the teacher who teaches mathematics, more specifically, and seeking elements that characterize a multiplication table to teach, as shown in Tables 2 and 3, below.

Table 2 - Characterizing elements of a multiplication table to teach (Group 1)

Knowledge for teaching	**Analysis of knowledge from teaching experiences**	**Characterizing elements of a "teachable multiplication table**
Group 1-	Introduce students to the fundamental	Know the fundamental facts.

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Knowledge about the multiplication table to be taught	facts.	
	Present each fundamental fact in parallel to its inverse.	Know the operation and its inverse.
	Teach addition and then subtraction, relating fundamental facts.	Know the relationship between the operations of addition and subtraction.
	Teach the fundamental facts to the students by making use of the teaching units.	Recognize in operations the fundamental facts and their inverses, calling the set of fundamental facts and their respective inverses the "teaching unit".
	Teach the teaching units respecting order to be taught, with the exception of zero in the unit.	Know the order to be taught and its exceptions.
	Present the teaching units, obeying the order of difficulty of the fundamental facts.	Know the order difficulty of the facts to organize the units to be taught.
	Distribute the fundamental facts into groups of teaching units, in the order they are to be taught.	Organize groups of fundamental facts according to order of difficulty in teaching operations.
	Get students to understand numbers before facts.	Teach numbers before the fundamental facts.

Source: Prepared by the researchers.

Tables 2 (previous) and 3 (next), present elements characterizing a multiplication table to teach, related to two groups of knowledge, respectively, specific knowledge about the multiplication table to be taught and knowledge about the practices of teaching multiplication tables. The division was made by observing the predominant features in the elements of these characterizations.

Table 3 - Characterizing elements of a multiplication table to teach (Group 2)

Knowledge for teaching	**Analysis of knowledge from teaching experiences**	**Characterizing elements of a "teachable multiplication table**
Group 2- Knowledge about teaching practices	Get students to give quick and correct answers.	Exercise orally and in writing the fundamental facts with the students.
	Use methods that facilitate the memorization of the fundamental facts.	Know and use new methods to facilitate memorization.
	Present the fundamental facts by means of a real situation.	Relate the facts to real situations.
	Present facts with the help of objects.	Teaching, with the use of objects, from counting to fundamental facts.
	To lead students to understand the meaning of operations.	Make relationships between the operations to be taught with real situations and using objects.
	To get the student to fix the fundamental facts with practice and exercises such as games and problems.	Know exercises, games, and problems for practicing the fundamental facts.

DOI: 10.20396/zet.v30i00.8667600

	Teach counting, review them orally, and practice repeating the combinations of the fundamental facts through exercises, games, and problems, avoiding mistakes.	Plan games, exercises, problems to aid memorization. Repeatedly and creatively exercise the key facts.
	Practice the most difficult facts with the students by means of exercises games and problems.	Prioritize in problems and games the teaching of the most difficult facts.

Source: Prepared by the researchers.

In the first group, the elements of the characterizations of knowledge about the multiplication table to be taught or about the fundamental facts to be taught are presented. We identified that many of them fit the methodological criteria for analyzing knowledges, proposed by Maciel (2019), "Presentation, Explanation, Grading, Articulation, Evaluation." How would the fundamental facts and operations be presented? Which facts would be taught first? What devices could contribute to the teaching of the multiplication table? How would graduation take place in the teaching of the contents? How would the facts and operations be articulated? How would the follow-up of what was taught occur? What parameter should be used to know if the student has learned the multiplication table? The characterizing elements of the first group indicate knowledge interpreted, based on the guidelines for teaching multiplication tables in Albuquerque's textbook (1951).

It is observed that some characterizing elements fit the methodological criteria proposed by Maciel (2019), such as "Presentation", the teaching of the fundamental facts would come after the teaching of numbers and would be presented through a real situation; "Explanation", the teaching of the facts should take place with the use of objects and in an order to follow; "Graduation", the teaching graduation would be organized by "didactic units" that would compose groups of fundamental facts, in an organized order to teach, divided into groups, with facts organized from the easiest to the most difficult (detailed in Albuquerque's(1951) guidelines); "Articulation", the teaching of a fact would occur in articulation with its inverse, and the operations of addition and subtraction should also be taught in an articulated manner - teaching the multiplication table in a way to establish relations would facilitate learning; "Generalization", repetition would lead to the generalization of relations between facts and operations; "Evaluation", evaluation would occur through repetition of facts, with different methods, including problems, games, giving correct answers, quickly and orally, would be an indication of learning.

And what about the knowledge about the practice of teaching multiplication tables? These were characterized in Group 2. The main elements characterizing a multiplication table to teach, knowledge about teaching practices, can be summarized as: exercising orality and writing, repeating the facts (aiming at "generalization"); using new

DOI: 10.20396/zet.v30i00.8667600

methods to teach, teaching using objects, real situations (aiming at "presentation, explanation, articulation"); using problems and games to teach the most difficult facts, because the context would provide meaning to the facts (aiming at explanation and grading); repeating and exercising the fundamental facts for memorization and reduction of errors (aiming at generalization and evaluation).

This concludes the search for the characterization of a multiplication table to teach from the analyses of the guidelines for teaching multiplication tables in Albuquerque's (1951) textbook. As already explained, the characterization of knowledge depends on the social historical and cultural researcher and his research problem, internal and external tensions and intentions. Thus, the characterization of a multiplication table to teach is not exhausted in the characterized elements, not even in this Manual.

Final considerations

The purpose of this text was to characterize elements of a multiplication table to teach, based on the guidelines proposed to the teaching of fundamental facts by Irene de Albuquerque, in her pedagogical manual, Methodology of Mathematics, published in 1951. To characterize a multiplication table to teach in a given time and context is, in a broader sense, to contribute to the characterization of a professional knowledge of the teacher who teaches mathematics.

The text, initially, tried to show that, even though it is present in a shy way in the current curricular guidelines, the multiplication table has always been present in school, since the nineteenth century, according to the textbooks. Moreover, recent studies have shown changes in the trajectory of multiplication table teaching in the early twentieth century, breaking with the traditional "memorizing the multiplication table", supported by the Pythagorean Table, for the use of materials that facilitate memorization and apprehension of the fundamental facts and the multiplication table (Parker's Cards and Calculation Tree). Finally, with the advancement of studies on the professional knowledge of the teacher who teaches mathematics, conducted by several researchers, members of the Associated Group of Studies and Research on the History of Mathematics Education (GHEMAT - Brazil), a study was started to continue those developed or in development, in which we intended to characterize a multiplication table to teach, based on the exercise of interpreting the guidelines of Albuquerque (1951) in his Manual.

In this study, the guidelines for teaching multiplication tables were detailed, so that knowledge could be analyzed and, from there, elements of a multiplication table to teach

DOI: 10.20396/zet.v30i00.8667600

could be characterized, in the light of the theoretical and methodological references, results that were achieved. The multiplication table to teach was thus characterized by a set of specific knowledge about the multiplication table to be taught by the teacher (knowing the fundamental facts; knowing the operation and its inverse; knowing the order to be taught and its exceptions; knowing the difficult order of the facts to organize the units to be taught, etc.) and knowledge about the practices of teaching multiplication tables.) and knowledge about the practices of teaching multiplication tables (exercising the fundamental facts with the students orally and in writing; knowing and using new methods to facilitate memorization; relating the facts to real situations; teaching, with the use of objects, from counting to the fundamental facts; knowing exercises, games and problems to practice the fundamental facts, etc.). The intention of this text was to show the movement from the reading and analysis of the information to the characterization of knowledge. It is worth mentioning that other paths for the characterization of a multiplication table to teach can be developed, as well as, in a broader sense, there can be further development of this study towards the objectification of a mathematics to teach in elementary school.

Acknowledgments:

I would like to thank to Capes Cofecub. To the research groups, GEMAIS/UFJ and GHEMAT-Brazil. To my advised in the Pedagogy course/UFJ, Sarah Sena C. Morais, for the work done. To the reviewer of this article, Vera Bonilha. To Silvia Iacovacci, the translator of this article.

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