Mathematical Knowledge for Teaching Algebra: curricular analysis of a primary teacher pre-service education

Conhecimento Matemático para Ensinar Álgebra: uma análise curricular na Licenciatura em Pedagogia

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Abstract

This article aims to understand and characterize the professional knowledge for teaching Mathematics and Algebra that are present in the curricular documents related to pre-service primary teaching programs. To execute this study, a document survey was conducted, which observed that the curricular documents envisage the development of mathematical knowledge for teaching in pre-service primary teacher education, but it does not describe the theme units presents in the Brazilian National Common Core Curriculum. Regarding the teaching of Algebra, during the analysis of the general guiding documents, it was observed the absence of this theme unit explicitly in these documents, which indicated that evidence of how this unit is worked in reality can only be observed during the teacher's planning.

Keywords: Algebra Teaching; Pre-service Primary Teaching Education; Teacher Instructor; Document Survey.

Resumo

No presente artigo tem-se por objetivo compreender e caracterizar o conhecimento profissional para o ensino da Matemática e da Álgebra que estão presentes nos documentos curriculares vinculados à Licenciatura em Pedagogia. Para a operacionalização do estudo foi viabilizada uma pesquisa documental que reconheceu que os documentos curriculares preveem o desenvolvimento do conhecimento matemático para o ensino na Licenciatura em Pedagogia, mas não caracteriza as unidades temáticas presentes na Base Nacional Comum Curricular. No que tange ao ensino da Álgebra, durante a análise dos documentos norteadores gerais nota-se a ausência dessa unidade temática de forma explícita nesses documentos, o que indica que evidências do que realmente é e como é o trabalho com essa unidade só pode ser observada no planejamento do formador.

Palavras-chave: Ensino de Álgebra; Licenciatura em Pedagogia; Formador de Professores; Pesquisa Documental.

Introduction

In the first grades, especially for mathematical teaching, the National Common Core Curriculum (BNCC[3]) brought along considerable changes, especially with the inclusion of

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Algebra as a theme unit (MEC, 2017). Before that, Algebra contents for first grades were presented in the national curricular documents along with number and operations' properties, with more incidence to relational thinking (Ferreira, 2017).

The spotlight given to Algebra, through the promulgation of the BNCC, has demanded a special attention to primary teacher education, adding to the challenge of rethinking the mathematical education of these professionals. It is important to highlight that tensions in Mathematics teaching, with emphasis to Algebra, are not new, given that they have existed since the beginning of Normal Schools[4] (Basei & Valente, 2019). Algebra was recognized in the normal formation only in 1890, before that, the axis worked in these courses were Arithmetic and Geometry, which were included in primary teacher education in 1946, in Brazil (Basei & Valente, 2019).

Based on the reflections presented by Curi (2020) on the importance that should be given to the Mathematics to be taught in pre-service primary teacher education (PSPTE) programs, in addition to the significant results of the study by Castro and Fiorentini (2021), which informed us on the low workload destined to the mathematical approach to teaching, in PSPTE programs in Brazil, this article aims to understand and characterize the knowledge for teaching Mathematics and Algebra that is present in (Brazilian) curricular documents related to pre-service primary teacher education. For that end, the following question is asked: *how do curricular documents propose the development of mathematical knowledge in pre-service teacher education programs and how is Algebra presented on these documents?*

To execute this objective, a document survey was conducted, starting with BNCC, the National Curricular Guidelines for the initial teacher formation (NCG[5]), the National Common Base for the initial teacher formation (NCB-Formation[6]) and the National Curricular Guidelines for pre-service primary teacher education programs (NCGPSPTE[7]).

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[3]A document that defines the organic and progressive set of essential learnings that all students must develop through the phases and modalities of Basic Education, so that they have their right to learning and development guaranteed, according to what is established by the Brazilian National Education Plan (PNE) (Brazil, 2017, p. 7).

[4] Institutions responsible for the education of primary teachers between 1834 and 2006, in Brazil.

[5] They are guidelines that institute general skills for the country's pre-service teacher education programs. All skills described in the NCG for the prospective teacher education are related to general skills of the BNCC. Despite choosing the 2018 prospective teacher education NCG (current version), we understand there are critiques to it, based on its prescriptive and mercantile characteristics in which the initial teacher formation if standardized and teacher subjectivity is removed from the process (Simionato & Hobold, 2021). However, it is necessary to highlight that, from the point of view of the contents, articulated to the objective of this article, the changes in it were not substantial.

[6] It is a base that covers a set of professional skills that all teachers must develop during their prospective teacher education to be able to put the general BNCC skills in practice. Just as the NCG, the NCB-Formation is also criticized for prescribing the idea of curriculum as a grid that boosters a formatted formation (Simionato & Hobold, 2021, p. 74).

[7] Instituted in 2006 based on the NCG, they are specific guidelines that regulated the PSPTE programs in Brazil.

Besides that, this study also analyzed the Pedagogical Project (PP[8]) of a PSPTE course of a public Brazilian university, and a Teaching Program (TP[9]) of a Mathematics discipline referring to the PP[10]. The choice for these last documents will be discussed throughout the text.

Theoretical Framework

Historically, teacher education courses would not present Algebra contents explicitly. In Brazil, in Normal Schools, for example, the presence of Algebra was associated with the intention of bringing teacher knowledge closer to what was taught in the classroom (Basei & Valente, 2019). With the development of BNCC, at the end of 2017, and the inclusion of Algebra as a theme unit to be taught in primary school, these contents started to be required from first grades' teachers. As a consequence, PSPTE programs had the need to develop the knowledge on algebraic contents and the specialized knowledge of these contents reinforced (Curi, 2020; Castro & Fiorentini, 2021).

This section presents a panorama of what is understood as Algebraic Thinking[11] and of the structure called Mathematical Knowledge for Teaching (MKT), in which one of its domains is directed towards common mathematical knowledge and another towards the specialized knowledge of the mathematical content (Ball; Thames & Phelps, 2008).

Algebraic Thinking: definition

Algebraic Thinking is defined as "a process in which students generalize mathematical ideas from a set of particular instances" (Blanton & Kaput, 2005, p. 413). Generalization, in this perspective "might be expressed in words or in symbols and could be based on the student's observation of a recursive pattern or a functional relationship" (Blanton & Kaput, 2005, p. 13). Specialized literature highlights two strands of Algebraic Thinking, the first relates Algebra as Generalized Arithmetic (Canavarro, 2007; Trivilin & Ribeiro, 2015; Barboza, Ribeiro & Pazuch, 2020) and, the second one, is based on the idea of Algebra through Functional Thinking (Kaput, 2008; Blanton, Brizuela, Gardiner, Sawrey & Newman-Owns, 2017; Ferreira, Ribeiro & Ponte, 2021).

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[8] It is a tool planned to regulate a determined course. This tool must contain all norms referring to the functioning of the course.

[9] Designed by the instructor, it is a discipline guide that must contain all the content that will be worked on the discipline, the way it is going to be worked and how many lessons it will take.

[10] The choice for the course's PP and for the analyzed TP is justified by the fact that this study is a phase of a doctorate research, in which, after this document survey, an empirical survey will be conducted (intervention phase of the mentioned thesis), having as a formation space the pre-service primary teacher education program whose documents will be analyzed here. Therefore, it is important to know the curricular and disciplinary principles before planning the survey's empirical phase, and, as a consequence, to portray the study of Resolutions that base the curriculum of teacher education programs in Brazil. It is necessary to highlight that, due to data confidentiality reasons, we will not identify the university and the instructor.

[11] BNCC presents four thinking forms (Arithmetic Thinking, Algebraic Thinking, Statistical and Probabilistic Thinking, and Geometrical Thinking) to work with theme units (Brazil, 2017).

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Discussing the first strand, Canavarro (2007, p. 89) highlights that "it is based on the structure of Arithmetic that the syntactic aspects of Algebra can be build, which implies on analyzing arithmetic expressions not in terms of the numerical value obtained by the calculus, but in terms of its form". Immersed in this strand are the comprehension of operations properties and the ability to generalize about them. Blanton and Kaput (2005, p. 414) highlight that the use of arithmetic to formalize generalizations (Generalized Arithmetic) means "reasoning about operations and properties associated with numbers or understanding equality as a relation between quantities". Understanding the meanings of the equal sing beyond the operational, as it is often worked during first grades, is an example of contents of the algebraic thinking present in the Generalized Arithmetic strand (Trivilin; Ribeiro, 2015; Barboza et al., 2020).

Regarding the second strand, called Functional Thinking, Canavarro (2007, p. 89) defines it as the one that "involves generalization through the idea of function, which can be seen, for example, as describing the variation of instances in a part of the domain". The characteristics of Functional Thinking are the description of regularities by using symbols and the work with sequences and patterns, as evinced in the tasks presented by Ferreira et al. (2021). Functional Thinking also involves the ability to describe regularities by using symbols. Among the Functional Thinking skills are "the ability to symbolize amounts and operate with symbolic expressions, to use graphs for representation, to discover relations between functions, to discover an unknown result by using what is known, and to identify and illustrate numerical and geometrical patterns" (Canavarro, 2007, p. 90).

Mathematical Knowledge for Teaching

Studies such as the ones by Curi (2020) and Castro and Fiorentini (2021) indicate how the scientific community has been dedicating itself to understand the structure of the knowledge that must be developed by future teachers for mathematical teaching. The MKT, presented by Ball et al. (2008) is defined as "mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students (p. 399)". This type of knowledge is shaped by a group of six domains that are set in a structure. Despite understanding the importance of all domains for the teachers' work, in this article, we draw attention to the study of two MTK domains: the *content common knowledge* and the *specialized (mathematical) content knowledge*. The choice for these two domains was propelled, as aforementioned, by the studies conducted by Curi (2020) and by Castro and Fiorentini (2021), in which the authors highlight the need to develop the (common and specialized) knowledge of teaching objects, based on the curriculum of the first grades of primary education, by future teachers of these grades.

The domain called Common Content Knowledge (CCK) refers to the mathematical knowledge used in a wide range of contexts. CCK is a knowledge non-exclusive to teaching, something every person can develop (Ball et al., 2008). Directed to mathematical content, this domain implies the teachers' knowledge of what they will teach students. Besides that, a

deep knowledge of the contents they teach allows teachers to know different strategies and to not restrict student learning.

The second domain, called Specialized Content Knowledge (SCK) refers to the skill of unpacking the common mathematical knowledge and making it accessible to students. To exemplify these two domains, their relations and distances, Ball et al. (2008, p. 401) affirm that:

> […] recognizing a wrong answer is common content knowledge (CCK), whereas sizing up the nature of an error, especially an unfamiliar error, typically requires nimbleness in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of specialized content knowledge (SCK).

Future teachers enter PSPTE programs with a certain (common) mathematical content knowledge (CCK), it is then necessary to direct Mathematics disciplines in these courses to the development of the specialized teaching content knowledge (SCK) (Curi, 2020). Besides that, according to Ball et al. (2008), common content and specialized content knowledge are necessary, but not enough to practice teaching, thus reinforcing the need for developing other MKT domains.

In sum, this article aims to look at these two MKT domains (CCK and SCK) seeking to analyze how they are developed in PSPTE programs, based on the guidance and recommendations of curricular documents, which regulate the existence of these programs.

Methodology

This article is qualitative-interpretative (Creswell, 2014) and documental research (Sá-Silva, Almeida & Guindani, 2009). The documents were analyzed by using the content analysis methodology (Bardin, 2016). As a procedure, documents were first read, beginning with the BNCC, which impact directly on teacher education. After that, the study of curricular documents that are close to PSPTE programs was initiated, such as the NCG and the NCB-Formation. Lastly, this phase was finished with the study of the NCGPSPTE.

After going through this course, it was observed the need to conduct a more specific study on and how the mathematical knowledge approached by these documents were practiced in PSPTE programs. Given that, a study of a course PP focused on the syllabuses regarding mathematical disciplines was carried out so to understand how the syllabus indications were seen in the instructor's planning.

To better organize the data, the next section is dedicated to presenting the documents used and then, the data analysis itself will be presented. For that, Figure 1 displays the organizations of the documents according to their presentation.
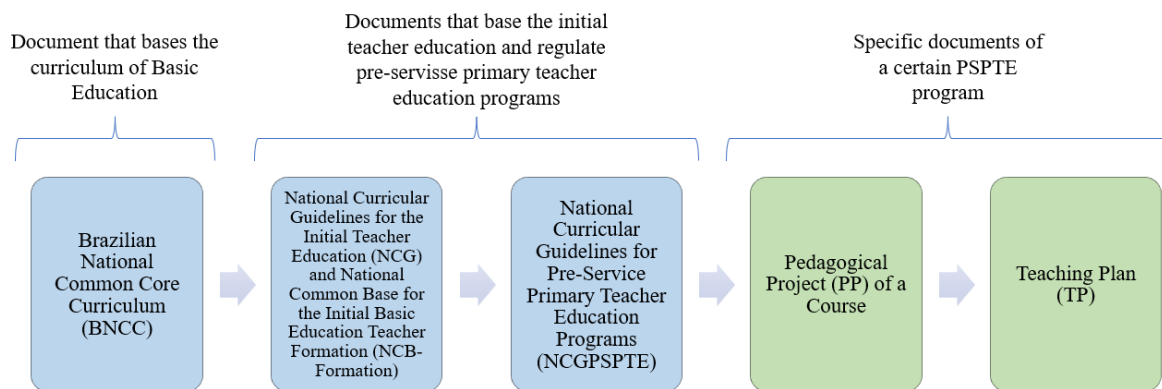
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Figure 1 – Order of data presentation

Source: Designed by the authors (2022)

Analysis procedures for data and their respective categories

Using a deductive analysis process, the starting point were the theoretical references studied to structure categories that could base the analysis (Bardin, 2016). It was possible to identify and show, in the studied documents, excerpts that are related to CCK and SCK (Ball et al., 2008) and the two strand of Algebraic Thinking, namely: Generalized Arithmetic (Canavarro, 2007; Trivilin & Ribeiro, 2015; Barboza et al., 2020) and the Functional Thinking (Canavarro, 2007; Blanton et al., 2017; Ferreira et al., 2021). The evidence found is an opportunity to develop both knowledge domains since, just by looking at the documents, it is not possible to guarantee that the teacher who go through the initial education will have already developed these domains.

Considering that, the data analysis categories were divided into two groups, preceded by the name of the "formation space", reinforcing the fact that the documents suggest working this knowledge in the prospective teacher education, but to evince them in practice, it is necessary to conduct a deeper and directed empirical study. The two groups were called: (i) Formation space directed to the development of basic Mathematical content (FSBM) and (ii) Formation space directed to the development of teaching Mathematical content (FSTM).

To organize the categories, a set of knowledge considered essential to the teaching practice was considered (Curi, 2020), among them is the knowledge on the concepts indicated to the first grades that, specifically for this study, branches into two new subcategories related to the strands of Algebraic Thinking (Table 1).

Table 1 – Data analysis groups, categories e subcategories

Groups	Categories; [code]	Subcategories; [code]
FSBM	Knowledge of the concepts indicated to the first grades (Curi, 2020, p. 15); [FSBM1]	Formation Space destined to the knowledge of Algebraic Thinking – **Generalized Arithmetic**; [FSBM1GA] (Trivilin & Ribeiro, 2015; Barboza et al., 2020)

		Formation Space destined to the knowledge of Algebraic Thinking – **Functional Thinking**; [FSBM1FT] (Ferreira et al., 2021)
	Knowledge of the role of Mathematics in the modern world (Curi, 2020, p. 16); [FSBM2]	
FSTM	Knowledge on the didactic treatment proper to the content and grade at hand (Curi, 2020, p. 15); [FSTM1]	
	Knowledge on the nature of Mathematics and of the internal organization of the area (Curi, 2020, p. 15); [FSTM2]	
	Knowledge of the mathematical doing, including problem solving, investigation activities, hypothesis identification, argumentation, communication, and the mathematical discourse (Curi, 2020, p. 15); [FSTM3]	

Source: Designed by the authors (2022)

The categories do not seek to exhaust the complete set of knowledge indicated by literature as essential to the teacher's work, but to deepen the knowledge related to content, be it the common (CCK) and/or specialized one (SCK). Besides that, when dealing with the common content, this article is related to the two strands of Algebraic Thinking.

Document presentation

The Brazilian National Common Core Curriculum

The BNCC is a guideline for the design of basic education curriculum all over the country and, consequently, it impacts policies for the pre-service and in-service teacher education. This document is recognized as a set of essential learnings that must be developed during basic education through ten general skills (MEC, 2017).

Each knowledge area (Languages, Mathematics, Natural Sciences, Human Sciences, and Religious Teaching) presents its specific skills that aim to have developed, by the end of basic education, the ten general skills presented in the document. The knowledge areas have theme units that include the goals of the knowledge, containing the skills to be developed in each grade of the basic education.

For the Mathematical knowledge field, the essential ideas are equivalence, order, proportionality, interdependence, representation, variation, and approximation. These fundamental ideas are distributed into the five theme units that compose Mathematics for

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Elementary School (Numbers, Algebra, Geometry, Quantities and measures, and Probability and statistics) (MEC, 2017).

In the theme unit of Algebra for Elementary School, BNCC proposes the development of Algebraic Thinking and highlights that it "must emphasize the development of a language, the establishment of generalizations, the analysis of quantity interdependence, and the resolution of problems through equations or inequations" (MEC, 2017, p. 270). The fundamental ideas corresponding to the Algebra theme unit are: equivalence, variation, interdependence, and proportionality.

Algebra contents for the first grades are presented through knowledge objects, such as: patterns, regularities and sequencies; equality properties; relations between addition and subtraction, and between multiplication and division; notion of equivalence; and directly proportional quantities. Each knowledge object branches into skills that, in a wider manner, direct toward what must be developed with students in basic education.

Documents for the preservice teacher education

The NCG and NCB-Formation are displayed in the CNE/CP Resolution no. 2, from December 20th, 2019, and are presented as general guides for all initial teacher education programs in Brazil. The NCGPSPTE, displayed in the CNE/CP Resolution no. 1, from May 15th, 2006, are a specific document that guide PSPTE programs in the country.

NCGs are responsible for generally organizing the curriculum of higher education teacher formation program, outlining parameters for the formation in second pre-service teacher education programs, regulating the pedagogical formation of major degree holders, regulating the action in pedagogical and management activities, and guaranteeing parameters for the internal and external evaluation of pre-service teacher education programs. The NCB-Formation ensures that the curriculums of preservice teacher education courses must be based on the BNCC and, so that all essential learnings are worked on them, the Resolution establishes the general and specific skills for teachers (CNE/CP 2019).

Table 2 – Dimensions of the Initial Teacher Education

Dimensions of the teacher education	Specific Skills
Professional Knowledge	Mastering knowledge objects and knowing how to teach them;
	Demonstrating knowledge about students and about how they learn;
	Recognizing the students' contexts of life;
	Knowing the structure and governance of educational systems.
Professional practice	Planning teaching actions that result on effective learning;
	Creating and knowing how to manage learning environments;
	Evaluating the development of the student, the learning and teaching;
	Conducting pedagogical practices of knowledge objects, skills, and abilities.

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Professional engagement	Being committed to their own professional development;
	Being committed to the students' learning and putting to practice the principle that everyone is able to learn;
	Participating in the school's Pedagogical Project and in the construction of democratic values;
	Professionally engaging with families and the community, aiming at improving the school environment.

Source: Designed by the authors (2022), based on CNE (2019)

NCGs stablish a minimum of 3,200 (three thousand and two hundred) hours directed to the preservice teacher education, which must consider the development of the professional skills established in the NCB-Formation. The total course load must be distributed into three Groups, according to Table 3.

Table 3 – Curricular division of the teacher education programs

Group I *800 (eight thousand) hours*	Group II *1,600 (a thousand and six hundred) hours*	Group III *800 (eight thousand) hours*	
Scientific, educational, and pedagogical knowledge.	Learning specific contents, components, theme units and knowledge objects of the BNCC.	*400 (four hundred) hours*	*400 (four hundred) hours*
		Supervised internship	Practice of the curricular components of Group I and Group II.

Source: Designed by the authors (2022), inspired by CNE (2019)

Among the skills that must be developed in the preservice teacher education, we highlight: "Displaying knowledge and understanding of the concepts, principles and structures of the area of teaching, content, of the phase, of the component and the knowledge area in which one is being trained to teach" (CNE/CP 2019, p. 15), and "mastering the Pedagogical Content Knowledge (PCK), using as reference the skills and abilities expected for each year or phase" (CNE/CP 2019, p. 15).

PSTE programs are destined to the formation of teachers to work in early childhood, in the first grades of elementary school, in high school courses, in the Normal modality, in Professional Education, in the area of school services and other areas in which pedagogical knowledge is foreseen (CNE/CP 2006). The egress of the PSTE program must be apt to teach Portuguese, Mathematics, History, Geography, Arts, Physical Education, in an interdisciplinary manner proper to the distinct phases of human development (CNE/CP 2006, p. 2). The NCGPSPTE guarantee that the structure of PSPTE programs must be constituted, among other things, by:

> Study of Didactics, pedagogical theories and methodologies, and of processes for the organization of teacher's work; decoding and using the codes of different languages

> used by children, besides the didactical work with contents, pertinent to the first schooling years, related to Portuguese, Mathematics, Sciences, History, Geography, Arts, Physical Education (CNE/CP 2006, p. 3).

To understand how curricular Resolutions are deprived in practice, we present a course PP, focusing the attention on understanding the development of mathematical knowledge for teaching, within said document.

The course PPC[12]

To organize the course PPC, a commission was instituted, which began its works in mid-2016, guided by the 2015 NCGs. The PSPTE program's general objective is directed to the formation of teachers to work in Early Childhood, in the first grades of elementary school with children, young, and adults, as well as in the educational management and coordination (CPP, 2018).

Since the initial proposal, there was a demand "for increasing the course load of the fields related to Mathematics, Sciences, History, and Geography" (PPC, 2018, p. 88). To meet this demand, the course loads of the components related to the teaching of Mathematics, Sciences, History, Geography, Didactics, Psychology, History of education, and Early Childhood were doubled (PPC, 2018, p. 90). The course PP is organized in Interdisciplinary Arguments[13], which are: (i) Epistemology and Education during the 1st and 2nd semesters; (ii) Human Rights and Inclusion during the 3rd semester; (iii) Epistemology and Teaching during the 4th, 5th, and 6th semesters; and (iv) Teaching and Society during the 7th, 8th, and 9th semesters.

Therefore, disciplines referring to the contents taught in basic education are disposed in the 4th, 5th, and 6th semesters, as well as Mathematics, displayed in the disciplines Teaching of Mathematics I (TMI) and Teaching of Mathematics II (TMII), present in the 5th and 6th semesters, respectively. These disciplines are the only ones in the course that focus on the content and/or teaching of Mathematics. Both these disciplines are mandatory and have a total course load of 60 (sixty) hours each.

The TMI discipline presents the following syllabus:

> Methodologies and auxiliary resources for the planning, evaluation, teaching and learning of Mathematics in Early Childhood and in the first grades of Elementary School with children, young, and adults. The mathematical language and its use in different social practices. Numbering system, numbers, and operations in the field of natural numbers. Information treatment: data collection, organization, and interpretation. The contextualization of different plannings necessary to teaching

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[12] To safeguard the course's identity, we will use the acronym PPC, followed by the year of its approval and page, whenever needed.

[13] Interdisciplinary Arguments are elements that articulate the development of curricular work through the following Mandatory Curricular Components: 1) Discipline and Modules; 2) Academic Activity of Individual Guidance, referring to the course completion work; 3) Academic Activity of Collective Guidance, organized as teaching internships in Early Childhood, in the first grades of Elementary School, and in Education Management (PPC, 2018, p. 76).

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> Mathematics in Early Childhood, Youth and Adult Education, and in non-scholarly contexts. Application, in the classroom, of the studies conducted in the curricular component through an activity of the Practical Interdisciplinary Survey of the Semester. Insertion of the curricular component in the Interdisciplinary Reading (Literary and Academic) of the Semester (PPC, 2018, p. 189).

As a complement, the TMII discipline, despite being related to the previous one, has its own characteristics:

> Methodologies and auxiliary resources for the planning, evaluation, teaching and learning of Mathematics in Early Childhood and in the first grades of Elementary School with children, young, and adults. The mathematical language and its use in different social practices. Numbers and operations in the field of absolute rational numbers. Plan and tridimensional geometric forms. Spatial organization: representation, interpretation, localization, and movement of objects in space. Symmetry. Quantities and Measures. Application, in the classroom, of the studies conducted in the curricular component through an activity of the Practical Interdisciplinary Survey of the Semester. Insertion of the curricular component in the Interdisciplinary Reading (Literary and Academic) of the Semester (PPC, 2018, p. 200).

Discipline planning: TMI[14]

This subsection presents a TP of the TMI discipline, whose aim is understood as the study of basic mathematical concepts worked during Early Childhood and in the first grades of Elementary School, and the study of the main didactical resources directed to teaching these concepts.

Regarding the program's content, the TP presents the study of the Mathematics Curriculum for Early Childhood and the first grades of Elementary School, just as it highlights some methodological axes for teaching Mathematics and the theme units of Numbers, Algebra, and the content of Information Treatment, according to Figure 2.

Program Content:	Mathematics Curriculum for Early Childhood and First Grades of Elementary School.
	Problem solving as a methodological axis of the teaching of Mathematics. The history of Mathematics as a didactical resource for teaching Mathematics.
	Numbers: decimal number system, operations (algorithms and problem solving), rational and decimal numbers, didactical resources for teaching Numbers.
	Algebra for the First Grades.
	Information Treatment: data collection, organization, communication, and

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[14] The choice for the Teaching of Mathematics I discipline was due to the instructor's availability to provide the Teaching Plan for analysis after a talk with the instructor, who reported working with three theme units during the TMI discipline and two theme units during the TMII discipline, considering that the Algebra unit would be in TMI.

	interpretation,	

Figure 2 – Program Content of the TMI discipline[15]
Source: (TP, 2021, p. 1)[16].

The teaching methodology indicated in the TP is based on cooperation, autonomy, and interaction among participants through basic activities, such as reading and interpreting texts, participating in classes, and performing written activities. About the classes, they seem to have been conducted online and, given the schedule displayed in the TP, eight classes have been reserved for each theme unit. In the schedule (Figure 3), it is possible to observe the lack of information on the algebraic contents worked how the were/would be taught:

07/07	Online class via Google Meet Algebra in the First Grades	4 classes
14/07	Online class via Google Meet Algebra in the First Grades	4 classes

Figure 3 – Schedule
Source: (TP, 2021, p. 2)

Data analysis

This section will be divided into two subsections, one focused on the general mathematical formation in the PSPTE program, and the second focused specifically on whether and how Algebra is covered in the presented curricular documents.

Mathematical education in Pre-Service Primary Teacher Education

Beginning the journey through the analysis of BNCC, one can see that the knowledge related to Mathematics existing in said document proposes a minimum set of contents to be worked during elementary school, aiming at guaranteeing the students' basic formation. This document is presented as a guide for the design of curriculums in basic education all over the country, impacting teacher education programs, especially with the implementation of the NCB-Formation.

The work with BNCC during the initial education makes it possible for future teachers to develop their common content (mathematical) knowledge (FSBM) and their specialized content (mathematical) knowledge (FSTM). For example, by proposing a study of BNCC and its skills, future teachers, with the aid of the teacher educator, can develop the content knowledge that must be worked during basic education (FSBM1) and the knowledge of the role Mathematics play in the world (FSTM2). The study of BNCC during the pre-service education can also make it possible for future teachers to know the internal organization of

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[15] The excerpts were highlighted by the authors to emphasize the contents that refer to the theme units that are worked on the Teaching of Mathematics I discipline.

[16] As in the course PP, we chose to refer to the Teaching Plan by using its acronym, followed by the year and page whenever needed.

the mathematical content (FSTM2), which evinces seven fundamental ideas: equivalence, order, proportionality, interdependence, representation, variation, and approximation (MEC, 2017, p. 267). These fundamental ideas are distributed among the five theme units that compose Mathematics for Elementary School (Numbers, Algebra, Geometry, Quantities and measures, and Probability and statistics).

The knowledge collected in the NCGs, NCB-Formation, and NCGPSPTE must be understood as specific/specialized knowledge of the teaching profession (FSTM). The NCGs aim at establishing minimum parameters directed to the formation of teachers, but they reaffirm the commitment to the development of a set of knowledge needed for the teaching practice:

> The formation of teachers demands a set of knowledge, skills, values, and attitudes, which are inherently based on practice, which must go far beyond the moment of the mandatory internship and should be present since the beginning of the course, both in the educational and pedagogical contents and in the specific contents of the field of knowledge to be taught (CNE/CP, 2019, p. 4).

The NCGs do not specify the course load to be destined to each discipline, it only marks the minimum 1,600 (one thousand and six hundred) hours for learning the specific contents, components, theme units, and knowledge objects of the BNCC (CNE/CP, 2019, p. 6). Therefore, it is possible to affirm that the document suggests working with scholar mathematical knowledge (FSBM) through the concepts indicated to be worked in first grades (FSBM1). Besides that, the NCGs keeps 800 (eight hundred) hours for scientific, educational, and pedagogical knowledge (CNE/CP, 2019, p. 6), enabling the opportunity to develop the knowledge of the didactical treatment of the content (FSTM1) and the knowledge of methodologies that may contribute to the teaching of Mathematics (FSTM3). In addition, the NCB-Formation establishes the specific dimensions and skills to be developed during the initial education of teachers. In the professional knowledge dimension, we highlight the specific skill that indicated the need to master the knowledge objects and know how to teach them (CNE/CP, 2019, p. 15). Mathematics corresponds to an area present in PSPTE programs, such specific skill provides for the opportunity to have spaces destined for the development of mathematical knowledge (FSBM) and spaces destined for the knowledge of how to teach Mathematics (FSTM).

Another specific skill referring to the teachers' knowledge involves "Mastering the Pedagogical Content Knowledge (PCK), using as reference the skills and abilities expected for each year or phase" (CNE/CP, 2019, p. 15). This skill is directly related to the specialized content knowledge (FSTM) and its categories are directed to the pedagogical treatment (FSTM1) and to gathering methodologies that must be used in teaching Mathematics (FSTM3).

As a consequence, the NCGPSPTE direct their look to the NCGs and establish specific guidelines for PSPTE programs, guaranteeing:

the study of didactics, of pedagogical theories and methodologies, of processes for the organization of the teaching work; decoding and using codes of different languages used by children, besides the didactical work with contents pertinent to the first schooling grades, related to Portuguese, Mathematics, Sciences, History, Geography, Arts, Physical Education (CNE/CP, 2006, p. 3).

Therefore, the mentioned excerpt also indicated that the NCGPSPTE guarantee the mathematical formation for future first grades teachers through a didactical work with the contents pertinent to the first schooling grades. This excerpt indicates a direction towards the development of specialized knowledge of the teaching profession (FSTM), being extended to the didactical treatment of the mathematical content (FSTM1) and to the presentation of methodologies for teaching this content (FSTM3).

The TMI and TMII disciplines, as observed in the syllabuses, mention the work with "methodologies and auxiliary resources for planning, evaluation, teaching and learning of Mathematics in Early Childhood and in the first grades of Elementary School for children, the young, and adults" (CPP, 2018, p. 189), reinforcing the development of knowledge for the didactical treatment (FSTM1) and of the knowledge of methodologies for teaching Mathematics (FSTM3).

Besides that, the syllabuses also provide for the work with the mathematical language and its use in different social practices (CPP, 2018, p. 189), enabling the knowledge of the role Mathematics plays in the world (FSBM2). Lastly, the syllabuses bring some mathematical contents, such as numbering system, numbers and operations in the field of natural numbers, and information treatment: data collection, communication, and interpretation (CPP, 2018, p. 189), enabling the opportunity to develop the knowledge of the concepts that will be taught in the first grades (FSBM1).

Algebra in curricular documents

Working with this area's contents must be included in the mathematical education of future primary teachers. However, it is not possible to find explicitly any mentions to the theme unit of Algebra within the NCGs, NCB-Formation or NCGPSPTE, but it is necessary to reinforce that the documents establish a workload to cover the knowledge objects and skills of the BNCC.

It is observed that the BNCC presents Algebra contents related to the first grades with emphasis to regularities, pattern generalization, and equality properties. Since it is a guiding document for the curriculum of basic education schools, in the scope of teacher education, it plays the role of indicating the contents that must be worked with future teachers for the enactment of their profession, through skills that are part of the BNCC. Given that, for the process of teacher education, one can understand that the skills related to the Algebra theme unit, in the way they are organized, can enable the development of content knowledge in the two strands of Algebra, be it emphasizing regularities and pattern generalizations (FSBM1FT), or when it highlights equality properties (FSBM1GA). For this opportunity to

be materialized in practice, it is necessary to look again at the PPC, so to understand whether and how Algebra is present in the syllabus of the disciplines related to Mathematics.

It is possible to notice that the syllabus of the TMI discipline provides for the work with contents of "numbers and operations in the field of natural numbers" (CPP, 2018, p. 189), contents that enable the development of both strands of Algebraic Thinking (FSBM1GA) (FSBM1FT). The TMII discipline provides for the work with "numbers and operations in the field of absolute rational numbers" (CPP, 2018, p. 200), contents that also enable the development of Algebraic Thinking (FSBM1GA) (FSBM1FT).

However, it is possible to notice the mention to other theme units and contents, just as the absence of Algebra. The explicit absence of Algebra in the syllabuses can be justified by the fact that the beginning of the design of this PPC was previous to the approval of the BNCC, a document that came into force in 2017 and that establishes Algebra as a theme unit. However, to understand whether Algebra is implicit in the instructor's work, even though it is absent in the syllabuses, we turned to the analysis of a TP of the TMI discipline.

In this plan, as showed, the Algebra theme unit is present, along with the unit of Number and the content of Information Treatment (Figure 2). In the specific case of the analyzed TP, the instructor had the autonomy of including Algebra in her planning even though it was not explicit in the syllabus. The question that remains based on that is: *will other instructors include Algebra in their respective TPs, even though it is not explicitly mentioned in the courses' PP syllabus?* Besides, it is necessary to consider that, despite the foreseen work with Algebra being explicitly present in the TP, the document does not inform how the theme unit was worked and which contents were explored (Figure 3), such as if both strands of Algebra were covered, materializing both mentioned formation spaces (FSBM1GA) (FSBM1FT).

Discussion and Final Remarks

After analyzing the documents, it is possible to understand that there is an intention of developing the mathematical knowledge with future teachers, but it is also necessary to highlight that literature indicates the need for other knowledge, beyond the common and specialized mathematical knowledge to work with teaching (Ball et al., 2008).

The aspects indicated in the documents suggest that teachers should develop knowledge for teaching Algebra directed to the two strands of Algebraic Thinking (Blanton & Kaput, 2005). For example, regarding the Generalized Arithmetic, the study of the equal sign properties is recommended (Trivilin & Ribeiro, 2015; Barboza et al., 2020), whereas for the Functional Thinking strand, the study of regularities and pattern generalization is recommended (Blanton et al., 2017; Ferreira et al., 2021).

It is necessary to highlight the importance of the teacher educator's work since the design of a TP is seen as important for the decision-making throughout the whole discipline. The act of planning as a component of the teaching practice must be considered through a

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reflection that goes beyond the mere formality with which it is seen by some teachers. Serrazina (2017) reminds us that the planning can include the challenges and anticipations regarding possible student difficulties. Based on the results of this study, it seems that the teacher educator responsible for the elaboration of the discussed TP has considered the contents of curricular documents related to basic education and the formation of teachers in her planning actions. We also infer that, by planning the discipline guided by these documents, the teacher educator may have enabled the development of the knowledge needed by future teachers for teaching Mathematics (*MKT*) (Ball et al., 2008).

It was possible to observe, through the legislation, that teacher education programs for the early grades provide, in their core, spaces for the mathematical education and for the development of the common and specialized content knowledge (Curi, 2020; Castro & Fiorentini, 2021). In opposition, Algebra only appears explicitly, as a theme unit, in the document referring to the first grades of elementary school (BNCC) and in the TP of the discipline. Here, we draw attention to the fact that, *if this theme unit is present in the curricular document of basic education, should it not be, by itself, materialized as a formation requirement for PSPTE programs?* After all, as well indicated by Shulman (1986), one cannot teach what one does not know.

In summary, the documents referring to the education of early grades teachers do not mention specific contents of any knowledge areas (theme areas), as it was also observed in the study by Castro and Fiorentini (2021). Therefore, it is up to the PSPTE programs to develop the syllabuses in the course PP. Hence, the role of teacher educators, through the planning of their disciplines' TPs, is essential to evince the knowledge referring to Mathematics and Algebra that will be provided to future first grades teachers.

Therefore, to conclude, it is worth to highlight the importance of the teacher educator role, who, in the real case studied here, was able to use her autonomy to insert the Algebra contents in the disciplines of Teaching of Mathematics. However, we suggest that, for this to happen more naturally in other institutions, teacher educators – responsible for the mathematical disciplines in PSPTE programs – must be involved in formative processes that are focused only on the development of Algebraic Thinking and Mathematical Knowledge for Teaching Teachers (MKTT) (Zopf, 2010), a type of knowledge that, besides containing the MTK domains (Ball et al., 2008), also considers the specificities of the teacher's work (Jaworski, 2008; Li & Superfine, 2014).

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