

## Mathematics Teachers and Educators knowledge

### Conhecimento do professor e do Formador de professores de/que ensinam matemática

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#### Abstract

Research on teacher knowledge has assumed, in the last 4 decades, a special focus of interest, and more recently several conceptualizations on mathematics teachers' knowledge have been emerging. This special issue of *Zetetiké* opened a space for discussing *Kindergarten, primary and secondary mathematics teachers and teacher educator's knowledge*. The 24 papers focus mainly, but not exclusively, the specificities of teachers' knowledge for the professional practice to be developed and they can be grouped in four groups discussing: (prospective) teachers or teacher educators' knowledge in the scope of different mathematical topics; different contexts to access, evaluate and develop such knowledge; resources and strategies in and for teacher education and different theoretical and methodological perspectives on teachers' knowledge.

**Keywords:** Teacher knowledge; Teacher educators' knowledge; Mathematics teacher; professional knowledge.

#### Resumo

Os estudos sobre o conhecimento do professor têm assumido, nos últimos 40 anos, um lugar de destaque e, mais recentemente, surgiram diversas conceitualizações do conhecimento do professor especificamente interessadas em discutir o conhecimento para a prática profissional do professor de matemática. Neste volume da *Zetetiké*, foi aberto um espaço especial para o tema do *Conhecimento do professor e do Formador de professores de e que ensinam matemática*, tendo sido aprovados 24 estudos que focam maioritariamente, mas não exclusivamente, as especificidades do conhecimento do professor que ensina matemática, os quais contemplam quatro perspectivas que discutem: o conhecimento de (futuros) professores ou formadores no âmbito de tópicos matemáticos específicos; diferentes contextos para aceder, avaliar e desenvolver esse conhecimento; recursos e estratégias na e para a formação de professores; e diferentes perspectivas teóricas e metodológicas relacionadas ao conhecimento profissional do professor.

**Palavras-chave:** Conhecimento do professor; Conhecimento do formador de professores; Professor de matemática; Conhecimento profissional.

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This special issue of the journal *Zetetiké* is associated with a call on the theme of mathematics Teacher's and Teacher Educator's Knowledge and aimed to bring together a set of papers focusing on the knowledge and practices of teachers and teacher educators – having as scope the education of mathematics teachers, both initial and continuing. The 24 published papers have the participation of national researchers in 18 texts and international researchers in six texts.

The knowledge of teachers and teacher educators has been the focus of attention in Mathematics Education for some time now from different perspectives, from a more general educational view to one that considers the specificities of the teacher's professional practice to teach mathematics (Ribeiro, 2017). In this general perspective, the specificities of this knowledge for the professional practice of teaching mathematics are not considered since the conclusions obtained are so general that they can be valid for the professional practice of a teacher of any area of knowledge (Fiorentini & Crecci, 2017) - thus, the researchers assume that the context in which the practice occurs already leads to talk about the knowledge of the mathematics teacher or teacher educators, which corresponds to the discipline that is addressed in this context.

Considering the specificities of teacher knowledge for the professional practice of teaching mathematics, or the practice of educators for that professional practice, research has shown the need to refine, specify, and gain a deeper knowledge and understanding about the content of that knowledge (e.g., Ball, Thames, & Phelps, 2008; Carrillo, et al., 2018; Rowland et al., 2005), with an emphasis on its role and presence in teachers' practices and the educational process, as well as in terms of its impact on and for teacher education.

For this special issue, the call for papers specifically integrated a focus on the latter way of understanding this teacher professional knowledge and the associated and required specificities for the professional practice to be developed. Thus, assuming the diversity of theoretical and methodological perspectives that can be considered in the different ways of understanding these specificities, in which each of them may have, at its core, a different dimension that shapes teachers' practice and the way in which teacher education can, needs, and/or should be framed given the most recent research findings with this focus. It becomes essential, then, to have a deeper and wider discussion about the core elements associated with teacher practice and teacher education, in order to be able to contribute to impact the development of teacher education courses and public policies that consider such specificities of teacher practice at its core and its development, as one of its main goals.

The 24 papers that make up the special issue can be grouped into four major groups, with some of them focusing in greater detail the theoretical and/or methodological discussions on the issues of the specificities of teacher knowledge for their professional practice of teaching mathematics, while a minority of others tend not to make this discussion explicit. We note that there are intersections between the different groups – and, in this sense, we do not perform a categorization of the papers presented, but we seek to emphasize the characteristics that stand out most in each of them. The four groups can be stated as:

*(i) Knowledge and practices of mathematics teachers and teacher trainers.*

This group integrates a total of 11 papers that are distributed along three dimensions: on the teacher educator (Coura & Passos, 2021; Ferretti, Martignone & Rodríguez-Muñiz, 2021; Polegatti, Camargo & Savioli, 2021); they explicitly discuss teacher knowledge on some mathematical topic (Alberca, M. & Contreras, 2021; Alencar, Diaz-Levicoy & Soares, 2021; Escudero, Muñoz-Catalán & Carrillo, 2021; Fabri, Panossian, Amin & Oliveira, 2021; Policastro & Ribeiro, 2021; Santos Junior & Maia, 2021; Teres & Grando, 2021) and discuss the relationship of professional knowledge and competence associated with instructional quality (Lindmeier, Seemann, Wullschleger, Meyer-Wyder, Leuchter, Vogt, Opitz & Heinze, 2021).

The papers focusing on the teacher educator discuss the educators' knowledge when (s)he assumes a role as a teacher researcher and takes teaching and teacher education as an object of study (Coura & Passos, 2021), concluding that being a educator requires a specialized type of knowledge. Ferretti, Martignone and Rodríguez-Muñiz's (2021) paper, on the other hand, based on the conceptualization of Mathematics Teachers Specialized Knowledge – MTSK (Carrillo et al., 2018), presents and discusses an extension of the MTSK model that can characterize the specialized knowledge of the teacher educator, focusing this discussion on the domain of PCK. The role of the teacher as an indigenous teacher educator of mathematics is the focus of attention and discussion in the paper by Polegatti, Camargo, and Savioli (2021).

Specific mathematical topics, and the (future) teachers' knowledge associated with, involved in, or required for practice or in solving tasks are the focus of five of the papers in this special issue. Alberca and Contreras (2021) discuss the knowledge of a primary teacher in the context of subtraction; Escudero, Muñoz-Catalán, and Carrillo (2021) present a discussion that discusses the knowledge of an kindergarten teacher in a context of practice related to geometric bodies; Fabri, Panossian, Amin, and Oliveira (2021) focus on teachers' manifestations of statistical knowledge in a continuous education context; Policastro and Ribeiro (2021) carry out a discussion focusing on teachers' knowledge on the topic of division and present a set of indicators of this knowledge; Santos Junior and Maia (2021) discuss how prospective teachers relate to non-Euclidean geometric models; Alencar, Diaz-Levicoy, and Soares (2021) present work related to teacher knowledge involved in the creation of stories for kindergarten associated with the teaching of decimal numeration; and Teres and Grando (2021) address teacher knowledge within the topic of Algebraic Thinking specifically associated with the topic of recursive sequences and associated generalizations, and discuss how the educational context contributes to teacher development.

- (i) The role, importance and impact of different contexts to access, assess and promote the knowledge development of mathematics teachers and teacher educators.*

This set composed of three papers discusses tasks for professional learning (Barboza, Pazuch, & Ribeiro, 2021); hypothetical learning trajectories (Oliveira & Ferreira, 2021) and the existence of a specific mathematics to teach (Grilo, Barbosa, & Maknamara, 2021).

Barboza, Pazuch and Ribeiro (2021) resort to professional learning tasks in the process of researching the construction of teachers' mathematical and didactic knowledge about the different meanings of the equality sign. Among the resources for teacher education are also the hypothetical learning paths that are the focus of attention in the paper by Oliveira and Ferreira (2021) in which the authors conduct a theoretical discussion focusing on the teaching of logarithms. Grilo, Barbosa and Maknamara's (2021) paper also assumes a perspective of theoretical discussion that aims to discuss the device of mathematics specificity operated by mathematics teachers, concluding about the existence of a specific mathematics to teach.

*(ii) The role of resources and strategies in and for teacher education (initial and continuous).*

Included in this group are three papers that discuss strategies (Schreibe & Porciúncula, 2021; Silveira, Santos, & Lawall, 2021) and technology (Gutiérrez-Fallas & Henriques, 2021).

In their paper, Schreibe and Porciúncula (2021) report a research that presents specific teaching knowledge about students and pedagogical strategies identified in teachers' narratives within statistical concepts; Silveira, Santos, and Lawall (2021) present and discuss the validation of a cooperative board game as a resource for teaching statistics in primary, this game being an educational product of a professional master's degree. The focus of the work by Gutiérrez-Fallas and Henriques (2021) discusses the development of technological pedagogical knowledge of prospective teachers, concluding that the educational experience promoted the development of technological competencies of the participants.

*(iii) Theoretical perspectives and methodological tools for studying and understanding mathematics teaching and its interaction with teachers' personal resources and other dimensions of professional knowledge.*

This group includes seven papers, each of which focuses on distinct theoretical or methodological dimensions essentially associated with complementary dimensions of our professional knowledge as mathematic teachers.

Costa, Silva and Gontijo (2021) present and discuss creativity workshops implemented with students and indicate some implications for the professional practice of teacher's who seeks to develop this creativity in students. In their study Melo, Giraldo and Rosistolato (2021) discuss, from a context of shared practice in higher education, some aspects of the teaching professional identity that emerge from the interactions between the actors involved in the shared teaching experience, observed from the dynamic relations between identity and otherness. A set of imbricated dimensions of teacher knowledge are associated with the promotion of inclusion and, in this scope, Ribeiro and Cristovão (2021) present an analysis

based on the MTSK that evidenced the importance of formative practices in the inclusive perspective so that prospective teachers can articulate mathematical and pedagogical knowledge with knowledge about Special Educational Needs, for an inclusive teaching of mathematics.

Moraes (2021) develops a work with future prospective kindergarten and primary teachers that, despite being of a more general scope and not focusing specifically on dimensions of the prospective teachers' knowledge, discusses the desire and willingness to enable the practices that they will implement so that their students do not face the same difficulties with mathematics that they themselves experienced.

Mancini Mocrosky, Orlowski and Kalinke (2021) present a research with primary teachers working at the early stage of developing students' numeracy highlighting ways of being a teacher through numeracy actions reflected in the daily teaching and present as a conclusion that the reading practices are shown as a formative possibility for primary teachers.

Barichello and Firer (2021) present a study that, despite not bringing general results that are new in the area, reinforces the worrying situation that the vast majority of prospective mathematics teachers arrive at the end of their teacher training program without adequate mastery over contents, notably upper secondary contents, something that research in Mathematics Education focused on teacher knowledge has been emphasizing as knowledge of the students' level and the need for us, as teachers, to be holders of a complementary mathematical knowledge specifically associated with the professional practice to be developed.

Moriel Junior (2021) conducts a review of publications on the Web of Science that have employed the MTSK conceptualization in their research, which allows us to gain insight into the scope and impact of this conceptualization of mathematics teacher knowledge in research focusing on mathematical knowledge and practices.

We hope that with this special issue of the journal Zetetiké dedicated to **Mathematics Teacher's and Teacher Educators Knowledge**, and the ways of understanding this knowledge and its specificities, or not, we hope to contribute to make it possible to deepen, broaden, and detail the elaboration of possible proposals that could contribute to unpack the work of the practices of mathematics teachers and teacher educators, leading to increase the community's understanding of the factors that would contribute most to the improvement of the quality of the teaching process and what this improvement would look like.

The 24 papers published here lead us to think about a set of possible theoretical and methodological paths to take so that we can refine our look at the specificities of mathematical practices in multiple contexts and the specificities of teacher knowledge associated with these practices in order to enable students to effectively understand mathematics. We hope, therefore, that reading these papers will help us – as a community of researchers in Mathematics Education, and in particular, that we develop our research

focusing on some dimensions involving the teacher when in specific training contexts or others that aim at the improvement of training and practice – to (re)think our research and educational practices, to (re)think our focuses of attention so that we can propose and implement the necessary (R)Evolution in research and teacher education that would lead to the improvement of students' results, through mathematical understanding.

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