Zetetiké

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Proposal of a Didactic Model for the development of Creative Thinking in Mathematics

Proposta de um Modelo Didático para o desenvolvimento do Pensamento Criativo em Matemática

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Abstract

In view of the various global issues, research results report that creativity is of fundamental importance in personal and interpersonal relationships with objects of study during knowledge construction processes to obtain possible answers. To contribute to this scenario, the objective of this work refers to the construction of a Didactic Model for the Development of Creative Thinking in Mathematics (DM-CTM), based on investigations regarding **creativity** and its dimension in teaching and learning processes in mathematics. These investigations have enabled analysis and interconnections with fundamental elements of didactics, with the Anthropological Theory of Didactics and, more specifically, with the Study and Research Path. The DM- CTM is a theoretical construct that opens a range of possibilities for research on didactic phenomena having creativity as its aspect.

Keywords: Creativity; Didactic model; Creative thinking in mathematics; Study and research path.

Resumo

Diante das diversas questões mundiais, os resultados de pesquisas relatam que a criatividade é de fundamental importância nas relações pessoais e interpessoais com objetos de estudo em meio a processos de construção de conhecimentos para a obtenção de possíveis respostas. De modo a contribuir com este cenário, o objetivo deste trabalho refere-se à construção de um Modelo Didático para o Desenvolvimento do Pensamento Criativo em Matemática (MD-PCM), a partir das investigações a respeito da criatividade e sua dimensão em processos de ensino e aprendizagem em matemática. Essas investigações possibilitaram análises e interconexões com elementos fundamentais da didática, em particular com a Teoria Antropológica do Didático e, de forma mais específica com o Percurso de Estudo e Pesquisa. O MD-PCM é um construto teórico que abre um leque de possibilidades para a investigação a respeito de fenômenos didáticos tendo como vertente a criatividade

Palavras-chave: Criatividade; Modelo didático; Pensamento criativo em matemática; Percurso de estudo e pesquisa.

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Introduction

This theoretical article aims to present a Didactic Model for the Development of Creative Thinking in Mathematics (DM-CTM), based on the results of authors who have produced understandings about the notion of **creativity** and on the Anthropological Theory of Didactics - ATD - (Chevallard, 1999, 2006).

With regard to the ATD, the theoretical constructs praxeological organization (or simply praxeology), Study and Research Path (SRP), milieu, didactic system, cognitive universe, among others, are fundamental in our proposal of DM-CTM. These theoretical constructs, although some of them appear at the beginning of this article, will be taken up and characterized in different parts of the text and in the section reserved for the discussion of the ATD.

Creativity is an important theme in the training of students as citizens of tomorrow, since being able to grasp the complexity of our world by fully mobilizing their creative faculties seems to be an essential condition for taking on the individual and social challenges that each of us encounters in our lives.

To present understandings of the notion of creativity, we make some reflections in the next section. These understandings underpin our problematic and the characterization of our research objective.

Problematization

Creative individuals are becoming increasingly urgent and essential in all areas of society. Disasters occur frequently and problem-solving demands creativity and expertise.

Numerous job vacancies[3] require creativity due to the shift towards technology that enables robotic labor to perform repetitive and mechanical tasks.

However, before delving into the topic of creativity, it is crucial to establish a definition.

Young (1985) argues that the term **creativity** originates from the Latin word *creare*, which means **to make**, and the Greek word *krelnein*, which signifies **to fulfill**. The author suggests that creativity can be approached in one of two ways. Creativity involves either constructing or creating something. By employing their imagination, people can **invent** something novel and improve upon what already exists. As per Young's perspective (1985, p.77), the creator transforms the old to new and adds their unique stamp to everything they do. They strive to be innovative, leaving behind traditional methods in favor of fresh improvements.

[3] https://revistacapitaleconomico.com.br/7-principais-exigencias-do-mercado-de-trabalho/

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Based on this definition, creativity is not about having an extraordinary idea unexpected. Instead, it centers on **the ability to partially recreate the ideas of others.**

As stated by Franken (1994, p.36), creativity is the inclination to produce or identify concepts, choices, or prospects that might be advantageous in resolving issues, communicating with others, and amusing ourselves or others.

These two definitions of creativity align with the historical progression of mathematics, which demonstrates that it evolved from a collection of concepts that were created and developed by previous civilizations based on the context of the time. These creations and mathematical breakthroughs determined the concept of mathematics held by these societies at that time. For instance, Devlin (2002, p.7-9) reports that for Egyptian and Babylonian civilizations, mathematics was a study of numbers, and the creations of these societies served practical purposes. In Greek civilization, mathematics was the study of numbers and shapes.

They created figures by combining numbers with geometric shapes. The Greeks had a geometric approach to mathematics. This perspective on mathematics remained largely unchanged among other cultures that developed mathematics (such as the Chinese, Indians and Islamic peoples) until the mid-seventeenth century. It was then that Newton and Leibniz introduced the concept of calculus. Soon after the introduction of calculus by Newton and Leibniz, mathematics expanded its scope to include the study of numbers, form, movement, change, and space. The prevailing viewpoint since the 20th century is that mathematics is the science of patterns, which are created, invented, and/or discovered through the expertise of innovative thinkers.

It can be concluded that creativity is inherent in our actions, in our know-how, in our praxeologies.

Praxeology, a concept introduced by ATD (Chevallard, 1999, 2006), is an approach to understanding human activity that combines a practical block, or praxis, with a theoretical block, or logos. Praxis refers to the practical dimension or **know-how:** it is formed by types of problems or **tasks** to be studied and a set of techniques built to accomplish them. The second **block**, logos, pertains to the descriptive, organizational, and justificatory part of the activity. This **block** includes the technological discourse (the logos of the techniques), which is composed of descriptions and explanations that facilitate comprehension of the techniques and allow for the creation of new ones. This block is formed by another level of justification, called theory, gives meaning to problems, allows for interpretation of techniques, and serves as the basis for technological descriptions and justifications.

Praxeologies are intrinsic to any theory in mathematics didactics. The various types of praxeologies reveal human creativity and/or creative thinking.

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Mathematics didactics perspective suggests that by developing and observing praxeologies in a didactic situation[4], we can identify present didactic phenomena. Praxeologies can only exist in a didactic situation when problems, questions, and different study paths leading to different solutions arise.

The definition of mathematical creativity at the school level is founded on a praxeological organization. According to Liljedah and Sriraman's (2006) definition, mathematical creativity is:

> (a) the process that results in unusual (new) and/or insightful solutions to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be viewed from a new angle that requires imagination (p. 19).

We infer, therefore, that mathematical creativity can develop, in the case of the classroom, in student-knowledge-teacher interactions in a didactic situation, the *milieu*[5] being provided with a certain type of praxeologies that favor mathematical creativity and/or the development of creative thinking in mathematics.

Margolinas (2009, pp. 13-14) explains that "according to Brousseau, the subject-milieu interaction is the smallest unit of cognitive interaction. A state of equilibrium in this interaction defines a state of knowledge, while the subject-milieu disequilibrium produces new knowledge."

Based on this definition, we infer that the **driving spring** for the development of creative thinking in mathematics depends on how the subject-*milieu* interaction is established for a certain milieu equipped with praxeologies that contribute to its development. Therefore, the aim of this paper is to present explicitly the Didactic Model for the Development of Creative Thinking in Mathematics (DM-CTM) based on ATD. Furthermore, the DM-CTM seeks to uncover the potential links between ATD and creative thinking in mathematics.

The fundamental structure of DM-CTM consists of support pillars related to the dimension of creativity, including four main aspects: a) the concept of creativity; b) category related to what is creative; c) the concept of creative mathematics; and d) the concept of creative thinking in mathematics. These aspects will feed back interaction between the

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[4] Brousseau (2011, p. 2) distinguishes two types of situations in the Theory of Situations: The mathematical situations in which no didactic intervention is foreseen and the didactic situations, which include a mathematical situation, embedded in a system of conditions that lead the subject to the direct adoption of the behaviors determined only by the intervention of the teacher, whether or not the student perceives the mathematical necessity. Mathematical situations aim to represent the minimum conditions necessary to explain or justify the implementation of a mathematical statement by an agent or a group of agents, without external didactic intervention.

[5] For Chevallard (2007, p. 344) the *milieu* is any system devoid of intention in the answer it can bring, explicitly or implicitly, to a given question. This *milieu* can only be understood in a dialectic with the media, defined very generally as any system of representation of a part of the natural or social world for a given audience. This position thus places the teacher and documentation systems outside the milieu and results from an analysis of oppositions and an overcoming of these oppositions. The *milieu* is devoid of intentions and the media has the intention to inform. For a process of study and research to engage in genuine questioning without being subject to the authority of an institution, a vigorous (and rigorous) dialectic must exist.

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student, teacher, and knowledge where the *milieu* is provided with didactic intentions[6] that encourages questioning, research, invention, and innovation, thus promoting the development of creative thinking in mathematics.

In the following section, we will describe the theoretical foundation of mathematics didactics, which underpins the DM-CTM, namely the Anthropological Theory of the Didactic.

Anthropological Didactic Theory (ADT)

The ADT is a theory created by Chevallard. According to Chevallard (2009a), this theory introduces certain fundamental notions which are discussed in this paper.

The primary fundamental notion of the theory is that of an object, any entity, material or immaterial, that exists for at least one individual. The second fundamental notion is the personal relation between an individual x and an object *o*, which is denoted by *R (x; o)*. If the personal relation of an individual *x* to an object *o* is other than empty, denoted by $R\ (x;\ o) \neq \varnothing$, we state that *o* exists for *x*. The third fundamental notion, that of person, is then the couple formed by an individual *x* and the system of his personal relationships *R (x, o)*, at a given moment in *x*'s history. The word person, as used here, should not be misleading: every individual is a person, including the very small child. Of course, in the course of time, *x*'s system of personal relationships evolves objects that didn't exist for him begin to exist; others cease to exist; for others, *x*'s personal relationship changes. In this evolution, the invariant is the individual; what changes is the person. Throughout this evolution, the individual remains constant while the **person changes**. The fourth definition is the Institution (*I)*, which explains the formation and evolution of the cognitive universe of an individual (*x*). An institution (*I*) acts as a social device in which **person** (*x*) subject themselves to various positions (*p*) offered by the institution, bringing into play their own ways of doing and thinking - that is, their praxeologies.

The concept of praxeology is central to the ADT. This concept generalizes several common cultural ideas - those of knowing[7] and know-how. A praxeology ($\wp$) is made up of

[6] For Sensevy (2010, cited in Aldon, 2011, p.40-41) didactic intentions are linked to a problem and to the *milieux* of preparation and action, which are to be described at different levels, general intentions linked to a personal epistemology (a practical epistemology) and local intentions constructed for a specific purpose. As a result of the interactions of the teacher with the construction *milieu* in a curriculum-related situation and of the teacher in a construction situation in interactions with the didactic milieu, intentions are faced with contingency and can be modified locally in a didactic situation due to interactions with the reference environment. Similarly, the teacher-observer confronts his/her intentions with the feedbacks of the students' goal-*milieu*. In this scheme, the position of the students must be added as actors of the didactic situations having, in the same way, intentions to learn what needs to be considered from a global point of view, in connection with a general position towards the School and locally, in connection with the particular context of a lesson. Thus, the close relationship between intention and milieu is particularly sensitive in innovative environments, and in particular in computerized environments.

[7] In this text we differentiate between knowledge and knowing, as defined by Margolinas (2014, p. 15): Knowledge is what realizes the balance between the subject and the milieu, what the subject puts into play when investing a situation. Knowing is a social and cultural construction, which lives in an institution, and which is by

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the praxis block [*T*, *τ*], know-how, and the logos block [*θ*, *Θ*], which includes speeches about practice. *T* refers to the type of task that includes at least one t task, *τ* refers to the technique or method of carrying out a task of type *T*, *θ* is the technology, which is a speech about the technique, and *Θ* is the theory or justification of the technology.

Praxeologies exist within the educational systems present in didactic situations. The didactic situation refers to the set of relationships established within a didactic system *S(x, y, o) - x*, where *x* refers to a student or a group of students, *y* is the director of the study who could also be the teacher, and *o* is the mathematical object being studied. The relationships include $R_I(p;o)$, which is the ideal relationship that each subject *x*, in position *p* within the institution *I*, should maintain with object o and *R(x; o)*, which is the relationship that each subject has with object *o*. Brousseau (1978) states that these relationships are established explicitly or implicitly between a student *x* and/or a group of students (*X*), a certain *milieu*, and an educational system led by the teacher (*y*) so that the students can acquire a knowledge that is constituted or in the process of being constituted.

Chevallard (2009b) proposes a new perspective on how praxeologies can be exist in didactic systems. This new perspective involves planning didactic situations in advance to develop a Study and Research Path (SRP). The aim of this approach is to promote an investigative process to obtain an answer ($R^{\heartsuit}$) to a question (*Q*) potentially generating other questions. This investigation process consequently mobilizes the development of new praxeologies that bring together old and new knowledge, thus expanding the praxeological equipment of the subjects involved.

The praxeological equipment is, for Chevallard (2007), the set of praxeologies that a person *x* has in relation to an object of knowledge and, which are explicit in their records regarding technique, technology, and other relevant aspects. The praxeological equipment of a person *x* is closely related to his subjections to the institutions through which he has passed and to his relations with *o*, which is adjusted/remodeled in a Study and Research Path (SRP), for example.

The SRP is based on the paradigm of questioning the world, proposed by Chevallard (2015), based on the Herbatian scheme: $S\ (x;\ y;\ Q) \curvearrowright M] \hookrightarrow R^{\heartsuit}$, in which the study and research process take place around the questioning *Q*. *M* is the didactic milieu, which in the course of the journey changes and/or expands according to the questions derived from *Q* and/or the student's answers $R^{\lozenge}$ printed by the institution, which *x*, with the help of *y*, manages to make explicit. The milieu *M* is the set of all useful resources activated by *x* for the construction of $R^{\heartsuit}$.

nature a text (which does not mean that it is always written materially). Knowing is depersonalized, decontextualized, de-temporalized, formulated, formalized, validated, and memorized. Knowledge therefore lives in a situation, while knowing lives in an institution. To define knowledge, it is necessary to describe the situations that characterize it. To define a knowing, it is necessary to determine the institution that produces and legitimizes it, which sometimes leads to considering several institutions and their possible conflicts (our translation).

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In this work, the SRP is a determining factor for the proposed DM-CTM.

Basic Structure of the Didactic Model for Creative Thinking in Mathematics (DM-CTM)

The DM-CTM was developed based on certain pillars. These pillars include: a) the concept of creativity, b) category related to what is creative, c) the concept of creative mathematics, and d) the concept of creative thinking in mathematics. The interconnections between these pillars and some constructs of mathematics didactics based on the ATD were also considered. In the structure of the DM-CTM, these pillars serve as the support for a didactic situation regarding the sphere of creativity.

Revealing the existence of these pillars, their meanings and the interconnections between them, contributes to the development of a feedback system of knowledge and/or knowing, which can be fundamental to signal the need for significant changes in the didactic system and /or interactions between subjects present in a didactic situation.

A feedback system refers to a system, which, armed with knowledge/knowing regarding the pillars and their interconnections, triggers feedback and/or action mechanisms, when necessary, to train those who teach someone to learn something. This system promotes adjustments to the **milieu** and/or the equipe of didactic intentions so that the adaptation/imbalance processes necessary for the development of creative thinking in mathematics to occur and, consequently, mathematical knowing to be unveiled, explored, constructed, innovated in a learning environment between subjects.

The basic structure of the DM-CTM, its components, interconnections, and feedback system are shown in Figure 1. The feedback system, represented by double-directional arrows that connect to the milieu, flows from the DM-CTM's base structure to the base of the didactic triangle, which includes knowing, student, and teacher providing feedback to the entire system and, consequently, changing the way in which interactions are established between student-teacher and the knowing at stake, the **milieu**, the didactic situations and, consequently, the way in which the praxeologies are established. From this feedback system, a natural process of expansion of the milieu can occur, which transcends the didactic triangle, represented by the cylindrical dotted structure. This cylindrical structure arises from the base, since the expansion is the result of a process of interconnection of the pillars CRC (Category that refers to what is creative: Person/Impression/Process/Product), C (Creativity), CTM (Creative Thinking in Mathematics) and MC (Mathematical Creativity) with some constructs of didactics of mathematics based on the Anthropological Theory of Didactics (ATD).

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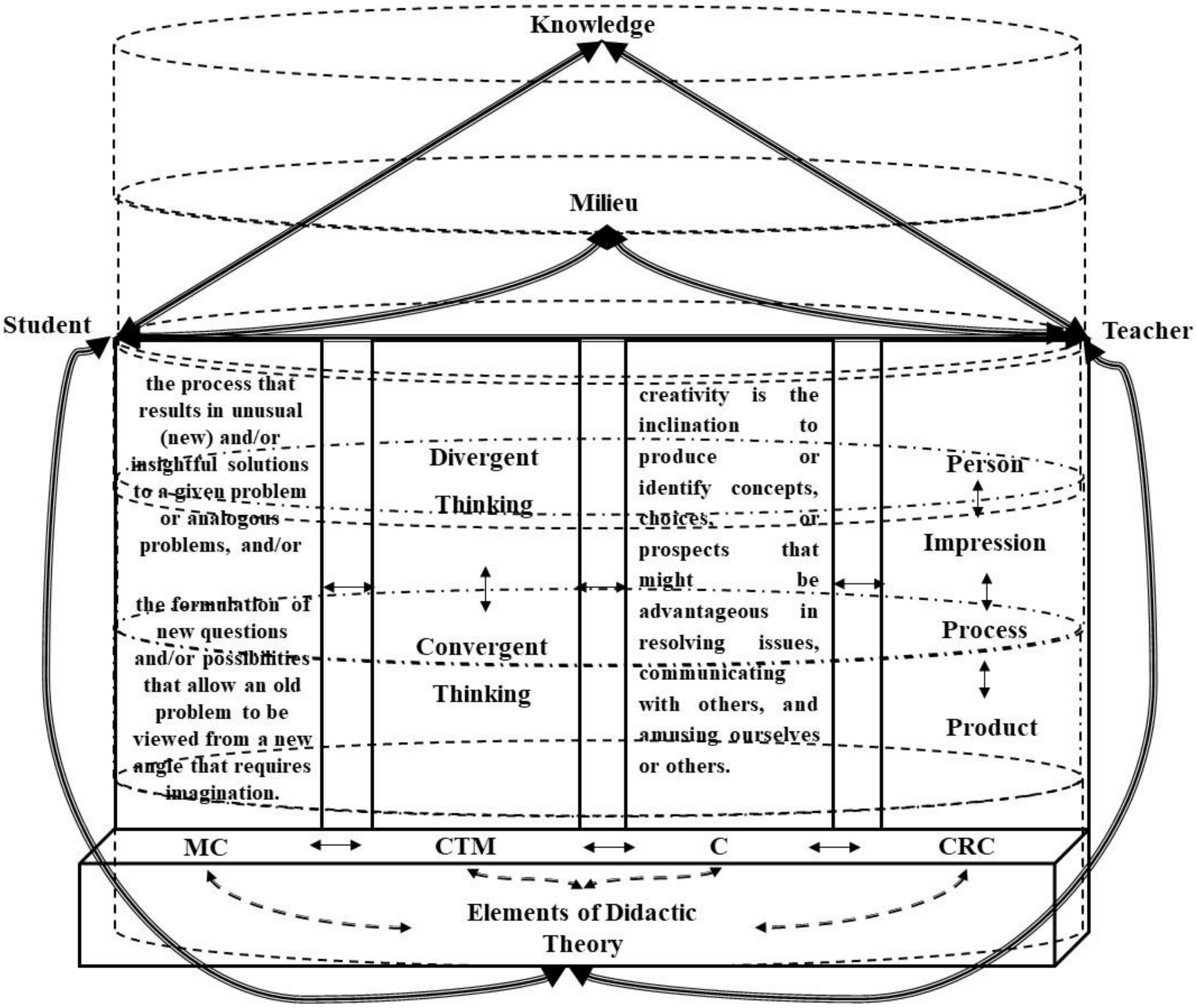

Figure 1 - Basic structure of the MD-PCM

Source: Authors' own elaboration

But what are the pillars of support and what interconnections are established between them? The pillars refer to the columns in Figure 1 and they were listed from a process of study and research on the dimension of creativity in teaching and learning processes.

Figure 2 presents the process of interconnections of the pillars and their influence on the cognitive universe of the subjects and, consequently, on the core of their relationships and/or interactions in didactic situations in which someone teaches someone to learn something.

A subject, according to Chevallard (2009a), is endowed with a Cognitive Universe, which we denote by $UC(x)= \{(o, R\ (x; o))\ /R\ (x; o) \neq \varnothing\}$, which is constantly changing, because the personal relationship $R\ (x; o)$ is built, if it does not already exist, in which case $R\ (x; o) = \varnothing$, or can change. And this change occurs according to the institution to which the subject is subjected and, depending on the position p he occupies, be it student, teacher, coordinator, etc.

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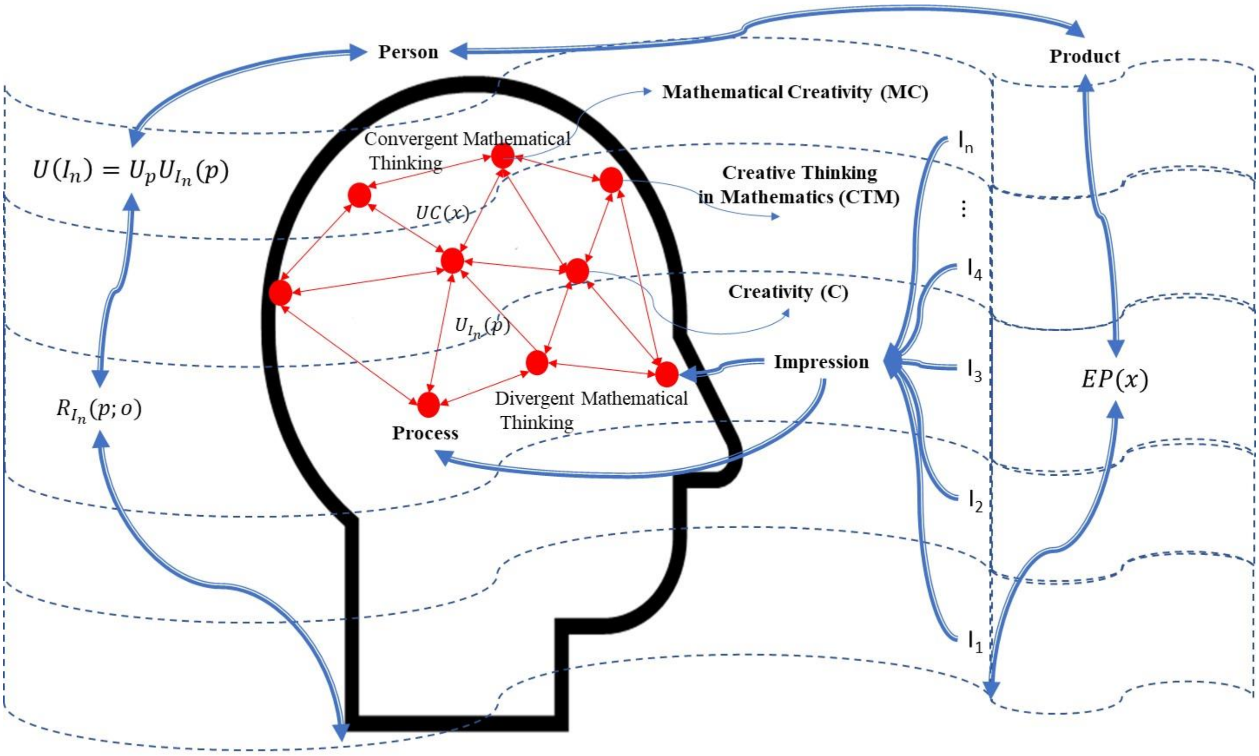

Figure 2. Cognitive Universe and the Interconnections of the DM-CTM Pillars

Source: Authors' own elaboration

Chevallard (2009a) asserts that from birth to adulthood we are part of institutions to which we subject ourselves in some way. Therefore, these Institutions (I_1, I_2, I_3, I_4, ..., I_n) (Figure 2) produce impressions in the individuals' cerebral system and, consequently, in their cognitive universe, *UC(x)*. According to these impressions, the tendency of the personal relationship *R (x; o)* is to be remodeled by the institutional relations, R_{In} *(p; o)*, of the subject in position p with the object o and, in a certain way, resembles it, or that is, $R(x; o) \simeq R_{In}(p; o)$, where ≅ represents the conformity of the personal relationship of x with the institutional relationship of the subject in position p.

Overall, the cognitive universe *UC(x)* of a subject is the outcome of their past and present experiences, influenced by the impressions they have received from the institutions to which they have been subjected. Rhodes (1961) identified these impressions as one of the components that determine the kind of environment that fosters or stifles creativity.

As shown in Figure 2, the word **Impression** establishes a link (blue arrow) with brain connectors (in red) and, as described by Rhodes (1961), it can either foster or hinder creativity in subject (*x*), triggering in their mental processes, the creation of ideas, which are generated within the subject's cognitive universe *UC(x)*, more specifically with regard to the subject's cognitive universe in position p in I_n, *n=1 ,2, 3,4, ...,*n, i.e., $U_{In}(p) = \{(o, R_{In}(p; o)) / R_{In}(p; o) \neq 0\}$, which is inserted in the cognitive universe of *In*, *U(In) = Up UIn(p)*.

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Boaler (2018, p.22) explains that an electric current fires in our brains, passing through synapses and connecting different brain areas during the idea generation **process** or when we process a new idea. The author mentions that when we learn something in depth, synaptic activity forms structural pathways, which can create lasting connections. If we only superficially visit an idea, synaptic connections can be erased, which depends on the level of the relationship *R (x, o)* and the way the cognitive universe *UC(x)* develops in the course of the subject's trajectory. The level of depth of *R (x; o)* is closely related to **Impression**.

CTM (Creative Thinking in Mathematics) is triggered by these mental processes due to the interconnection between Divergent Thinking and Convergent Thinking. Divergent Thinking involves generating ideas, such as problem definitions, strategies, or solutions from a specific starting point, while Convergent Thinking involves selecting and evaluating ideas to arrive at the best possible solution (Brophy, 2001, cited in De Vink et al, 2022). De Vink et al (2022, p.136) suggest that divergent and convergent thinking can be intertwined throughout the mathematical creative process, as using both modes of thinking can help children generate different possible solutions or strategies, select the most appropriate one, and evaluate its quality (Tabach & Levenson, 2018, cited in De Vink et al, 2022, p.136).

However, for **CTM** to occur and, *x* to develop **Creativity (C)** as defined by Young (1985) and Franken (1994), this concept and its practice must exist in the Institutions. Subjects *x* must have impressions in the sphere of creativity in such a way as to have skills to produce **Mathematical Creativity (MC)**, which Chamberlin et al (2022, p.116) define as a psychological construction, with four associated attributes: fluency, flexibility, originality and elaboration (Kwon, Park & Park, 2006; Nadjafikhah & Raftian, 2013 cited in Chamberlin, 2022, p.116).

Chamberlin asserts

> Fluency (Imai, 2000; Krutetskii, 1976) is considered the ability to think in various ways in an attempt to generate as many problem solutions as possible. Flexibility is considered the ability to think in various ways, such as in various mathematical subdomains, in an attempt to overcome fixity (Kim, Cho & Ahn, 2003). Originality (Chassell, 1916) refers to the ability to contribute new and unique answers or products and comes from creative thinking. Elaboration is considered the ability to extend initial prototypes of a project into a more complex one (Chamberlin et al, 2022, p.116-117).

Developing the four attributes enables the subject to create the CM, resulting in the **Product**. According to Rhodes (1961), the **Product** (Figure 2) results from ideas typically expressed in language or craftsmanship. From the perspective of ATD, the Product is revealed in the of praxeologies, through the praxeological equipment (*EP(x)*) of *x*.

Therefore, what we learn and/or the experiences we go through in the ***n*** Institutions can determine the level of the relationship *R (x; o)*, that is, how much x knows or does not know *o* e, the way *x* knows e, consequently this level and form of relationship between ***x* and *o*** impacts a subject's level of creative production, in a know-how that promotes innovation

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To address the uncertainty around the consequences of institutional subjections, we established a signaling parameter (SP) for the DM-CTM, which will act in the feedback system at the base of the didactic triangle. The SP is based on the concept of creativity at school level, as described in this work. Its aims to strengthen the way in which interactions between student-teacher and the knowing at stake are established, the **milieu**, the didactic situations so that potentially creative praxeologies are established.

This parameter is based on the concept of brain plasticity, which was reported in Boaler's research (2016). Brain plasticity is defined as the ability of the brain to grow and change in a short period (Abiola & Dhindsa, 2011; Maguire, Woollett & Spiers, 2006; Woollett & Maguire, 2011, as cited in Boaler, 2016). Boaler (2016, p.26) states that "new evidence from neuroscience reveals that everyone, with the right message and teaching, can succeed in math and everyone can have high levels of learning at school."

This paper associates the definition of success in mathematics with potentially creative individuals.

Considering the information on brain plasticity, we believe in the potentiality of the signaling parameter (SP) of our DM-CTM feedback system, which can be used both by researchers in the analysis of teaching phenomena and/or for the planning of didactic situations potentially creative, but also in the sense of bringing to discussion its importance for the development of creative didactic situations, both from the perspective of teachers in training courses and in discussions in continuing education activities.

The foundation of this parameter will be discussed later in the proposal of the DM-CTM based on the ATD.

Didactic Model for Creative Thinking in Mathematics (DM-CTM)

The DM-CTM framework resulted from a research process on teaching and learning to develop CTM (Aljarrah & Babb, 2022; Bicer et al, 2022; Bruhn, 2022, De Vink, 2022). According to Aljarrah and Babb (2022, p. 59), providing subjects in a didactic situation with learning environments that stimulate creativity and, consequently, creative acts is essential. According to Bicer et al (2022, p.95), another implication is to encourage students to generate more ideas and representations, asking them questions that stimulate creativity (for example, did anyone draw or visualize differently?) and letting them work in collaborative classroom environments.

In his research involving elementary school children, Bruhn (2022, p.102) discusses the development of tasks that can foster creativity-related skills, including fluency, flexibility, originality, and elaboration through (meta)cognitive stimuli. According to the author, the teacher should assume a questioning and guiding position to stimulate and develop creativity-related skills and foster creative behavior.

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Regarding the types of tasks to be proposed, De Vink (2022, p.136) suggests that open tasks are considered the most suitable for CTM, as they generally allow multiple responses and can take different forms.

An SRP was incorporated into the basic structure of the DM-CTM based on the researchers' considerations. This was done because the dimension of creativity can be developed through questioning the knowing at stake, including during the planning of didactic situations by teachers. The SRP includes the dimension of creativity (pillars of support) in the relationships and interactions between the subjects involved in didactic situations (in the classroom or research groups) to obtain the **Products**. These Products, according to Rhodes (1961, p. 309), refer to reflecting on “unusual (new) and/or insightful solutions to a given problem or analogous problems, and/or the formulation of new questions and/or possibilities that allow an old problem to be seen from a new angle that requires imagination”. This is the definition of creativity according to Liljedah and Sriraman (2006, p. 19).

The SRP incorporates, according to Chevallard (2001, 2007, 2009a) three structuring principles that focus on creativity. These are (1) the SRP is organized around a generative question; (2) it is fundamentally organized around five tasks: **observe, analyze**, **evaluate the** $R^{\lozenge}$ responses, **develop, disseminate, and defend the** $R^{\heartsuit}$ response, (3) need for SRP piloting, regulating the study and research processes.

When developing an SRP, learners can employ knowledge from other areas to find solutions to the general question before returning to the main topic at hand (Chevallard, 2007). Furthermore, the author emphasizes the need to examine a broad range of knowledge areas in a study process to determine the relevant information needed to solve a problem. This process facilitates the determination of which knowing is relevant when answering the generating question.

Figure 3 shows the internal structure of the DM-CTM, where we made an analogy to a piping system, which starts from the foundation, which is supported by the Anthropological Theory of Didactics (ATD), in which the dimension of creativity, support pillars of the structure is substantiated.

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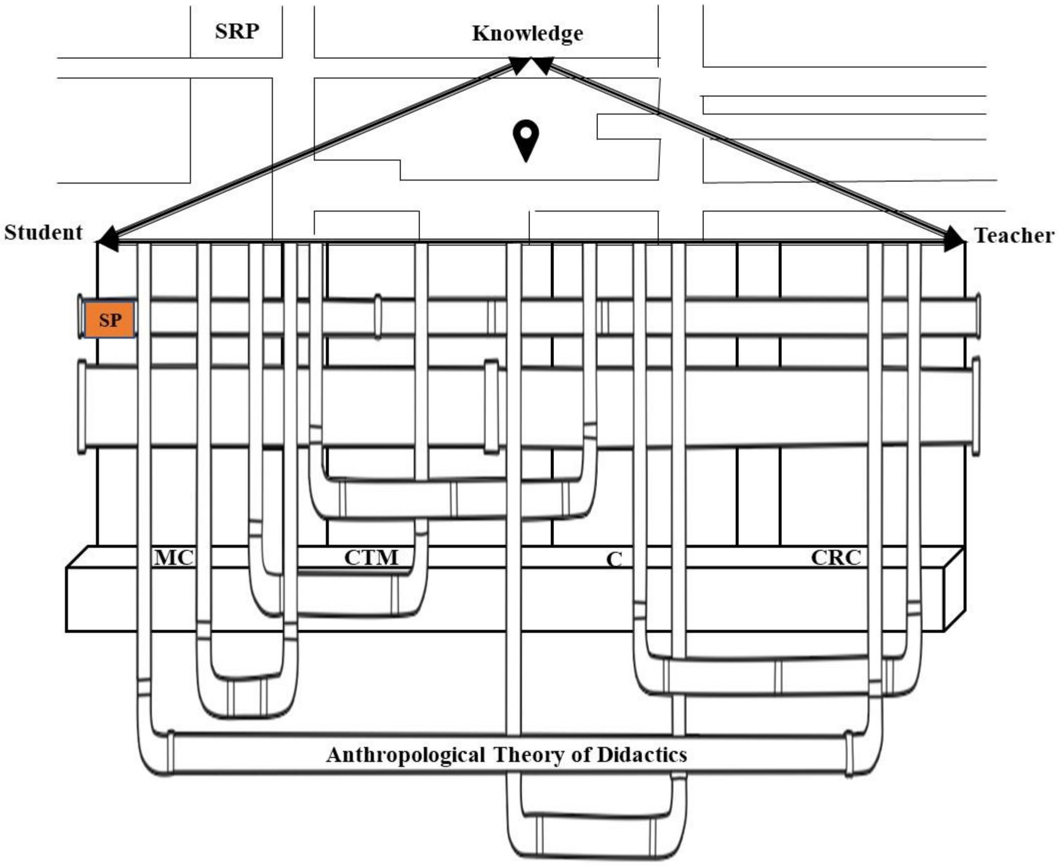

Figure 3 - Internal structure of the DM-CTM

Source: Authors' own elaboration

The piping system is fed back in such a way that the intensity of the flow is determined by the signaling parameter, (SP), which is evaluated and re-signified from the analysis of the SRP established in a didactic system *S (x, y, Q)*, where *x* is someone who learns something, *y* is someone who teaches and *Q* the didactic stake.

By its focus on the study and research of the answer to a generative question promoting the paradigm of questioning the world, the function of the SRP in the structure of the DM-CTM is to contribute to the expansion of the milieu (represented by the dotted cylinder, Figure 1), from a process of investigation that goes beyond the boundaries of the classroom, in the sense of its investigative nature from the paradigm of questioning the world. Thus, the insertion of an SRP in the structure of the DM-CTM aims to deepen and/or make meaningful the relationship *R (x, o)*, present in the didactic system represented by the didactic triangle (evidenced in the form of a scheme in the structure of the DM-CTM, Figures 1,2 and 3).

The products of the SRP must be analyzed and/or re-signified in order to emit to the SP, the necessary demands (interconnections between the ATD and the pillars of support) for the feedback of the system so that the functioning of the DM-CTM is effective, in the sense

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of imprinting in the cognitive universe of the student (*x*) and/or the teacher (*y*), *UC(x/y)* the importance of the development of the CTM.

The SP parameter shows the plasticity capacity of the brain both from the perspective of the subjects in the student position and in the teacher position. For both subjects carry the impressions of the institutions through which they have passed.

The structure of the DM-CTM and its operating system allow new impressions to be acquired in the subjects with the objective of developing of the CTM and, consequently, of potentially creative praxeologies that contribute to the development of the cognitive universe (*UC(x)*) of the subjects, in which the relations *R (x; o)* become increasingly significant.

Final considerations

The study and research on the dimension of creativity enabled us to make interconnections with fundamental elements of didactics and, more specifically, with the Anthropological Theory of Didactics. The results of the research and these interconnections were fundamental for the construction of the DM-CTM.

The DM-CTM is a theoretical construct that opens up a range of possibilities for research into didactic phenomena with the dimension of creativity as its focus. Bringing to light the pillars of support of this model and the possibilities of communication of these from the elements of the ATD, presents a universe of possibilities for the planning of didactic situations in which potentially creative praxeologies are developed, thus stimulating the creative mathematical thinking of students as well as teachers.

The insertion of the SRP in the DM-CTM imprints the culture of questioning in the relationships and/or interactions between teacher-knowing and student, establishing environments that enable the subjects involved to create, innovate from a process of research and investigation of the knowing at stake. That is, the culture of SRP in institutions imprints in these environments spaces of open questions that promote the development of a scientific posture in the subjects regardless of their positions.

From the questions and discussions made so far, mathematical praxeologies have been developing throughout the history of mathematics and remain in evolution. That is, praxeological equipment has expanded over time and, from contributions in the collective in universes of doubts, assumptions, during a **know-how** full of experiences and knowledge exchanges, in constant adjustment in the coupling of praxis and logos.

And all this was only possible due to problem situations in all spheres of society that promoted possibilities for the development of the subjects' cognitive universes, the establishment of new personal relationships and even discoveries regarding the plasticity of the brain.

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The DM-CTM brings a whole apparatus in its structure in order to reveal a possibility of interconnection between the dimension of creativity and ADT in order to contribute to the development of CTM.

References

Aldon, G. (2011). *Interactions didactiques dans la classe de mathématiques en environnement numérique*: construction et mise à l'épreuve d'un cadre d'analyse exploitant la notion d'incident. (Thèse en Education: Didactique des mathématiques). Université Claude Bernard Lyon 1, 694p. Available in: https://theses.hal.science/tel-00679121v2/document.

Aljarrah, A., & Babb, A.P.P. (2022). Shifting metaphors as a trigger for creativity: a review of students' mathematical creative acts. In S. A. Chamberlin (Ed.), *Proceedings of the 12th International Conference on Mathematical Creativity and Giftedness (MCG 12)* (pp.59-65). Available in: https://d-nb.info/1268386723/34.

Bicer, Ali, Bicer, Ay., & Chamberlin, S. A. (2022). Young students' mathematical creative thinking processes. In S. A. Chamberlin (Ed.), *Proceedings of the 12th International Conference on Mathematical Creativity and Giftedness (MCG 12)* (pp.93-96). Available in: https://d-nb.info/1268386723/34

Boaler, J. (2016). Mathematical Mindsets: unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching. Jossey-Bass (A. Wiley Brand).

Brousseau, G. (2011). La théorie des situations didactiques en mathématiques. *Éducation & Didactique*, 5(1), (pp.101-104). https://doi.org/10.4000/educationdidactique.1005

Brousseau, G. (1978). L'observation des activités didactiques. *Revue Français de Pedagogie*, 45, (pp.130-139).

Bruhn, S. (2022). Supporting first graders in showing their individual mathematical creativity through (meta-) cognitive prompts. In S. A. Chamberlin (Ed.), *Proceedings of the 12th International Conference on Mathematical Creativity and Giftedness (MCG 12)* (pp. 97-103). Available in: https://d-nb.info/1268386723/34

Chamberlin, S.A., Payne, A., & Kozlowski, J. (2022). The five legs of creativity. In S. A. Chamberlin (Ed.), *Proceedings of the 12th International Conference on Mathematical Creativity and Giftedness (MCG 12)* (pp.116-121). Available in: https://d-nb.info/1268386723/34

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en didactique des mathématiques*, 19(2), 221–266.

Chevallard, Y. (2001). Les TPE comme problème didactique. *Communication au Séminaire national de didactique des mathématiques* (pp.1-9). Disponível em: http://yves.chevallard.free.fr/spip/spip/article.php3?id_article=14.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), Proceedings of the IV congress of the European society for research in mathematics education (pp. 21–30). Barcelona: FUNDEMI-IQS.

Chevallard Y. (2007). "Passé et présent de la théorie anthropologique du didactique". In L. Ruiz-Higueras, A. Estepa & F. J. Garcia (Eds.), *Sociedad, Escuela y Matemáticas* (pp.

DOI: 10.20396/zet.v31i00.8671768

705–746). Jaén: Servicio de publicaciones de la Universidad de Jaén. Available in: http://yves.chevallard.free.fr/spip/spip/article.php3?id_article=134.

Chevallard, Y. (2009a). La TAD face au professeur de mathématiques. *Communication au Séminaire DiDiST de Toulouse* (pp.1-17). Available in: http://yves.chevallard.free.fr/spip/spip/article.php3?id_article=162.

Chevallard Y. (2009b). La notion de PER : problèmes et avancées. Texte d'un exposé présenté à l'IUFM de Toulouse (pp. 1-29). Available in: http://yves.chevallard.free.fr/spip/spip/article.php3?id_article=161.

Chevallard, Y. (2015). Teaching Mathematics in tomorrow's society: a case for an oncoming counter paradigm. In: *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer International Publishing. Available in: https://link.springer.com/chapter/10.1007/978-3-319-12688-3_13.

De Vink, I. C., Willemsen, R. H., Keijzer, R., Lazonder, A. W., Schoevers, E. M., & Kroesbergen, E. H. (2022). Creative Mathematical Thinking Process. In S. A. Chamberlin (Ed.), *Proceedings of the 12th International Conference on Mathematical Creativity and Giftedness* (MCG 12) (pp. 135-141). Available in: https://d-nb.info/1268386723/34.

Franken, R. E. (1994). *Human Motivation*. Thomson Wadsworth (3th edition).

Devlin, K. (2002). *Matemática: A ciência dos padrões*. (Tradução: Alda Maria Durães). Porto Editora.

Liljedahl, P., & Sriraman, B. (2006). Musings on mathematical creativity. *For the Learning of Mathematics*, 26(1), 17-19.

Margolinas, C. (2009). *Points de vue de l'élève et du professeur*. (Essai de développement de la théorie des situations didactiques). Education. Université de Provence - Aix-Marseille Available in: https://tel.archives-ouvertes.fr/tel-00429580v2/document.

Margolinas, C. (2014). Connaissance et savoir. Concepts didactiques et perspectives sociologiques ? *Revue française de pédagogie*, 188, 13-22. https://doi.org/10.4000/rfp.4530

Rhodes, M. (1961). An analysis of creativity. *The Phi Delta Kappan, 42*(7), 305–310.

Sensevy, G. (2010). Formes de l´intention didactique, collectifs et travail documentaire. In G. Gueudet, & L. Trouche, L. (Ed.), *Le travail documentaire des professeurs en mathématiques*, (pp. 147-164), Ressources vives, Presse Universitaire de Rennes.

Young, J. G. (1985). What is Creativity? *Journal of Creative Behavoir*, 19 (12), 77-87.