Problem posing and creative thinking in Mathematics class

Proposição de problemas e pensamento criativo na aula de Matemática

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Abstract
Problem posing has been considered a cognitively demanding activity, with potential for the development and evaluation of critical and creative thinking. In the classroom it is considered one of the authentic forms of mathematical investigation in which students have the possibility to associate their experiences, interests and knowledge in the creation of problems. Contributing to this research strand, this paper aims to discuss the potential of problem posing for the development of creative thinking in mathematics classes. In particular, this qualitative study aims to analyze a problem posing activity carried out with a class of first grade elementary school children. The results indicate that problem posing has significant potential for the development of creativity, allowing children to assign meaning and critically analyze data, relating their experiences, knowledge, and interests.

Keywords: Creativity; Problem Posing; Mathematical Teaching.

Resumo
A proposição de problemas tem sido considerada uma atividade cognitivamente exigente, com potencial para o desenvolvimento e a avaliação do pensamento crítico e criativo. Em sala de aula é considerada umas das formas autênticas de investigação matemática em que os estudantes têm a possibilidade de associar suas vivências, interesses e conhecimentos na criação dos problemas. Contribuindo com essa vertente de pesquisa, este artigo tem como objetivo discutir potencialidades da proposição de problemas para o desenvolvimento do pensamento criativo nas aulas de Matemática. Em particular, este estudo, de natureza qualitativa, se propõe a analisar uma atividade de proposição de problemas realizada junto a uma turma de crianças do primeiro ano do Ensino Fundamental. Os resultados indicam que a proposição de problemas tem expressivo potencial para o desenvolvimento da criatividade, além de possibilitar que as crianças atribuam significado e analisem criticamente os dados, relacionando suas experiências, seus conhecimentos e interesses.

Palavras-chave: Criatividade; Proposição de Problemas; Ensino de Matemática.

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Introduction

Children, during their growth, experience many moments of creation and invention, in everyday games and play, with family and colleagues. And these activities are extremely relevant to their development, both cognitive and affective. And, in addition, in their school trajectory, they also come across proposals involving inventiveness from a very early age. Among the school experiences lived by children, one can mention, for example, the creation of stories, with the description of events of their daily lives or of the childish imagination and the sharing of these stories with their colleagues, through conversation circles. However, in mathematics classes, and even in other areas of knowledge or curricular components, as children advance in schooling, these moments of invention and creation give way to more directed and systematized activities. Ellerton (1986) has already drawn attention to the fact that, in particular, in the context of problem solving in mathematics classes, opportunities for students to invent their own problems are rare.

In contemporary mathematics education research, there is a growing interest in problem posing as an educational practice in which students create and present problems in mathematics classes (Cai, 2022; Possamai, Allevato & Strelow, 2023; Zhang & Cai; 2021).

In several countries, recommendations for school mathematics have associated posing with problem solving. The National Council of Teachers of Mathematics - NCTM, which is the largest mathematics teaching organization in the world and has influenced curriculum reforms in several countries, especially in the United States and Canada, brings this orientation in discussions about the mathematics curriculum. The document Principles and Standards for School Mathematics (NCTM, 2000) emphasizes the importance of regularly providing moments in which students create problems, both inside and outside the mathematical context.

In Brazil, the Common National Curriculum Base - BNCC (Ministério da Educação, 2018) is a normative document that guides the construction of curricula by defining the essential learning that all students must develop throughout Basic Education. The document brings problem solving as a privileged form of mathematical activity. But, unlike previous curricular guidelines, it is possible to see the importance attributed by the BNCC to the processes not only of solving, but also of elaborating problems, which is shown in the expressive number of skills recommending the actions of solving and elaborating, instead of just solving problems, emphasizing that “[...] this option broadens and deepens the meaning given to problem solving” (Ministério da Educação, 2018, p. 536).

The term problem development will be used here whenever the BNCC is used as a reference, as it is the way it is presented most often in the document. In our research, however, we use the term problem posing, considering what is shown in recent research, especially those at the international level, and understandings about its meaning, which will be clarified in the next section of this article.

Problem posing has been used as a context for measuring creativity, but also as an activity to promote students' creativity in the mathematics classroom (Bicer, 2021; Silver,
1994), since it is a generally challenging, cognitively demanding task that allows involving students' interests and daily experiences. In this respect, Bonotto (2013) emphasizes that problem creation, as well as problem solving, has the potential to promote creative thinking and also to evaluate it.

All students have the potential to generate creative ideas, “[...] however, time and effort to develop their creativity generally will not be invested unless teachers provide experiences that afford cognitively demanding and creativity-directed educational opportunities.” (Bicer, et al, 2020, p. 458), and problem posing is one of these possibilities in mathematics classes.

Thus, this article aims to discuss the potential of problem posing for the development of creative thinking in mathematics classes. In particular, this study proposes to analyze a problem posing activity carried out with a class of children in the first year of elementary school.

In the next section, the theoretical assumptions that underlie the studies on the problem posing in mathematics teaching will be presented. Then, the higher-order thinking skills will be presented, focusing on creative thinking, the methodological characterization of the research, the work actually carried out by the students, as well as the analysis of their productions.

**Problem Posing in the teaching of mathematics**

Initially, we explain the understanding assumed for the problem posing, which is constituted as an activity in which both students and teachers can be creators of mathematical problems. In particular, we are interested in discussions and reflections on educational practices in which the problem posing is performed by students, since this is a current demand of school mathematics curricula.

The portrait of Brazilian research, presented by Possamai and Allevato (2022), when analyzing 24 theses and dissertations produced until 2021, shows the use of the terms elaboration, formulation and problem posing to denote activities in which students are the ones who create mathematical problems, without, however, a clear positioning on the meaning of each of these terms. In this regard, we agree with the definition presented by the authors, who assume:

Based on the studies we have carried out based on Brazilian and international productions, and linguistic meanings inherent to the Brazilian language, we are using the expression **problem posing** to denote the whole set of ideas that constitutes the processes involving the **creation of problems**, which begins with the organization and construction of the first mathematical ideas and the structure of the problem constitution - **formulation**; and advances to its expression, in which the statement is established, associating the maternal and mathematical languages - **elaboration**. Then, the proposition proceeds to the presentation of the problem created to a potential solver (Allevato & Possamai, 2022, p. 156, emphasis added).
This definition converges with the understanding presented by Zhang and Cai (2021, p. 962) that “mathematical problem posing is a process of formulating and expressing a problem within the domain of mathematics”.

It should also be clarified that, in the context of teaching through problem solving, “mathematics problem is a task presented to students in an instructional setting that poses a question to be answered but for which the students do not have a readily available procedure or strategy for answering it” (Lester & Cai, 2016, p. 122) and whose learning objective, previously delimited by the teacher, is achieved in the course and discussions arising from the problem solving process experienced by the students.

In this sense, teaching through the problem posing resembles teaching through problem solving (Allevato & Onuchic, 2021), but the problem created may not be exactly a problem for the pose himself, but for the potential solver. In this case, the learning objective, intended by the teacher, is constituted in the creation of the problem and in the mathematical discussions and reflections triggered in the classroom, by and about the problems presented by the students. It should be noted that,

[…] problem-posing tasks are usually cognitively demanding, because problem posing often requires posers to reflect on the broader aspects of structure and goal. The high cognitive demand of problem-posing activities can provide intellectual contexts for students’ rich mathematical development (Zhang & Cai, 2021, p. 962).

Therefore, problem posing and problem solving are complementary and closely related mathematical activities, but different in nature (Cai & Leikin, 2020; Silver, 1994; Silver & Cai, 1996).

A problem posing tasks consists of (1) trigger elements, which include the context and data to be considered in creating the problem; and (2) a prompt, which indicates what is expected of the proposer (Cai, 2022). Based on these trigger elements and prompts, problem posing activities can be categorized as free, semi-structured or structured (Stoyanova & Ellerton, 1996).

In free situations, students create problems without restrictions, as occurs from images provided as a trigger element, being asked (prompt) to create a mathematical problem from it, or a difficult problem, for example. In semi-structured problem-solving activities, an open-ended situation is provided and students are invited to explore its structure, using knowledge (usually mathematical), skills, concepts and their experiences. Structured situations are those in which students create problems by reformulating other problems (Stoyanova & Ellerton, 1996).

Depending on the teacher's objective, he/she will choose trigger elements and a prompt that make the problem posing activity more or less structured. Also, depending on the elements of the research to be developed, if this is the case, some types of activities are more suitable than others. In particular, in this article we analyze a semi-structured problem posing activity, with a view to developing creative thinking, according to the theoretical assumptions discussed below.
From problem posing to developing creative thinking

Activities involving problem posing are generally considered intellectually demanding tasks. According to González (1998), intellectually demanding tasks are tasks of high cognitive demand, which involve intellectual effort and stimulate the activation of more sophisticated thought processes.

Such tasks should provide reasoning effort, that is, they should not be achievable with the mere exercise of memorization, nor with the mechanical use of algorithmic schemes, nor with the application of preconceived recipes; on the contrary, they should merit the realization of a certain intellectual effort. (Gonzáles, 1998, p. 67).

In contrast to memorization exercises and mathematical tasks involving the mechanical application of formulas and algorithms, intellectually demanding tasks presuppose the mobilization of more complex cognitive resources by students, thus implying the development of higher-order thinking skills. In this sense, the realization of this type of task is an auspicious learning opportunity.

Still on the nature of the tasks proposed in mathematics classes, Jesus, Cyrino and Oliveira (2018, p. 22) point out that “different types of tasks constitute different learning opportunities for the student, since some have the potential to mobilize them to complex forms of thinking and others do not”. The authors point out the importance of prioritizing, in the classroom, the work with cognitively challenging tasks, in view of their potential to promote the development of higher-order thinking skills.

The central idea of classifying certain thinking skills as higher order lies in the recognition that some learning models require more complex cognitive processes than others (Vieira & Allevato, 2021). While tasks of a procedural nature, such as those involving the application of formulas and algorithms, require less sophisticated cognitive processes, tasks that require the construction of their own resolution strategies, such as tasks involving the elaboration of problems, require more complex cognitive processes.

Higher-order thinking skills play a key role in intellectual development and the ability to solve problems effectively. These advanced cognitive processes, including analysis, evaluation, and synthesis, are essential for building deeper and more meaningful knowledge. As noted by King, Goodson and Rohani (2008), this broad category of skills encompasses several crucial facets of thinking, contributing to the formation of analytical minds and comprises critical, logical, reflective, metacognitive and creative thinking.

In the educational context, the dimension of creative thinking plays a fundamental role, transcending the conventional boundaries of teaching and learning. By promoting creativity in the school environment, teachers provide students with the opportunity to advance in their education as critical and autonomous thinkers, capable of building and using knowledge in a flexible and adaptable way to varied situations. Related to creative thinking, in the school setting, creativity can be defined as the process that results in a new solution or idea for a mathematical problem or in the formulation of new questions, produced by an individual or several individuals (Haavold, Sriraman & Lee, 2020).
Gontijo (2007) describes creativity in mathematics as

[... the ability to present numerous appropriate solution possibilities for a problem situation, so that they focus on different aspects of the problem and/or different ways of solving it, especially unusual ways (originality), both in situations that require problem solving and elaboration and in situations that require the classification or organization of mathematical objects and/or elements according to their properties and attributes, whether textually, numerically, graphically or in the form of a sequence of actions. (Gontijo, 2007, p. 159, emphasis added).

It should be emphasized, in the aforementioned definitions, the use of creativity in situations involving the processes of formulation and elaboration of problems. In fact, research related to the development of creative thinking in mathematics classes has pointed to the problem posing as a fertile ground for this purpose (Ayvaz & Durmus, 2021; Gontijo, 2007; Gontijo, et al, 2019).

Ayvaz and Durmus (2021) point out that in many studies, the problem posing in mathematics classes is associated with problem solving. This fact denotes the not uncommon difficulty in delimiting boundaries between the processes of posing and solving problems. Nevertheless, such studies reveal that by engaging and dedicating themselves to problem posing processes, students are more interested and self-confident, transfer what they learn in the classroom to real-life situations, improve their problem-solving skills, make fewer mistakes and develop flexible thinking skills, which is the basis of creativity.

However, although the process of problem posing is often related to the development of creative thinking, its use as a measure of creativity still lacks further investigation. Joklitschke, Baumanns and Rott (2019) question the viability of this perspective of using problem posing, pointing to the limited number of research published in the best-ranked journals in international databases.

Thus, the present study seeks to broaden the understanding of the relationship between problem posing and the development of creative thinking. The next section presents the methodological characterization of the research.

Methodological characterization

Although recent, research on problem posing has different objectives and, consequently, can be characterized methodologically in three ways: as a construct, variable and intervention (Cai & Hwang, 2020).

As a construct, the research aims to analyze students' ability to problem posing in different contexts, such as when they are related to levels of difficulty or in group activities, for example. It also analyzes problem posing as a teaching activity to be observed, described and understood. In this case, it is assumed that the construct is well defined, being used as a variable in the analysis of a larger phenomenon, or of its relations with other aspects. Problem posing can be used, for example, to analyze relationships with creativity or problem solving (Cai & Hwang, 2020).
As an intervention, the research aims to analyze how problem posing is used to promote better learning outcomes and improve creativity, among others objectives, making it possible to help teachers use problem posing in their classes (Cai & Hwang, 2020).

In the present study, we constituted the problem posing as a variable, aiming to analyze the development of creative thinking, from problems posed by 19 children, 6 and 7 years old, in the school stage of the 1st year of Elementary School, from a school in Blumenau, Santa Catarina. This is qualitative research, whose process is based on the interpretation of phenomena and attribution of meanings (Kauark, Manhães & Medeiros, 2010). It is understood that this research is characterized as a case study, because when conducting the investigation with this group, it is not intended to produce generalizations from the reported results. (Amado & Freire, 2014).

The data analyzed consist of written records of the problems created by the children and statements and clarifications obtained in interviews with the class teacher and one of the children. The analysis of these data considered that “the important thing is that the creativity and the voice of the author appear in the analysis, which will be dialoguing with the lenses of the theoretical voice and with the analyzed literature” (Borba, Almeida & Gracias, 2018, p. 81). Thus, to carry out the analysis, we looked at the duly documented research data, seeking an understanding of the texts produced according to their development and meaning, intertwining emerging aspects with the theoretical foundation.

The problem-solving activity was developed out of class by the children, for which they were provided, as triggering elements, an image that suggests a context and numerical data to be considered; and as a prompt, the request to create and solve the problem, as illustrated in Figure 1.

![Figure 1 - Problem posing activity](https://example.com/figure1.png)

This is an activity that was included in the class textbook and, according to the teacher, its objective was to analyze the different situations created by the children, highlighting the mathematics they built. Next, the analysis of these data is presented, with emphasis on the problems that allow discussing the mathematical knowledge constructed and the development of creative thinking.
The problems proposed by the children

It should be noted that this was the first problem posing activity carried out by the children, who had little experience with problem solving. Of the 19 children, only 3 managed to create a problem; the others described the situation, but without presenting a question. This is an aspect analyzed by Possamai, Allevato and Strelow (2023) in relation to the implementation of the problem posing in mathematics class. In this sense, the study developed attested that the students' previous experiences with problem solving and even with the problem posing are determinant in the results achieved and in the quality of the problems created by the children. And there are certainly other important aspects to consider. In the present article we will discuss the data with an eye to the development of creative thinking.

From the scene provided by the image (Figure 1), the most immediate description suggests buying an aquarium and this is the situation presented in the creations of 12 children. Figure 2 illustrates the posing presented by one child.

![Figure 2 - Aquarium purchase situation](source: Research collection)

This is an expected description from the elements that have been provided and, although it does not include a question that sets a problem to be solved, it already reveals a creative thinking process, since:

[...] the connection to creativity lies not so much in problem posing itself, but rather in the interplay between problem posing and problem solving. It is in this interplay of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity. (Silver, 1994, p. 76).

In this articulation between posing and solving the problem, some children present the solution in the description of the situation itself. The children’s lack of experience with problem solving leads to a lack of skill in knowing that it is necessary to pose a question to the problem. The question is possibly established in the formulation process, and in the elaboration it is configured as the motto that will trigger the resolution process and lead the solver to portray the solution (Possamai, Allevato & Strelow, 2023).
The plot developed by the child is consistent with the initial command and suggests working with the additive field involving magnitudes of the National Monetary System, although the mathematical operation is not explicit due to the absence of a question.

It should also be noted that the children, subjects of the research, were in the process of acquiring the Alphabetic Writing System, a fact that may have constituted an obstacle to the registration of the problem, that is, to the elaboration. Even so, it is worth emphasizing the importance of the writing produced by the children as a process of developing creative thinking in mathematics that constitutes “[...] generate new mathematical ideas, processes, or products that are new to the students but may not necessarily new to the rest of the world” (Bicer, 2021, p. 253).

Some proposals present the reason for buying the aquarium, relating the two scenes. Five children described this situation in more detail, one of which is illustrated in Figure 3.

![Figure 3 - Reason for buying an aquarium](source: Research collection)

Although this record also does not present a question, there is a greater wealth of detail, including the concern to present a motivation for the purchase of the aquarium. This fact gives the posed situation greater plausibility and verisimilitude with real-life situations. These aspects are in line with the studies by Ayvaz and Durmus (2021); the authors suggest that working with problem posing contributes to the process of transposing the knowledge learned at school to real-life situations.

Also noteworthy in the child’s record is the way in which she represents monetary values, with the symbol of the current currency written after the monetary value, indicating that she is still appropriating the symbols and conventions of the National Monetary System.

Regarding the motivation presented by the child for the purchase of the aquarium, another aspect deserves to be considered: the replacement of a small aquarium by a larger one. In his creative process, the child resorts to volume-related estimates to justify the need to purchase a new aquarium.

Bicer et al. (2020, p. 477) point out that “when students are provided with these cognitively challenging opportunities through problem posing, they have more opportunities
to construct deep and meaningful conceptual understandings of mathematics, and this may encourage students to manifest their mathematical creativity”.

Some children even assigned names to the characters in the situation, as illustrated in Figure 4.

![Assigning names to characters](source: Research collection)

Among the children who posed only a trivial situation of buying the aquarium, only one assigned names to the characters. However, of the five children who detailed the reason for the purchase, four named the person making the purchase and some also included the name of the seller.

The wealth of details in the problems posed by some children suggests that the process of creating the problem is, at this stage, closely related to the experiences lived by the children and their cultural repertoire. In addition to the plot created for the situation, this record also highlights the presentation of the mathematical sentence inherent to the proposed problem: $10 - 5 = 5$. The “problem” created and its consequent solution move between records in the mother tongue and records involving mathematical symbols.

Among the problems posed, there were also two children who created situations without involving the purchase of the aquarium: one of them involved the purchase of a fish and the other described the purchase of feed for the fish.

In addition, one of the children changed the values indicated in the problem, putting the value of the aquarium as R$15,00, having offered R$20,00 to make the payment. When questioned, the child justified that she had already experienced the purchase of an aquarium with her mother and that aquariums are expensive. Thus, she considered the value of R$15.00 attributed by her as more appropriate and necessary for the purchase situation considered.

This is an important aspect of higher-order thinking, in which the child critically analyzes the situation and creatively modifies it according to their life experience. In this sense, Bicer et al. (2020) point out that creative thinking includes affective dimensions, articulating imaginative ideas and reality.
In this activity, which refers to a semi-structured problem posing (Stoyanova & Ellerton, 1996), the elements that constituted the problem posing required the children to consider: an image, numerical data, a context, a supposed solution to the problem (“change” of 5 reais) and also to think about solving the problem. Given the little experience of these children with posing and solving problem tasks, and given that they are still in the literacy process, an activity with fewer elements and possibly with teacher mediation could be more conducive to the development of creative thinking.

Some problem posing activities give students more freedom to develop creative thinking (Bonotto, 2013). More structured activities, either because they offer more elements or because they are more targeted to the type of problem that can be created, can restrict the type and degree of complexity of thinking employed by students.

Nevertheless, it was possible to perceive, in the problems posing by the children, elements that refer to creative thinking, such as the elaboration of plots and justifications consistent with the situation associated with the images; and even the suggestion of situations with values different from those presented in the order, which reveal creativity and criticality in relation to the values initially posed.

With a view to developing creative thinking, for educational practices and research that consider the problem posing as a variable, the use of freer elements and less structured situations is suggested.

Final considerations

This article aimed to analyze the potential of problem posing for the development of creative thinking in mathematics classes. To this end, the problems created by children of the 1st year of elementary school were analyzed, based on a semi-structured problem-posing activity.

The development of children's creative thinking was verified in the textual productions by describing the context related to the data used, by assigning names to the characters and by using knowledge and experiences from the real world, as well as adding elements that were not present in the trigger element.

The problem posing has the potential to develop creativity, enabling students to attribute meaning and critically analyze data, relating their experiences, knowledge and interests, without limits to inventiveness, thus favoring the development of skills that involve higher-order thinking.

In addition, in agreement with other researchers, references for the present study, we believe that less structured problem-posing activities could foster flexible thinking more intensely, expanding the possibilities of developing creativity and criticality.

As future work, it is suggested to evaluate the results by offering only the image as a trigger element of the activity, at different levels of education, to analyze the level of complexity of the problems created when considering a freer structure for the problem posing.
References


