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Critical and creative thinking in mathematics: assessment rubrics

Pensamento crítico e criativo em matemática: rubricas avaliativas

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Abstract

Curricular policies have shown concern about developing critical and creative thinking in mathematics in Basic Education. However, many of them do not include aspects related to how to monitor their development with students; and how to verify the potential of teaching materials for this purpose. This investigation sought to structure two evaluative matrices for critical and creative thinking in mathematics. The first aims to assist teachers in monitoring the development of each student; and the second focused on helping education professionals. A documentary analysis was adopted, a meta-synthesis based on a set of evaluation rubrics on the topic. Based on a systematic interpretation of the content of these rubrics, we proceeded with the structuring of two matrices that can be used as instruments by different actors in the educational scenario.

Keywords: Critical and creative thinking in mathematics; Creativity and critical thinking in mathematics; Assessment of critical and creative thinking in mathematics; Evaluation Rubrics.

Resumo

Políticas curriculares têm evidenciado preocupação em desenvolver o pensamento crítico e criativo em matemática na Educação Básica. Todavia, muitas delas não contemplam aspectos relacionados a como acompanhar seu desenvolvimento junto aos estudantes; e a como verificar a potencialidade de materiais didáticos para tal. Esta investigação buscou estruturar duas matrizes avaliativas para o pensamento crítico e criativo em matemática. A primeira com vistas a auxiliar professores no acompanhamento do desenvolvimento de cada estudante; e a segunda com foco em auxiliar profissionais da educação. Adotou-se uma análise documental, metassíntese a partir de um conjunto de rubricas avaliativas sobre o tema. Com base em uma interpretação sistemática do teor destas rubricas, procedeu-se com a estruturação das duas matrizes que podem ser utilizadas como instrumentos pelos diferentes atores do cenário educacional.

Palavras-chave: Pensamento crítico e criativo em matemática; Criatividade e pensamento crítico em matemática; Avaliação do pensamento crítico e criativo em matemática; Rubricas de avaliação.

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Introduction

Mathematics Education is dedicated to studies related to mathematics teaching and learning (Fiorentini & Lorenzato, 2006). It is a growing area of research and encompasses investigations at different levels, stages, and modalities of education, using theoretical, practical, or theoretical-practical investigations that take the form of a variety of research techniques, such as observation, interviews, document analysis, among others (Lozada & Nunes, 2018).

Among the techniques mentioned above, document analysis is relevant to this article, as it supports the production and analysis of information, for example, from school curricula and assessment guidelines (with their matrices and scales) - precisely what was sought in this research.

An example of the potential of document analysis in mathematics education research can be seen in the work of Fonseca and Gontijo (2020a), who used it to investigate the presence and meanings attributed to the words creativity and criticality in normative documents of Brazilian education. The analysis, according to the authors, identified that until recently, there was no objective conceptualization of the construct in the primary normative documents referring to Brazilian Secondary Education: (a) National Curricular Parameters, part III - area of Natural Sciences, Mathematics and its Technologies (Ministério da Educação, 2000); and (b) National Common Curricular Base (Ministério da Educação, 2018).

It is worth noting that the number of studies carried out in Brazil on the field of critical and creative thinking in mathematics has been growing in recent years. However, this number is minimal when the focus is on the initial and continuing education of mathematics teachers. Gontijo and Fonseca (2020) observed a total of 17 theses and dissertations on creativity in mathematics defended in Brazil between 2010 and 2020. Of these works, only 3 had teacher training as the object of analysis. A question that remains open in this field concerns the availability of instruments to help teachers monitor their student's progress in terms of critical and creative thinking in mathematics.

Another example of the relevance of documentary analysis in understanding the elements that can interfere with the students' formative process is in the investigation of the teaching materials used at school, especially the textbook, as these may not present activities that stimulate these types of thinking. Thus, documentary analysis can ascertain how these materials are structured and enhance critical and creative thinking in mathematics. However, for this to be ascertained, there is once again a need for instruments that contribute to a specific analysis of the subject.

This article was therefore developed with the aim of presenting two matrices with descriptors for assessing critical and creative thinking in mathematics. Concerning the first, it is hoped that the instrument will contribute to its use in school routines, helping teachers understand each student's development and signaling elements that contribute to the structuring of creative feedback. As for the second, it is hoped that the instrument will

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contribute to the undertaking of specific analyses on the development of critical and creative thinking in mathematics, supporting education professionals, both those who work with the development and analysis of teaching materials and teachers who, in addition to producing and selecting various materials for their classes, need to indicate the textbooks that will be made available to students in their schools.

Critical and Creative Thinking in Mathematics: definitions and forms of assessment

The guiding documents for curriculum policies, notably the Common National Curriculum Base (BNCC) (Ministério da Educação, 2018), when dealing with the field of mathematics, refer to various forms of "thinking": numerical thinking (p. 517), algebraic thinking (p. 517), geometric thinking (p. 517), proportional thinking (p. 518) and computational thinking (p. 518). According to the BNCC (Brazil, 2018), these different ways of thinking are integrated and make up a set of ideas that produce links between the various fields - Arithmetic, Algebra, Geometry, Probability and Statistics, Quantities and Measures - and which are essential for the development of mathematical thinking. In addition to these fields, the BNCC highlights critical thinking and creativity as essential elements for doing mathematics in basic education. Regarding these latter elements, Roy and Schubnel (2017) state that they are a constitutive part of mathematical thinking.

The BNCC's emphasis on critical thinking and creativity is supported by publications from international organizations that work in education. The Organization for Economic Cooperation and Development (OECD) (Vincent-Lancrin *et al*., 2019), for example, points out that critical and creative thinking is becoming increasingly important, both because of issues related to the economy, which orbits around innovation, and for personal life, since they contribute to individual well-being and the proper democratic functioning of society.

These are some of the reasons why several countries have included the development of critical and creative thinking in their educational curricula. In some countries, even most people believe "that schools should help students become 'independent thinkers' rather than just impart knowledge" (Vincent-Lancrin *et al*., 2019, p. 20).

However, in the curricular guidelines of some countries, there is a lack of clarity about this construct, which requires reformulations of the documents and, at the same time, initial and continuing teacher training actions on the subject. In the case of Brazil, Fonseca and Gontijo (2020a) warned that the BNCC does not explain what critical and creative thinking means, nor does it provide guidance on how to stimulate this way of thinking in basic education students. The lack of clarity about critical and creative thinking can hinder teachers' planned and intentional pedagogical work with their students.

The lack of clarity between the terms can be seen from two perspectives: the first based on common sense, with no explicit definitions, which allows for "reading between the lines." In the second, it fluctuates under the different concepts constructed, given the

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existence of a plurality of definitions over time (Roy & Schubnel, 2017).

As such, the OECD presents its versions in the quest to highlight a common definition that can contribute to education today. The organization points out that creativity "aims to create new and appropriate ideas and products." In contrast, critical thinking "aims to evaluate carefully and judge statements, ideas, and theories regarding alternative explanations or solutions, to reach a competent and independent position - possibly for action" (Vincent-Lancrin *et al*., 2019, p. 20).

From this perspective, creativity is associated with imagination and the generation of ideas, while critical thinking enjoys inquiry and a more analytical dimension. However, these associations do not detract from the fact that during the creative process, other elements are judged in order to generate something new, while critical thinking also involves imagining and creating alternative theories and reasons. From this perspective, there is a constant alternation between actions that jointly involve critical thinking and creative thinking, as pointed out by Lipman (2003).

For this research, we adopted the definition presented by Fonseca and Gontijo (2020a) about what characterizes critical and creative thinking in mathematics. It should be noted that this definition is aligned with the perspective presented by the OECD but with a scope and focus that is more adjusted and operationally constructed for analyzing student productions in mathematics. According to the authors (Fonseca & Gontijo, 2020a, p. 971), critical and creative thinking in mathematics is characterized as

> the coordinated action of generating multiple and different ideas to solve problems (fluency and flexibility of thought) with the decision-making process in the course of developing these ideas, involving analysis of the data and evaluation of evidence that the proposed paths are plausible and appropriate to reach the solution, arguing in favor of the best idea to achieve the objective of the problem (originality or appropriateness to the context). In other words, the use of critical and creative thinking in mathematics is materialized through the adoption of multiple strategies to find the answer(s) to the same problem associated with the ability to reflect on the strategies created, analyzing, questioning, and interpreting them in order to present the best possible solution.

This concept, in addition to considering latent traits that can be identified and measured for the purposes of teacher monitoring and/or academic research, also characterizes critical and creative thinking in mathematics as an alternation between actions. The infographic shown in Figure 1 helps to exemplify the concept.

Figure 1 shows clues as to how pedagogical work can contribute to developing students' critical and creative thinking in mathematics. Provocative questions that stimulate the generation of new answers can be understood as part of what Bezerra, Gontijo, and Fonseca (2021, p.94) have called creative feedback, which according to the authors, is "feedback whose intention is to develop creative potential" and which has "in addition to the particularities of effective formative feedback" the following characteristics:

> 1) stimulating the development of creative thinking skills, such as fluency, flexibility, and originality, as well as analysis and judgment of one's ideas; 2) promoting the development of self-perception of creative capacity; and 3) boosting or maintaining intrinsic motivation (Id.).

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It should be noted that although the nomenclature used by the authors only highlights creative thinking, its characteristics include elements of critical thinking, such as analysis and judgment of one's ideas.

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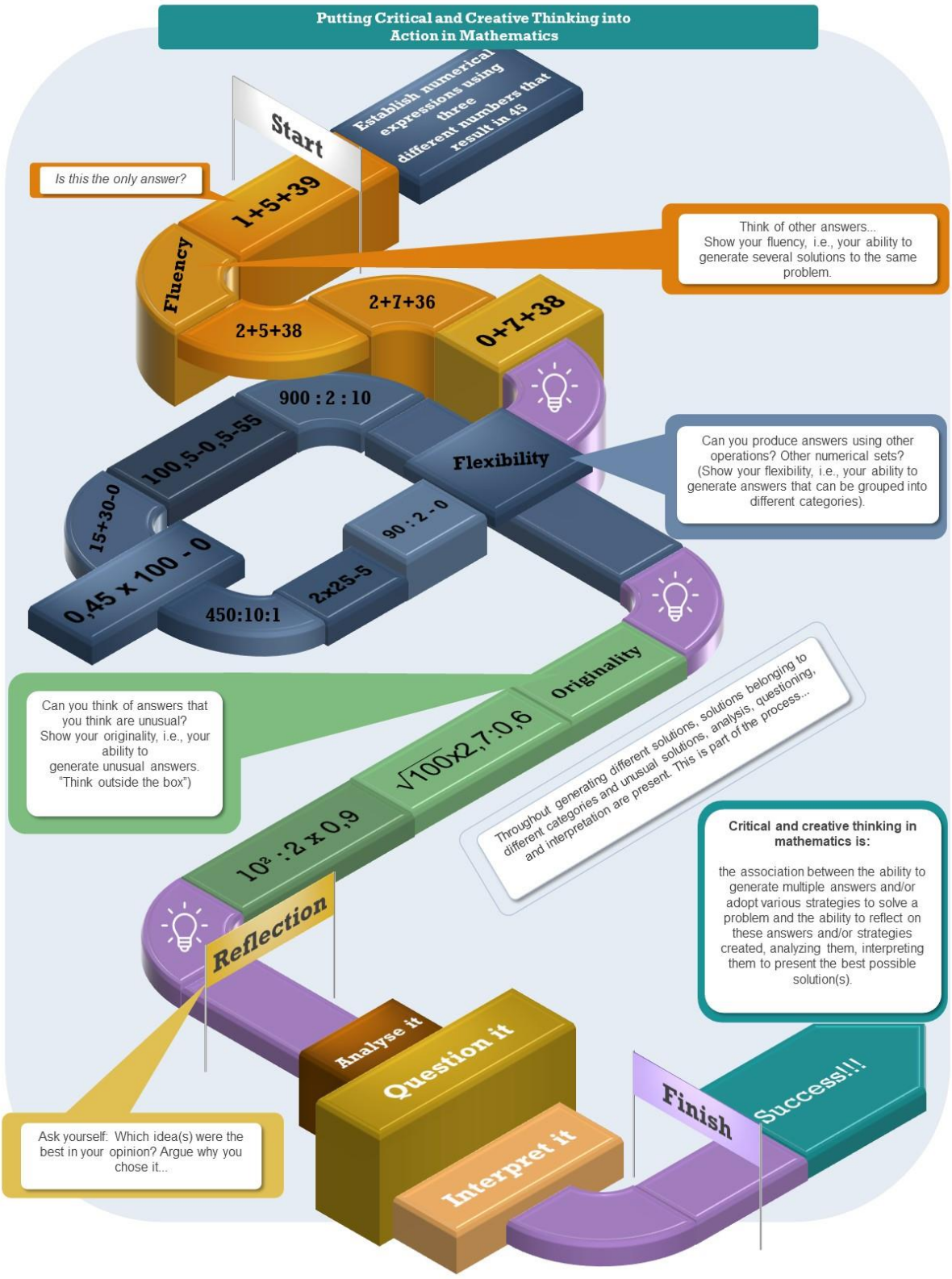

Figure 1: Infographic "Putting Critical and Creative Thinking in Mathematics into Action"

Source: Fonseca and Gontijo (2020b).

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It is worth highlighting that although there is no agreed definition of critical and creative thinking in mathematics, there is some convergence between researchers on specific points. This makes it possible to identify common dimensions on the subject, which makes it possible to structure assessment rubrics, i.e., the production of descriptors that synthesize, based on a conceptual translation, the traits, actions, and behaviors linked to what is being assessed (Vincent-Lancrin *et al*., 2019). In this way, assessment rubrics for critical and creative thinking in mathematics can become an instrument that helps teachers and other education professionals understand each student's development and select and choose appropriate teaching materials for this purpose.

Methods

This is a descriptive study, intending to make an inventory of rubrics that can be used to assess students' development and the potential of teaching materials related to critical and creative thinking in mathematics, i.e., to describe characteristics linked to this theme using rubrics.

In addition, as initially mentioned, this is a documentary study, which according to Gil (2018, p.29), "makes use of all kinds of documents prepared for various purposes." This type of research is sometimes confused with bibliographical research. However, this research was based on documentary research due to the nature of the publication analyzed, which, in addition to providing informative data, also presents a series of data compiled from different institutions/countries. The documentary analysis is characterized as a meta-synthesis, which in simplified form, can be presented as a qualitative alternative to meta-analysis, i.e., an interpretation based on data from another primary source (Bicudo, 2014; Matheus, 2009).

The selected and analyzed document is Fostering Students' Creativity and Critical Thinking: What it Means in School (Vincent-Lancrin *et al*., 2019). Specifically, as an object of analysis, the annex of this document was analyzed, the content of which presents materials produced by governments and/or institutions from 26 locations in different countries, all referring to the assessment or analysis of critical and creative thinking[4].

Results and Discussions

The document's annex under analysis shows assessment descriptors from 26 different locations: 1. Australia Curriculum; 2. Galileo Education Network; 3. Achievement Charts; 4. Quebec Curriculum; 5. Chile: Educar Chile; 6. Creative Little Scientist; 7. India: Central Board of Secondary Education's Continuous and Comprehensive Evaluation; 8. Design for Change (India); 9. Israel - Assessment Tool of Creativity and Critical Thinking; 10. Japan-Human-Centered Innovation; 11. Netherlands - The Draft Report by SLO; 12. Quick Scan

[4] Available at: <https://www.oecd.org/education/Chapter-2-CCT-from-concepts-to-teacher-friendly-rubrics_web-annex.pdf>. Accessed on January 7, 2023.

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Creativity; 13. Puerto Rico- Critical Thinking Rubric, University of Puerto Rico Rio Piedras; 14. Scotland Curriculum for Excellence; 15. Spain: Innovation and Creativity Evaluation Template, Universidad de Europa Madrid; 16. Spain: La Batería PIC (Test of Creative Imagination); 17. United Kingdom: progression in student creativity in school: first steps towards new forms of formative assessments (Lucas *et al*.); 18. US- Buck Institute for Education; 19. Culture of Creativity; 20. Dschool; 21. United States-Holistic Critical Thinking Scoring Rubric; 22. Partnership for 21st Century Skills; 23. Value Rubric; 24. US-Illinois Curriculum; 25. USA- Critical and Integrative Thinking Rubric, Washington State University; 26. International.

The document is structured based on a list of information about each institution, giving their name, authorship, access link, and what they represent in the respective educational structure with which they are associated. It also records the name of the object being assessed, the school years for which it is intended, and the definition of progress in the construct. It then lists the domains, subdomains, and levels of progression—finally, the type of assessment.

It should be stressed that the document analyzed was prepared by Vincent-Lancrin and collaborators (Vincent-Lancrin *et al*., 2019), synthesizing assessment rubrics associated with creativity and critical thinking in the general domain, i.e., they can be applied to any area of knowledge. These rubrics help identify students' creativity and critical thinking skills and can be used to reflect on and plan the strategies and activities to be adopted. It is worth noting that this list is not intended to be used to assess grades or even to analyze compliance with a list of skills. Its purpose is to provoke teacher reflection to encourage discussion between teachers and students to clarify the subject and what is being developed. The document also points out that it can be adapted to take account of specific contexts.

It should be noted that the literature suggests that assessment rubrics need to include both descriptions of what is being assessed and ways of observing/measuring it (Fernandes, 2020) - elements that can be observed in the assessment rubrics for the development of critical and creative thinking in mathematics provided by the OECD document under analysis (Vincent-Lancrin *et al*., 2019), available in Table 1.

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Table 1: Rubrics for assessing creativity and critical thinking

	CREATIVITY	**CRITICAL THINKING**
	To produce new ideas and solutions	To question and evaluate ideas and solutions
INVESTIGATING	• Feel, empathize, observe, and describe experiences, knowledge, and relevant information • Make connections with other concepts and ideas, integrate other disciplinary perspectives	• Understand the context, structure, and limits of the problem • Identify and question assumptions, check the accuracy of facts and interpretations, analyze gaps in knowledge
IMAGINING	• Explore, search for, and generate ideas • Stretch and play with unusual, risky, or radical ideas	• Identify and review alternative theories and opinions and compare or imagine different perspectives on the problem • Identify strengths and weaknesses of evidence, arguments, claims, and beliefs.
DOING	• Position and propose how to solve a scientific problem in a personally innovative way	• Justify a solution or reasoning based on logical, ethical, or aesthetic criteria
REFLECTING	• Reflect on and evaluate the novelty of the chosen solution and its possible consequences • Reflect on and evaluate the relevance of the chosen solution and its possible consequences	• Evaluate and recognize the uncertainty or limits of the solution or position endorsed • Reflect on the possible bias of their own perspective compared to other perspectives

Source: Vincent-Lancrin *et al*. (2019, p. 23) - with adaptations.

Consistent with the text under analysis, which already recognized the possibility of adapting such rubrics to specific contexts, the OECD offered a framework that constitutes a rubrics model for assessing creativity and critical thinking, specifically in mathematics (see Table 2).

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Table 2: Rubrics for creativity and critical thinking in mathematics

	CREATIVITY	**CRITICAL THINKING**
	Produce new ideas and solutions	Questioning and evaluating ideas and solutions
INVESTIGATING	Makes connections with other mathematical concepts or ideas from other disciplines.	Identifies and questions generally accepted assumptions and ways of presenting or solving a math problem.
IMAGINING	Generate and play with various approaches to proposing or solving a math problem.	Considers various perspectives on how to approach a math problem.
DOING	Visualizes and proposes how to solve a math problem meaningfully and personally.	Explains the strengths and limitations of different ways of presenting or solving a math problem based on logical and/or other plausible criteria.
REFLECTING	Reflects on the steps taken to propose and solve a math problem.	Reflects, based on possible possibilities, on the mathematical approach chosen and the solution found for the problem situation.

Source: Vincent-Lancrin *et al.* (2019, p.117) - with adaptations.

Vincent-Lancrin *et al*. went on to present rubrics that can be explicitly used to assess creativity in tasks in one or several curricular components, including recognizing that it can be used in "any exercise with technical or disciplinary requirements that includes space for students to demonstrate their creative thinking skills. It can be used to provide formative or summative feedback" (Vincent-Lancrin *et al*., 2019, p. 56). The Institution offered a similar presentation for the critical thinking rubrics (p. 57), considering the same list of criteria.

To this end, they define what they call "product" and "process":

> "Product" refers to a student's final visible work (for example, the answer to a problem, an essay, an artifact, or a performance). These criteria are intended to assess the student's work, even if the learning process is not observable or fully documented. "Process" refers to the learning and production process observed by the assessor or documented by the students: the process may not be evident in the final product. Typically, the process may show a higher level of skill acquisition than the product (Vincent-Lancrin *et al*., 2019, p.55).

The product and the process need to be monitored to develop pedagogical work that encompasses critical and creative thinking in mathematics.

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Matrices for assessing critical and creative thinking in mathematics: monitoring students and analyzing teaching materials

The matrices were initially produced using the matrix presented by Vincent-Lancrin *et al*. and adapted for the Brazilian context. As a second step, a comparison was made with each of the institutions cited in the text under analysis in search of complementary elements that could be inserted, especially considering the Brazilian conceptualization adopted.

In addition to the aspects mentioned above, Graham Wallas' (1926) model of creativity was used to create the matrix, considering that the four phases have connections not only with creativity but also with criticality. The four phases served to inspire the central axes of the assessment, which brought the rubrics together: (a) initial; (b) reflective; (c) testing; (d) checking. These, in turn, were constructed by writing simple sentences, each showing traits linked to critical and creative thinking in mathematics.

The first matrix is designed to help classroom teachers understand their students' development and guide them in developing feedback. The proposed levels of progress were constructed and qualitatively separated into three levels: (a) developing; (b) satisfactory; and (c) adequate. This assumes that everyone can improve their critical and creative thinking in mathematics (see Table 3).

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Table 3: Follow-up matrix - developing critical and creative thinking in mathematics

DIMENSIONS	SUBDOMAIN - CREATIVITY	CRITICAL THINKING SUBDOMAIN	PROGRESSION LEVELS	STRUCTURING QUESTIONS FOR FEEDBACK
	To Produce new ideas and solutions	To question and evaluate ideas and solutions		Examples of questions and/or guidelines that can stimulate the student to develop each of the items mentioned
INITIAL	Generates and/or plays with various approaches they consider possible before proposing a solution (debating, writing, drawing, role-playing...) Structures strategies (action plans) before starting to solve the problem	Adequately synthesizes the information and elements needed to solve the problem Questions the conventional forms of solving strategies adopted and/or solutions found	- Development - Constant encouragement is needed. Identify and act on the items that need to be improved.	*"What does this problem bring to mind: some content, some situation, some memory...?"* *"What do you think would be needed to solve this problem?"* *"Does the problem have all the information you need to solve it?"* *"Is this the only possible strategy/answer to the problem?"*
REFLEXIVE	It makes associations with other concepts, whether from other areas or mathematics itself Formulates/reformulates problems by highlighting similar situations	Considers different perspectives on how to approach a math problem Hypothesizes the implications for the problem/solution of changes in one or more of its elements	- Satisfactory - Continued encouragement is needed. Encourage the strengthening of those items that can still be improved.	*"Have you solved similar problems before?"* *"In what other situations do you think this type of problem could occur?"* *"Can you deal with the problem in different ways (algebraically, geometrically, etc.)?"* *"What are the implications of changing something in the problem?"*
TESTING	Proposes solution(s) to the problem meaningfully and personally Solves the problem using different strategies and/or finds different solutions	Argues about the strengths and weaknesses of each strategy adopted and/or solutions found Suggests inferences based on the context, strategies adopted, and/or solutions found in the problem		*"Do you understand each step of the strategy adopted and/or see any meaning in the solution found?"* *"Can you adopt different strategies and/or find different solutions?"* *"How do you rate the strategies adopted and/or the solutions found? Is one better than the other?"* *"Write down everything you can conclude from this problem."*
VERIFICATION	Evaluates the steps taken to propose and solve the problem Looks for counterexamples to verify the strategies adopted and/or solutions found	Evaluates the mathematical strategy adopted to optimize it Evaluates the mathematical strategy adopted and/or the solution found, arguing about its plausibility	- Adequate - Encouragement needs to be maintained. Encourage the habit to be applied to tasks in other areas as well.	*"How would you present the strategy adopted and/or a solution found to someone else?"* *"Are there any specific situations that could prevent or hinder the strategy you adopted and/or the solution you found?"* *"Can you summarize the path taken to solve the problem?"* *"Does the strategy adopted and/or solution found make sense (procedures adopted, type of response and its context, etc)?"*

Source: Elaborated by the authors.

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The second matrix (Table 4), in turn, refers to elements suggested for evaluating teaching materials to identify their potential for stimulating students' critical and creative thinking in mathematics.

The matrix comprises 15 sentences subdivided into three groups: idea generation, idea evaluation, and classroom climate. Each sentence is associated with a 6-point scale, where 1 represents the absence or unsatisfactory presence of the object under consideration, and 6 represents the fully satisfactory presence of the object under analysis.

The scale also includes a value referring to the absence of meaning of the sentence about the material analyzed, characterized by the acronym "na" (not applicable).

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Table 4: Matrix for analyzing teaching materials - developing critical and creative thinking in mathematics

EVALUATION ASPECTS	DESCRIPTION *THE MATERIAL UNDER ANALYSIS OFFERS ACTIVITIES AND INVITING SPACES THAT:*	EVALUATION
IDEA GENERATION	*admit and suggest the adoption of multiple resolution strategies, even in algorithmic situations;;*	○1 ○2 ○3 ○4 ○5 ○6 ○na
	allow and suggest multiple answers, including original answers, through open and closed problems;	○1 ○2 ○3 ○4 ○5 ○6 ○na
	suggest the construction of hypotheses and inferences about different situations linked to a problem;	○1 ○2 ○3 ○4 ○5 ○6 ○na
	encourage connections with other mathematical concepts or ideas from other disciplines;	○1 ○2 ○3 ○4 ○5 ○6 ○na
	encourage the development and redefinition of mathematical problems;	○1 ○2 ○3 ○4 ○5 ○6 ○na
	encourage the expression of different ways and approaches to solving a problem (debating, writing, drawing, role-playing...);	○1 ○2 ○3 ○4 ○5 ○6 ○na
	promote habits of mathematical investigation.	○1 ○2 ○3 ○4 ○5 ○6 ○na
	Allow/encourage the improvement of an idea or product.	○1 ○2 ○3 ○4 ○5 ○6 ○na
IDEA EVALUATION	*suggest identifying and questioning conventional and unconventional ways of solving a problem;*	○1 ○2 ○3 ○4 ○5 ○6 ○na
	stimulate reflection on the implications of a problem once it has been restructured;	○1 ○2 ○3 ○4 ○5 ○6 ○na
	encourage students to synthesize information systematically;	○1 ○2 ○3 ○4 ○5 ○6 ○na
	encourage students to question and evaluate the strategies adopted and/or the solutions found to a problem (ready-made and their own).	○1 ○2 ○3 ○4 ○5 ○6 ○na
CLASSROOM ATMOSPHERE	*encourage active student participation;*	○1 ○2 ○3 ○4 ○5 ○6 ○na
	encourage collective work;	○1 ○2 ○3 ○4 ○5 ○6 ○na
	promote student engagement with mathematics;	○1 ○2 ○3 ○4 ○5 ○6 ○na
	contribute to minimizing feelings of anxiety in mathematics.	○1 ○2 ○3 ○4 ○5 ○6 ○na

Source: Elaborated by the authors.

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Both matrices are subject to improvement, as in any scientific construction. It should be noted, however, that they include the main elements that characterize critical and creative thinking in mathematics, in line with the literature in the area.

Final considerations

To make progress in the planning of teaching strategies, as well as in the development and/or selection of teaching materials that contribute to the development of critical and creative thinking in mathematics, it is necessary for teacher training to include these elements. However, before that, training those who need to be trained is necessary. Moreover, before that, theorizing or constructing knowledge based on scientific research is necessary.

Concerning teacher training, we highlight Beghetto (2017), who discusses the actions of these professionals based on three forms of teaching involving creativity: a) teaching about creativity, b) teaching for creativity, and c) teaching with creativity. For the author, teaching about creativity consists of including this topic as an object of study in training programs and discussing the theoretical and practical aspects of developing creativity. Teaching with creativity, in turn, involves using methods, techniques, and strategies to teach creatively, while teaching for creativity involves promoting creative thinking through the development of systematically planned activities for this purpose.

Taking Beghetto (2017) as a reference, the instruments presented here are essential tools to support pedagogical work aimed at promoting teaching for creativity, as they help to identify elements that can stimulate students' critical and creative thinking. In this sense, the matrices are models contributing to student monitoring and analyzing different teaching materials.

The limitations of this article include the fact that although the matrices were built using meta-synthesis and theoretical support, there was no field testing. Furthermore, this limitation opens the way for future research, in which they can be tested and improved.

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