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Creativity in the education of teachers who teach mathematics: a case study

Criatividade na formação de professores que ensinam matemática: um estudo de caso

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Abstract

This investigation is part of a research project on the development of mathematical reasoning through problem solving and posing tasks from the perspective of the Exploratory Problem Solving Model and presents the results of an experiment carried out in the teachers preparation who teach mathematics. The study is qualitative and interpretative with an empirical basis, supported by design-based research. The main objective is to implement problem-solving instructional tasks in the formation of teachers who teach mathematics in order to build up a knowledge base on problem-solving and creativity. The freedom allowed to teachers and future teachers, both in the development of mathematical reasoning and in mathematical communication, seems to have an influence on the promotion of personal, innovative and creative mathematical solving.

Keywords: Design-based research; Exploratory problem-solving model; Mathematical creativity.

Resumo

Esta investigação faz parte de um projeto de pesquisa sobre o desenvolvimento do raciocínio matemático através de tarefas de resolução e proposição de problemas na perspectiva do Modelo Exploratório de Resolução de Problemas e apresenta resultados de uma experiência realizada na formação de professores que ensinam matemática. A estudo tem cunho qualitativo e interpretativo com base empírica, apoiada na pesquisa baseada em design. O objetivo principal é implementar tarefas instrucionais de resolução de problemas na formação de professores que ensinam matemática para a constituição de uma base de conhecimento sobre a resolução de problemas e a criatividade. A liberdade permitida aos professores e futuros professores, quer no desenvolvimento do raciocínio matemático, quer na comunicação matemática parece ter influência na promoção de resoluções matemáticas pessoais, inovadoras e criativas.

Palavras-chave: Pesquisa baseada em design; Modelo exploratório de resolução de problemas; Criatividade matemática.

Introduction

In today's ever-changing world, it is difficult to estimate the importance of creativity. The development of creativity, in general, and mathematical creativity, in particular, is important today to strengthen people's ability to adapt to new and challenging situations, which is essential for the well-being of each individual and acts as a basic mechanism for social, technological and scientific development. According to Leikin (2018), one of the

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central functions of the education system is to enable people to adapt fluently to these changes and innovations. Mathematics is the basis of progress in technology, and school education must guarantee, along with mathematical knowledge, students' creative abilities. Cognitive skills intertwined with intrapersonal and interpersonal skills and supporting each other are important for the development of creativity. Interpersonal skills, such as teamwork and collaboration, develop alongside cognitive and interpersonal skills. Intrapersonal skills include positive self-evaluation.

Cai and Leikin (2020) consider challenging Problem Solving (PS) to be an activity aimed at creativity. Mathematical creativity is generally associated with problem solving and problem posing. In this way, PS can be used to develop mathematical creativity (Elgrably & Leikin 2022). Mathematics teaching and learning has been shifting from a traditional model, in which the teacher presents most mathematical ideas and procedures through traditional, teacher-centered instruction, towards student-centered, inquiry-oriented instruction. In this sense, it is important to implement research-based innovations in ME during the training process that open up paths and opportunities for learning and developing creativity through problem solving.

There have been efforts to incorporate problem solving as an instructional approach and creativity at different levels, including the training of teachers who teach mathematics. This study looks at creativity through problem solving in the training of teachers who teach mathematics. It is important to recognize that there is a scarcity of studies in this area involving the training of teachers who teach mathematics, creativity and problem solving.

This investigation is part of a research project on "The development of mathematical reasoning through problem solving and posing tasks" from the perspective of the Exploratory Problem Solving Model-MERP (Koichu, 2019) and presents a discussion of a classroom implementation, carried out in the training of teachers who teach mathematics on Problem Solving (PR), supported by *Design-Based Research* - DBR (Cobb et al, 2003) in an interactive perspective that combines educational research with instructional practice (guided by theory) , being necessary to understand the students' thinking in order to decide how to act proactively.

The aims of this research are:

a) to implement PS tasks in the teachers preparation who teach mathematics, with the intention of promoting creativity in mathematics;

b) to contribute to the creation of a knowledge base on problem solving and creativity in the training of teachers who teach mathematics.

There is a tendency in Mathematics Education (ME) research for accumulated knowledge in terms of research results and theoretical references not to find its way into classrooms. Implementation of Research Results in Mathematics Education' is understood, according to Jankvist et al (2021), as an ecological perturbation of a given educational system through the gradual support of innovation in conjunction with an action plan aimed at solving

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what is perceived as a problem by at least one of the parties. The operational definition of implementation research according to Century and Cassata (2016) is that of a systematic investigation of innovations implemented in controlled environments or in common practice. By innovation, the same authors define it as programs, interventions, processes, instructional approaches, methods, or policies that provide a change for the individuals who implement it and the end users. Two types of knowledge are associated with the implementation process: practical knowledge of how to use the innovation and an understanding of the principles and objectives essential to the innovation. In this study, the two are considered relevant.

Problem solving and creativity in teacher preparation

Cai and Leikin (2020) argue that problem-solving, which represents a cognitive challenge, and creativity are among the basic cognitive skills of the 21st century that determine intellectual development and career readiness and adaptation to change. At the same time, the same authors highlight socio-emotional, collaborative skills, social and ethical responsibility and commitment, openness and flexibility to diverse points of view, which are generally considered components of math teachers' professional development. These efforts indicate the interest of many professionals in making the "problem" a prominent feature in the classroom.

For Joklitschke, Rott and Schindler (2021), interest in creativity in mathematics education (EM) research is increasing, and the field is developing. In research on creativity in EM, there are many different aspects to be studied, as well as many underlying theoretical assumptions about creativity. Some scholars focus on creative products, while others focus on creative processes. There is research that draws on theories from the domain of psychology, and other research that draws on theories from mathematical stages. The body of studies on creativity in EM is rich and diverse.

Stylianides and Stylianides (2020) consider that research conducted in the world of practitioners, i.e. with attention to real mathematics classrooms and in close collaboration between ME researchers, future teachers and teachers, increases the likelihood that results will reach classrooms and be directly applicable in practice rather than potentially relevant. Research from this perspective should seek to develop empirically tested, theory-based solutions to alleviate students' learning problems in real classrooms by clarifying not only what works, but why and how it can work, which points can be relaxed about theory, and which are structural in the context of real classrooms.

Considering that research into problem solving and posing in EM has advanced in recent years (Cai and Hwang, 2020; Kontorovich et al 2012; Silver et al, 1996), we agree with Cai and Hwang (2020) that there is a disconnect between research and practice in ME. Mathematics teaching in the classroom, which is under the control of teachers, focuses on a narrow and unambitious set of learning objectives and remains traditional and expository teaching with few opportunities for learning and developing creativity.

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In PS teaching, learning takes place during the problem-solving process (Schoenfeld, 1985, 1989; Marcatto & Onuchic, 2020). As students solve problems, they take time to think about possible solutions and then use any approach they can think of, draw on any knowledge they have learned and justify their ideas in a way they find convincing.

The learning environment provided by teaching through PS is a natural setting for students to work, individually or in groups, on their resolutions and then present them to the whole class. By engaging in this process, they have opportunities to learn mathematics through social interactions, negotiating meaning and building shared understanding. Such activities help them clarify their ideas and gain different perspectives on the concept or idea they are learning.

It is based on the definition of Koichu, Cooper and Winder (2022), who consider PS as the involvement with mathematical situations to which the student attributes problematicity and does not have a readily available solution path but has an appropriate background to find the solution. In this way, problem solving is understood as a means of helping students learn to think mathematically and is therefore part of the mathematics curriculum in almost all countries, including Brazil. Here, PR is included in the National Common Curriculum Base (BNCC) as a perspective for developing skills and competencies (Ministério da Educação, 2018). However, it is still a source of difficulty for teachers and students.

The processes of PS can be characterized by their internal or external structure. According to Rott, Specht and Knipping (2021), the internal structure refers to metacognitive processes, such as heuristics, verifications, or the solver's beliefs about the topic. The external structure refers to observable actions that can be characterized in phases, such as understanding the problems, drawing up a solution plan and those involving solution.

However, real PS processes, as advocated by Rott, Specht and Knipping (2021), seem to happen in different ways to how they were idealized. They are not linear, as they contain deviations, errors and cycles that often do not follow a predetermined sequence, involving steps forward and back between analyzing, planning, and exploring a problem.

According to Gordeau (2019), the task of mathematical PS should provide the possibility of engaging in collaborative discussions, in which the teacher remains removed from the initial discussions, giving students the opportunity to experience sharing and interacting with others. Different approaches can emerge, multiple ways of seeing and analyzing the problem. Promulgating ideas about the problem during the collaborative discussion can promote progress where some students had difficulties. In discussions about the proposed task, some may express skepticism about the mathematical ideas presented by other colleagues. Generally, students seem to feel authorized to question their colleague, while most would probably not feel comfortable questioning an argument presented by the class teacher.

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In this sense, the Exploratory Model of Problem Solving (MERP) presented by Koichu (2018; 2019) was adopted, conceptualizing PS oriented towards mathematical discourse in a collaborative way. MERP, according to Koichu (2019), is a sequence of shifts in attention stimulated by the availability of three types of resources: (i) the solver's individual ones or the reorganization of existing resources through interaction with the problem; (ii) those of the whole class, i.e. connecting existing individual resources with those of other solvers of the same problem, always based on interaction with peers who share mathematical ideas and heuristics that complement each other in a productive way; (iii) those based on interaction with an external source of knowledge about the solution, for example by doing an internet search or a more advanced colleague in the course.

In addition to making a commitment to PS in the mathematics curriculum, teachers need to be strategic in selecting appropriate tasks and orchestrating classroom discourse to maximize learning opportunities. We also expect teachers to provide opportunities for the development of creative skills and abilities, as defined in the Common National Base for the Initial Teachers Formation of Basic Education (Ministério da Educação, 2019, p. 13) in its General Competence 2: "Research, investigate, reflect, carry out critical analysis, use creativity and seek technological solutions to select, organize and plan challenging, coherent and meaningful pedagogical practices".

According to Gontijo and Fonseca (2020), creativity in mathematics must be stimulated in teachers if we want to stimulate creativity in mathematics in students.

> To be able to stimulate their students' mathematical creativity, teachers must acquire adequate pedagogical knowledge during their initial training, and this must be improved throughout their careers through continuing education programs. Regarding critical and creative thinking, many teachers admit to a lack of previous experience or adequate preparation to stimulate this type of thinking in their students. Formation programs need to explicitly explore what it means to think critically and creatively so that teachers feel creative first, to feel able to stimulate critical and creative thinking in their students. (Gontijo & Fonseca, 2020, p. 745)

The authors' emphasis on the relevance of explicitly exploring teachers' critical and creative thinking in training programs corroborates what Flores (2022) says about teachers' interest being able to benefit not only their own teaching, but also their learning and motivation in teacher training. For this author, the focus of training should be related to the need to train critical and questioning math teachers.

There are variations in the structure, organization, and curriculum of the training of teachers who teach mathematics. According to Flores (2022), it is necessary to develop a systemic vision to understand the logic, curriculum, and objectives of teacher training, as well as their professional development. This vision must encompass the nature and objectives of the school curriculum, the conception of the teacher as a professional and their role in curriculum development and must also provide opportunities for professional learning and the reworking of beliefs about the nature of teaching and learning mathematics. Added to this is the political, economic, social, and cultural context in which the teacher or future teacher is

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inserted, forming a complex and interrelated set of factors. The development of mathematical creativity by primary school pupils, for example, refers to the teacher's pedagogical interest in the educational aspects of teaching.

Gontijo (2007) presented a conception of creativity in mathematics, characterizing it as

> as the capacity to present numerous appropriate solution possibilities for a problem situation, so that they focus on different aspects of the problem and/or different ways of solving it, especially unusual ways (originality), both in situations that require problem solving and elaboration and in situations that require the classification or organization of mathematical objects and/or elements according to their properties and attributes, whether textually, numerically, graphically or in the form of a sequence of actions. (Gontijo, 2007, p. 38)

In a recent study, Elbrably and Leikin (2022) investigated the relationship between mathematical creativity, expertise, problem solving and problem proposing, comparing two groups of student volunteers: medal winners in mathematical Olympiads and graduates who had excelled in university mathematics courses. The results were that: the Olympic medalists' problem-solving expertise significantly influences their investigations and explorations of problems, in relation to proof and creativity skills, compared to those of the graduates; university courses do not develop creative mathematical skills and competences.

Assuming that creativity is a skill that can be developed in all individuals, Sriraman (2009, p. 133) believes it is "sufficient to define creativity as the ability to produce something new or original" for the individual who devised it or for a group of individuals, for example in the case of a classroom, which is further compatible with the definition of mathematical creativity "as the process that results in unusual and insightful solutions to a given problem regardless of the level" (Sriraman, 2009, p. 133) or context in which it manifests itself.

In addition to novelty or originality, there also seems to be agreement on the conditions for obtaining creative results, which include knowledge, intellectual skills, motivation, the environment, and mastery of specific ideas, for example mathematics (Sriraman, 2009; Elbrably & Leikin, 2022). Thus, from the perspective of the development of creativity, there is a concern with the construction of instruments that make it possible to assess and/or highlight its development.

Leikin (2009) suggested a model for assessing creativity by analyzing the similarities and differences between the problem-solving strategies used. This model suggests assessing creativity through three categories suggested by Silver (1997): fluency, flexibility, and originality. Fluency is developed by generating several mathematical ideas, obtaining several resolutions to a mathematical problem (when one exists). Flexibility is promoted by generating new mathematical resolutions when at least one has already been produced. Originality is achieved by exploring many solutions to a mathematical problem, generating a new problem.

In the present study, these categories were considered suitable to serve as the basis for a description and evaluation of the mathematical creativity of teachers and future teachers of mathematics in problem solving.

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Research Methodology

Considering the objective, the research question and its qualitative and interpretative nature, this study has a methodological approach based on case studies (Yin, 2010; Ponte, 2006). Qualitative, because it values teaching processes in a natural environment (Bogdan & Biklen, 1994), and interpretive when it seeks to understand, in the context of teaching, the ways in which teachers and students constitute environments for each other (Erickson, 1986). This perspective is suitable for developing this study, as it enables theoretical teaching experiments to be transformed into practices that give meaning to learning, since its field of study is the classroom, seeking to discover what is essential and characteristic in it when dialog drives the construction of knowledge.

Qualitative research takes the view that things are studied in their natural environment, with reality being socially constructed not in a single way, but observed and interpreted in different ways, because "researchers don't find knowledge, they build it" (Cobb et al, 2003, p. 10). Interaction with others is extremely important for the formation of interchangeable meanings and senses, whether subjective or not, but which help us to understand the world in which we live and act.

The study described here is characterized as a case study and is based on Design-Based Research, with a focus on learning processes (Cobb et al, 2003). According to Yin (2010, p. 20), a case study is a type of investigation that allows the holistic and significant characteristics of real-life events to be preserved in the desire to understand complex social phenomena. Furthermore, "in any case study, attention must always be paid to its history (the way it developed) and its context (external elements, local reality, social or systemic nature that most influenced it)", as affirmed by Ponte (2006, p. 5).

The DBR was adopted in this study because it is a powerful research tool when you want to introduce new ideas and study their interactions. It allows you to propose tasks to create observations of related behaviors and refine tasks based on student response. It has a pragmatic and theoretical orientation with the aim of creating solutions to important challenges and subjecting these solutions to careful examination and review, generating contributions to learning theory.

Research based on design cycles is a process of investigation involving the person who knows (the researchers in question), the context in question (the training of teachers who teach mathematics) and the activity taking part (the design experiment), with the aim of studying learning processes (PS) or change and the way (PS tasks) of promoting them in natural contexts (real classrooms). We therefore justify the choice of this methodology because of our interest in an interactive process of refining and implementing new ideas.

The stages of this study followed the planning, intervention, and retrospective analysis phases. The unit of analysis considered here were the documented episodes in which the mathematical topic was the focus of the activity and the classroom discourse that was understood to be relevant to the context of the study. In this sense, the critical episodes for the

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analysis were those that supported or refuted the initial paradigm. These episodes may not seem important on their own, but they become critical when seen in chronological order with other episodes.

This study was conducted at a public university in the federal education system. The subject in which these studies were carried out was offered to undergraduate and postgraduate students enrolled in an initial training course for mathematics teachers and a postgraduate program in science teaching. The class consisted of 16 students, who were initially informed about the study in question and were asked to agree to take part in the study and, if they agreed, they signed an informed consent form.

Description and Analysis of Data

The course - Teaching Mathematics through Problem Solving - is part of the curriculum of a postgraduate program in Science Education and the undergraduate courses in Mathematics at a public university. This course took place over one semester in 2018 and its aim was to discuss Problem Solving as an instructional approach and to experience and reflect on its didactic-pedagogical possibilities for the classroom.

The activities planned for this course were divided between readings on Problem Solving as an instructional approach and experiences of instructional practice using PR. The texts contributed to understanding and reflection on the approach in question and the problem situations enabled discussions on math concepts and content and various ways of solving the same situation.

The class consisted of 16 matriculated students and was made up of heterogeneous backgrounds. Six students were from the postgraduate master's program, three from the undergraduate bachelor's degree in mathematics and seven from the bachelor's degree in mathematics. Of the six students in the master's program, four are teachers who teach mathematics, three of them in elementary school, Early Years and Final Years, and one teacher with a degree in Pedagogy, who teaches mathematics in the early years. One of them is a high school physics teacher. Another was also a high school philosophy teacher.

During the stage of planning and anticipating how the students would be involved in a task involving MERP, the importance of their involvement with the task and the characteristics of an exploratory approach, which were desired from the perspective of the discipline, were highlighted. The concern was that undergraduates and teachers should have opportunities to learn to teach mathematics in Basic Education, not through a teaching practice of direct instruction, disregarding important steps such as the discussion of ideas among students, but as a system of interrelated dimensions: (1) the nature of the classroom tasks; (2) the role of the teacher; (3) the classroom culture; (4) the mathematical tools to aid learning; (5) the concern for equity and accessibility (Lester & Cai, 2015). This perspective seems to be important as it articulates planning and activity in the classroom with subsequent reflection, which can support the implementation of PS in the curriculum.

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It is necessary to understand how to organize a lesson of this nature and understand the aspects involved in conducting it, from the choice of tasks and appropriate PR models to promoting an environment of discussion and learning in the classroom. Planning and conducting an exploratory lesson is even more challenging for teachers and future teachers who have no experience of teaching mathematics. It is essential that training and professional development courses create these opportunities for reflection on practice (Martins, Mata-Pereira & Ponte, 2021). It therefore became essential to discuss PR models that were appropriate to the context.

In this study, three main stages were defined from the perspective of MERP. The first involved planning the activity (by the trainer), studying the content (PS in teacher formation), selecting a task (Table 1) suitable for the classroom context, which considered the knowledge and interests of the undergraduates and postgraduates, and which should have an adequate cognitive demand and anticipate the main paths they could take to solve the task. The second stage covers the exploration of the problem and the trainer's moves towards the collaborative sharing of ideas and the promotion of mathematical discourse. The third stage presents a reflection on the development of the whole activity.

The problem addressed in this article is the Venusian problem. This problem and others discussed in the course have the same objective: to discuss Problem Solving as an instructional approach in real classrooms, to promote creativity, collaborative work and the discussion of mathematical ideas with each other and with the teacher.

Frame 1: The Venusian problem

Problem: How many fingers?

Suppose a probe lands on Venus and sends us the image below of a multiplication table written on a wall. If Venusians use a positional notation like us and a base system corresponding to the total number of fingers on their hands, how many fingers does a Venusian have on each hand?

$$\begin{array}{r} \Delta\ \phi \\ \Delta\ \phi \\ \hline \Delta\ \Theta\ \Delta \end{array}$$

Source: Adapted from Mathematics Teacher-NCTM (1992)

The students had seven days (one week) to explore and solve the problem and return for the collective discussion (all the students and the teacher trainer), which took place in person. During this week, students could discuss the problem by email and/or instant messaging application.

Several solutions were presented by the group, but only one student presented a complete solution, i.e., she explored the problem with arguments for each stage of her solution and obtained a solution to the problem (Chart 2). This article will focus on this resolution. The teacher with a degree in Pedagogy, whom we will refer to here as Teacher P, has been working in the early elementary school grades for almost three decades. As a student, she was confronted with difficulties arising from her training and professional practice. From there, she recounts a little of her journey in search of learning and reflection.

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My view of mathematics changed, because I thought that the mathematical register was just numbers, and I didn't think of dissertative problem registers. At first it seemed strange, but as the lessons went on, I realized that this strategy made me think more about the problem and the results of the answers became clearer. I could also see that the diversity of the problems and how they can lead students to investigate different answers produced an investigative and reflective dialog, leading to the introduction of content. Among the various problems we solved, one caught my attention and led me to research, deconstruct my concepts and develop a new concept to solve the problem. (Primary school teacher)

Frame 2: Resolution presented by the teacher.

Given the data in the problem, I related the number of Venusians' fingers to the non-decimal bases. Researching non-decimal bases, I found information about addition and multiplication that is a little different from ours. In this search I found a table of addition and multiplication in base 8, see how the facts are done in this base.

Base 8 table for addition and multiplication:

+	1	2	3	4	5	6	7
1	2	3	4	5	6	7	10
2	3	4	5	6	7	10	11
3	4	5	6	7	10	11	12
4	5	6	7	10	11	12	13
5	6	7	10	11	12	13	14
6	7	10	11	12	13	14	15
7	10	11	12	13	14	15	16

X	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	10	12	14	16
3	3	6	11	14	17	22	25
4	4	10	14	20	24	30	34
5	5	12	17	24	31	36	43
6	6	14	22	30	36	44	52
7	7	16	25	34	43	52	61

Having the results of addition and multiplication in non-decimal base, therefore base 8, I tested some values for the symbols and came to the conclusion that we have multiplication as follows:

$$\begin{array}{r} 42 \\ \times\ 42 \\ \hline 104 \\ 2010\ \\ \hline 404\ \ \end{array}$$

I'll explain how I thought of this multiplication. Assuming that the multiplication is like ours, so 42 x 42, then to begin with 2 x 2 (looking at the table) is 4. 2 x 4 (looking at the table) is 10. I'm considering the result 10 to be just one order. Thinking about the multiplication of the tens 4 x 2 (looking at the table) is 10 and 4 x 4 is 20, the same thing I consider the result 10 and the result 20 to be just one order.

Remember that when I multiply the tens I leave one order blank, let's say the order of simple units. Moving on to addition, we have 4 plus zero units, which is 4.

10 + 10 = 20, which is another order, since it's 8, the largest value considered for base 8, I need to go to the hundreds, giving 20 + 20 = 40, considering the value 40 will also occupy the tens order. Omitting this part of the operation, we get 42 x 42 = 404, which represents the image sent by the probe. We conclude that the Venusians have four fingers on each hand, eight in total.

Source: Researcher's archive (2018)

It is worth noting that the solution presented by teacher P may indicate that it is possible to produce mathematical knowledge, even if you don't have mastery of all the content related to this subject. This solution presents several ideas (fluency), different representations (for example, tabular and arithmetic, therefore flexibility) and a process that resulted in an unusual solution for the individual who came up with it (originality). In this way, the exploration of RP can lead the individual to make use of previous knowledge, but also allows them to develop their creativity in the search for solutions.

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Two students from the last term of the mathematics degree course presented their explorations of the problem, but without reaching a conclusion (a solution). Their explorations involved working out and solving systems of equations, i.e., observing the multiplication operation (provided through symbols) seems to have induced algebraic thinking, coding, and decoding the symbols provided in algebraic notation known to these undergraduates. Despite significant procedural knowledge, they were unable to complete the problem.

In general, among the students with a background in mathematics (ten undergraduates and four graduates), many tended to look for a generalized and formalized solution, representing and applying algebraic expressions and equations, while the two students (graduates in other areas) insisted on formulating hypotheses that were tested empirically or looked for numerical examples that satisfied certain constraints of the problem.

In the context of this study, although originality is considered the most important indicator for a resolution to be considered creative (Sriraman, 2009), it is not enough to describe the creative product. The concept of creativity is defined by combining the components of originality, fluency, and flexibility in an integrated system. Thus, it was considered that originality refers to the uniqueness of the resolution, fluency consists of mobilizing the necessary and appropriate mathematical knowledge to solve the problem and flexibility refers to the appropriate forms of representation to express the knowledge involved and which interferes in the mathematical communication of the solution.

Concluding remarks

In view of the results obtained and the objective of implementing problem-solving tasks in the formation of teachers who teach mathematics, it was possible to observe that those who have no training (or few years of training) in mathematics tend to spend more time exploring the problem, in contrast to those with training in mathematics who tend to demonstrate greater control over the execution and verification of the strategies employed to solve the problem.

While students with some mathematical training sought generalized and formalized solutions, teachers without mathematical training applied empirical and informal reasoning.

The freedom allowed to teachers and future teachers, both in the development of mathematical reasoning and in mathematical communication, seems to have an influence on the promotion of personal, innovative, and creative mathematical solutions. This freedom and the time spent in collective discussion, from the perspective of MERP, can provide communication of reasoning, emphasizing the importance of representing the ideas, concepts and mathematical processes involved in the resolution.

The use of previous mathematical knowledge and reliance on past experiences seems to be an obstacle to progress in solving a new problem. We suggest that identifying divergences between the problem at hand and past experiences is also important, as it can

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help the problem solver develop new strategies. It is also suggested that problem-solving models also emphasize the need to devote oneself to problems over a long period of time and even to failed strategies.

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