



Creative Strategies in the Problem Formulating and Solving in School Mathematical Discourse

Estratégias Criativas na Formulação e Solução de Problemas do Discurso Matemático Escolar

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Abstract

This paper presents and discusses the results of a research project that aimed to understand the strategies and what they reveal about the creative potential of students in activities of formulating and solving mathematical problems. The research is anchored in the Commognitive Theory, which considers mathematics a discourse, bringing a look at the System Perspective, for which the author points out that creative action does not occur in isolation, but in the relationship between three systems: individual, domain and field, as well as the verification of how students manifest elements of creativity in their productions in relation to Flexibility and Originality. It is a research with a qualitative approach, developed with students in the final years of elementary school in a public school in the municipality of Canaã dos Carajás in the state of Pará, Brazil. For the production of data, activities were applied that required students to formulate problems. The results point to the occurrence of creative expression involving flexibility and originality a routines of exploration, act or ritual.

Keywords: Mathematical discourse; Situation-problems; System perspective; Elements of creativity.

Resumo

Este artigo apresenta e discute resultados de uma pesquisa que teve como objetivo compreender as estratégias e o que estas revelam sobre a criatividade dos alunos em atividades de formulação e solução de problemas do discurso matemático escolar. Está ancorada na Teoria Comognitiva, que considera a matemática um discurso, na Perspectiva de Sistemas, para o qual a ação criativa não ocorre de forma isolada, mas da relação entre três sistemas: indivíduo, domínio e campo, bem como na verificação de como os alunos manifestam elementos da criatividade nas suas produções em relação à Flexibilidade e Originalidade. É uma pesquisa de abordagem qualitativa, desenvolvida com alunos dos anos finais do Ensino Fundamental de escola pública do município de Canaã dos Carajás no Estado do Pará. Para a produção de dados foram aplicadas atividades que demandavam dos alunos formulação de problemas e solução de problemas. Os resultados da pesquisa apontam para a ocorrência de manifestação criativa em relação à flexibilidade e originalidade em rotinas de exploração, ato e ritual.

Palavras-chave: Processos linguísticos; Criatividade; Problemas; Perspectiva de sistema; Produção textual.

Submetido em: 20/12/2022 – **Aceito em:** 31/07/2023 – **Publicado em:** 20/12/2023

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Introduction

We perceive the students' problems in subject of mathematics from the low indexes shown in external evaluations, such as the International Student Assessment Program (Pisa), a test applied to 15-year-old students to assess knowledge and skills related to reading, mathematics and science in more than 37 nationality members of the Economic Cooperation and Development (OECD) and more 42 of similar economy (INEP, 2021). Although we notice an increase in the average proficiency of Brazilians in the 2015 Pisa assessment with 377 compared to 384 in its latest 2018 edition, it does not guarantee a good placement in the world ranking, since Brazil scored below the OECD average in reading, mathematics and science, reaching only 2% the highest levels (Level 5 and 6). Furthermore, 43% of Brazilian students scored below the minimum (Level 2) proficiency level, against only 13% in the OECD.

This scenario, identified both by our practice and by the results of large-scale assessments, makes it increasingly clear that we need to think about teaching mathematics beyond what has long predominated in pedagogical practices, which is the focus on rules memorization and application. We understand that schools should start stimulating the development of creative thinking, starting from basic education up to the higher levels of education, the gradual improvement of this ability.

The Common National Curricular Base (BNCC), a document that guides Brazilian education, highlights among its general competencies the ability to,

Exercise intellectual curiosity and resort to the approach proper to the sciences, including research, reflection, critical analysis, imagination and creativity, to investigate causes, develop and test hypotheses, formulate and solve problems and create solutions [...] (Ministério da Educação, 2018, p. 9)

That is, these are skills that both in language teaching, mathematics, social or natural sciences, among others, can lead to the development of important skills for student learning. In mathematics classes, it enables them to expand their mathematical repertoire which can result in a much more refined understanding of working with problems, as well as to break the patterns of answers expressed in the textbooks.

Fonseca (2015) says that creativity is freedom of mind in search of new connections and the breaking of patterns of answers even if this requires exhaustive thinking and new experiences occur. This understanding converges with the thought of Sfard (2008) when he says that creativity occurs through routines, which despite paradoxical their meanings in the sense of the word, the author clarifies that for there to be creativity, is necessary knowledge of routines to be able to reflect on the mathematical discourse. However, Sfard, (2008, p. 219) says that to be creative "[...] it is necessary to be able to apply routines in a non-routine way. That is, creativity is not about reproducing a discourse, but reflecting on it to build new discourses.

For Sfard (2008), discourse is a distinct type of communication made by its repertoire of permissible actions and the way in which these actions are aligned with the interlocutor's

responses. Discourses can be identified based on the use of words, mediators, routines and narratives. School mathematical discourse is a combination of routines and narratives produced in the colloquial and literate spheres, but which gains specificities due to the interference of other fields, such as the pedagogical. These routines bring with them the possibility of constructing narratives about mathematical objects that can be validated depending on the argument's repertoire used to defend the objects raised. For the author, the construction of this repertoire is based on object-level rules, which deals with the properties of the objects of discourse, and meta-rules, which is the substantiation or validation of the discourse objects rules. It is in this perspective that we aim to understand the strategies and what they reveal about the creativity of students in formulating and solving of problems of school mathematical discourse.

The discussions around this problematic are based mainly on the assumptions of Sfard (2008) and Gontijo (2007), discussing the substantiation process of narratives based on creativity elements such as fluency, flexibility and originality.

Mathematics as a discourse

The approach to mathematics in studies and research that conceive it as a discourse is brought by Sfard (2008), for whom mathematics comprises a communication form and, therefore, can be materialized in the discourse form. Therefore, mathematics is considered a discourse capable of promoting communication within a given context. Communication takes place through social interaction, being able to produce knowledge from a certain discourse.

As for the elements that allow identifying a discourse and differentiate it from another, Sfard (2008) presents four

Use of words in mathematics are related to aspects of quantity and/or form (fraction, equation, mode, determinant, among others) as well as the meaning that those discourse objects will take in specific situations.

Visual mediators - are symbols that allow, among other possibilities, a better communication. In colloquial mathematical discourse people generally resort to mathematical representations through images, regardless of the layer of discourse. Academic discourse involve representations by means of symbols, such as: %, =, +, -, <, >, \sum , π , etc., created mainly to facilitate mathematical communication.

Endorsed narratives - consist of a sequence of verbal expressions whose purpose is to make the discourse objects description in question, highlighting the relationship between the objects and the processes by which the objects are constituted, such as theorems, definitions and axioms, among others, in mathematical discourse. This procedure will be subject to validation (endorsement) or rejection by an expert community in the field, whose procedural terms result from the discourse itself.

Routines - are ordered actions used by the discourses in the narrative construction. For this, they make use of words and typical visual mediators to structure the textual elements

used in the discursive construction. In mathematical discourse some of these routines are defining, demonstrating, proving, as well as formulating and solving problems.

For Sfard (2008), every discourse is constituted by standardized activities, governed by well-established rules, especially in its discursive formalization, which can provide solidity to the objects and identity to the discourse. As for mathematical discourse, he defines it as governed by two types of rules, object-level and meta-rules (or metadiscursive). The first is constituted by narratives about regularities in the behavior of the discourse objects, as in "the sum of the internal angles of a triangle is equal to 180° ", which defines a property of triangles. The second is focused on the actions of participants in a discourse, on the process by which mathematicians or mathematicians demonstrate or define rules that are at the object level.

Sfard (2008) states that the routines of mathematical discourse can be Exploration, Act, or Ritual.

Explorations are characterized by the ability to produce a narrative that is endorsable by expert at the end of a performance, and can be a construction narrative, which is the production of a narrative that does not yet exist in the discourse, that is, of an object-level rule; of substantiation, which are the mechanisms used to evaluate whether or not the narratives produced by discourses can be endorsed; or of recall, which is the process by which discourses seek to evoke narratives already endorsed in order to substantiate new narratives. Acts focus more on the production or physical transformation of objects in a routine than on the production of narratives at the end of the performance, although this is not abdicated. The rituals are focused more on the guarantee of maintaining interpersonal interactions in the routine than with the production of a narrative.

Creativity and mathematical learning

The focus of studies and research on creativity, at first, was based on the distorted view that only special beings classified as geniuses were endowed with creative abilities (Taylor, 1976). This perspective begins to be changed in the twentieth century, when creative ability is no longer seen as a special gift limited to the arts, but rather as a cognitive psychology focused on the study of mental processes, such as "creative processes" and "problem solving" (Sternberg, 2001). According to Gontijo (2007), research generally tends to focus on only one motivating element of creative production. Such action makes it possible to analyze creativity by means of categories, three of which are highlighted by Feldhusen and Goh (1995, apud Gontijo 2007): person, process and environment.

The focus on the person comprises an analysis directed at cognitive aspects linked to the person emotional and personality, as well as experiences lived on a daily basis, to identify if it reveals a new production and if it has social representativeness in the analyzed product. The focus on the process is linked to the product development as a creative action. The focus on the environment refers to the space in which the experience takes place and whether it can be a motivator or an inhibitor to creative abilities. In this respect, Gontijo (2007) points out

that more recent researches seek to link these categories mainly to the latter, believing that they are factors that may contribute to creative action and that if analyzed separately, they may provide an incomplete perception of an individual's creative action.

In this sense, one of the models that reflects this current view of approaching creativity is the system perspective presented by Csikszentmihalyi (1998), whose proposal for analyzing creativity is to take a historical approach, not limiting itself to individual aspects or metric studies of the individual's creativity, but also focusing on the individual's social-historical-cultural environment. To Csikszentmihalyi (1998, p. 47), "creativity is any act, idea or product that changes an existing domain, or that transforms an existing domain into a new one. And the creative person is: someone whose thoughts and acts change a domain or establish a new domain".

The relationship between being creative and creativity is directly linked to the relationship between the individual's action with the environment, since it must be evaluated all the internal and external variables to the individual so that a better understanding of why the idea produced occurs. However, Gontijo, Silva and Carvalho (2012) say that despite Csikszentmihalyi's model for studying creativity being applied in several areas, in mathematics the creativity study and analysis still prevails without taking into account external factors to the individual, focusing more on the internal factor, considering only the result of the individual's production.

Gontijo (2007) presents some authors who discuss creativity, such as Aiken (1973), Ervynck (1991) and Hadamard (1954), with different models of creativity perception.

The model advocated by Aiken (1973) presents an understanding of creativity that goes through two perspectives, one focused on mathematical production and the other on the product resulting from this process. The first focuses on the cognitive process, understanding the relations established mentally to solve a problem. This moment is marked by the possibility of alternating thoughts, outlining new strategies that will culminate in a better understanding of the problem. The second approach corresponds to the answer, that is, what is presented as a written result from the internal analysis of the data.

The model advocated by Ervynck (1991) presents mathematical creativity from three stages, classified by the author in a sequence from 0 to 2, where zero is characterized as the first stage and two as the last stage. Stage 0, linked to the school environment, is defined as the moment when students, in order to solve a problem, use mathematical strategies or techniques without showing mastery of the content on which the applied techniques are based. Stage 1 comprises the moment of mechanization, the application of algorithmic mathematical techniques through the repetition of formulas in the solutions to the activities. In stage 2 there is a detachment from the "ready-made formulas", since in this stage creativity truly emerges, because the student is able to establish relations between the acquired knowledge and the proposed problem and present an original solution.

Similarly, Hadamard (1954), inspired by Wallas' ideas, argues that creativity, according to Gontijo's (2007) interpretation, consists of understanding creativity in four

stages: initiation, incubation, illumination, and verification. Initiation corresponds to the moment when the student is free to solve a problem and uses the knowledge already acquired in his experience to solve a problem. Incubation corresponds to the stage in which during the presentation of the problem there is a detachment of attention on its resolution, the brain establishes connections with other knowledge in order to contribute to the solution of the problem or information that will culminate in a new restructuring. Illumination is characterized as the moment when an unexpected response to the problem arises. Verification corresponds to the crucial moment of the whole process, mainly because it validates and organizes the answers presented at the moment of illumination.

Despite this variety of definitions, Gontijo (2006) points out that they do not clash, they just highlight different aspects. Facing this range of knowledge presented around the characterization of mathematical creativity, Gontijo (2007) defines it as

the ability to present numerous appropriate solution possibilities for a problem situation, so that these focus on distinct aspects of the problem and/or different ways of solving it, especially unusual ways (originality), both in situations that require problem solving and elaboration and in situations that request the classification or organization of objects and/or mathematical elements according to their properties and attributes, whether textually, numerically, graphically or in the form of a sequence of actions (Gontijo, 2006, p. 4).

The author stresses that creativity is considered an ability that can be stimulated from the identification of positive relationships experienced by students at a given moment with mathematical objects, and it is up to the teacher to propose activities that enhance these relationships.

Studies since the 1950's have evaluated creative thinking based on three aspects: Fluency, Flexibility and Originality, based on the model proposed by Torrance (1966), who develops the Torrance Test of Creative Thinking (TTCT), with the aim of assessing creativity based on the three constructs above (Amaral, 2016).

Similarly, Gontijo (2007) believes that in creative production in mathematics these three elements plus elaboration are also implicitly present in the creative process. He defines fluency as the ability to present a diversity of different ideas produced for the same subject, flexibility as the individual's ability to modify thought or present distinct classes of answers and originality as the presentation of infrequent answers if compared to those of another group of individuals and elaboration as the ability to present a considerable number of details of the exposed idea. For Gontijo (2007), the existence of variables such as abstract thinking, inductive and deductive reasoning, analogical, metaphorical and inductive thinking contributes for students to present these elements.

The perspective that we adopted for creativity in this work is that of Gontijo (2007), focusing to identification of signs of Flexibility and Originality, since the students proposed only one solution for each problem presented. We also tried to evidence the concepts of the system perspective proposed by Csikszentmihalyi, in which he considers that creativity occurs in the interaction of three systems: domain (culture, set of rules), field (group that

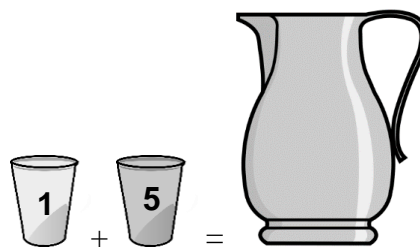
controls the domain) and individual (person who knows the domain), in order to a more coherent analysis regarding the identification of creativity in the students' production. This perspective is done by considering the routines of mathematical discourse, as proposed by Sfard (2008).

Method

This research has a qualitative approach, because the natural environment constitutes an important supplier of data and the researcher as the main instrument to extract them, through field research and the direct contact of the researcher is provided a descriptive character of this research model (Bogdan and Biklen, 1982). This reinforces the fact that all the information found is relevant to the understanding of the object of study. The authors argue that understanding the process of consolidating information from an investigated situation is more important than just seeing the product.

The data were produced with 20 students in the final grades of elementary school in a public Brazilian school. The research instrument corresponded to the application of two activities (Figures 1 and 2).

Juice Preparation Method:



Mix 1 part of concentrated juice into 5 parts of water

Figure 1: Activity 1

Source: Research Data



Figure 2: Activity 2

Source: <http://amavitaalimentos.com.br/site/refresco-uva-1kg/> Captured on Feb. 18, 2021.

Directions for 1 liter:

Mix 5 tablespoons in 1 liter of cold or ice-cold filtered water.

Stir and it is ready to drink. Yield: 5 cups of 200ml.

Preparation for 10 liters:

Mix the contents of this envelope in 10 liters of cold or ice-cold filtered water.

Stir and it is ready to drink.

There is no need to add sugar, Amavita is already sweetened.

The application of instrument occurred in 2 meetings, with approximately 2 and a half hours each, with 20 students present on the first day and only 14 on the second, resulting in 34 elaborate problems. On a sheet of paper, next to the picture, the student was verbally instructed to read the information in the image and, using all their creativity and mathematical knowledge, to create and solve a mathematics problem involving the elements of the figure given to them.

The analysis first involved identifying the type of routine that the student performed, according to the theoretical guidelines of Sfard (2008). Next, we sought to identify indications of flexibility and originality in each of them. In the discussions, we used the following identification code: P1A1T6, where "P" indicates the Problem, being P1 Problem 1, P2 for Problem 2 and so on (P1, P2, ..., P34) according to the order of the material collected in the research; the "A" is the identification of the activity referring to instrument 1 or 2; and the T refers to the series, which can be from 6th to 9th grade.

Analysis and Discussion

We organize this section based on the types of routines in mathematical discourse – explorations, acts and rituals (Sfard, 2008), elements that allow us to identify creativity – flexibility and originality (Gontijo, 2007) and the system perspective of Csikszentmihalyi (1998).

Exploration and Creativity

In P1A1T9, the student was concerned with relating the ideas used in the creation of the problem with the information in the image in the proposed activity, since the plot of the problem consists of a juice preparation as it is illustrated in the thematic image. To construct the narrative the student uses the idea of proportionality (Figure 3).

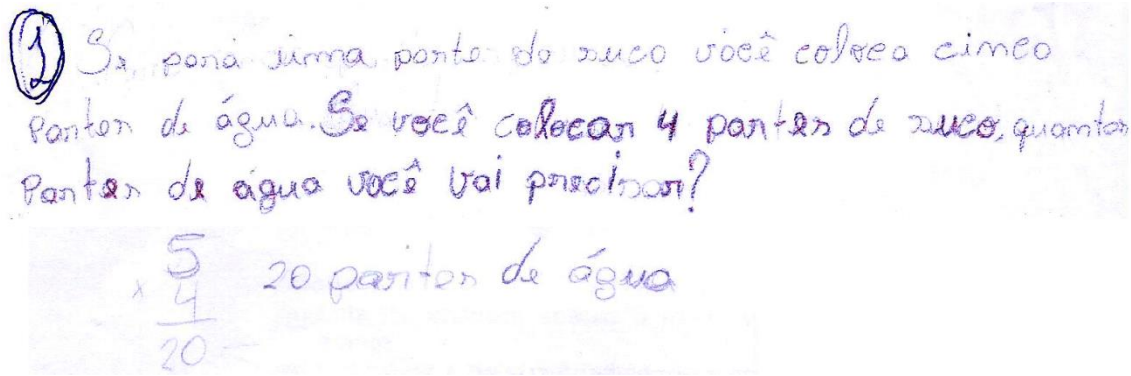


Figure 3- Problem-solving prepared by the student - P1A1T9

Source: Research data

1 If for one part of juice you put five parts of water. If you put four parts of juice how many parts of water will you need?

A: $5 \times 4 = 20$ 20 parts of water

Despite being a problem with a simple structure, we notice that the student had the concern to ensure an organization of information, presenting a clear and coherent description of his proposal with the central idea of the problem whose theme is proportion. We understand the production to be an exploration routine, since it correctly establishes a proportion relation between two magnitudes, "juice" and "water", as observed in the first part of the problem when writing "If for one part of juice you put five parts of water", and to conclude the problem it takes up again to two magnitudes mentioned proposing the expansion of the value of one, and asks how much will need the other, ensuring the same proportion established initially. The ability shown by the student in formulating the problem may be the result of experiences in school, since a 9th grade student reported having already participated in a similar activity.

For Csikszentmihalyi (1998), this ability of the student may be supported by a domain, that is, a set of rules and inherent to the activity of problem formulation, which, according to the author, constitutes one of the factors to which the creative action can be perceived, because when faced with the parameters governed in a certain domain, some participants of the discourse only reproduce what is transmitted to them, while others go beyond, presenting new possibilities and changing the domain. This action is characterized by the author as a creative process. We identify elements of flexibility in relation to the answer presented for the problem, because one of the ways that is expected as a resolution procedure for a proportion problem is the technique as it is put in the textbooks. However, the student solves with a direct multiplication between 4 and 5, obtaining as answer 20 parts of water. In other words, the student was able to present a solution method that was different from the expected one.

In problem P3A1T8, the student also uses the idea of proportionality in the narrative construction (Figure 4).

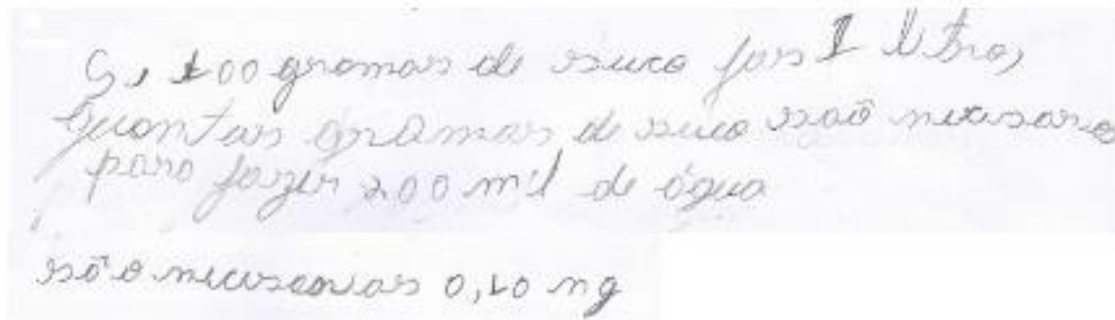


Figure 4- Problem elaborated by the student - AP3A1T8

Source: Research data

If 100 grams of juice makes 1 liter, how many grams of juice are needed to make 200 ml of water.

A: It takes 0.10 mg

There is some difficulty of the student in the use of language to express the organization of the ideas presented, but that does not compromise the understanding of the narrative, which probably would be an essay like 'If 100 grams of juice concentrate preparation (powder or liquid) makes 1 liter of drink, how many grams of concentrated preparation are needed to make 200 ml of drink? We attribute originality to the production, since the student elaborated the problem using new information, which was not present in the image, as well as identified unusual answers compared to the other productions analyzed.

In the construction of the problem, we infer that the way the measurement units are placed by the student may refer to situations experienced previously in his daily life. For the first data, the unit of measurement in grams is used to refer to the juice, possibly taking as reference the package of juice bought at the supermarket, which is given in this unit. Possibly, he also alluded to a jar and a glass with storage capacities in liters and ml, respectively, which may have motivated him to use this information in the construction of the problem. That is, according to Sfard (2008) and Gontijo (2007), the subject when exposed to certain situations tends to acquire experiences, becoming capable of creating new strategies when exposed to similar situations. Creative action is also confirmed in the perspective of Csikszentmihalyi (1998) when defining creativity as a change of domain (rules) or its readaptation to the situation, a fact perceived when the student uses units of measurement seen in school or in textbooks and relates them to the preparation of a juice. Therefore, when building a narrative bringing elements such as gram, liter and milliliter, the student seems to remember a field of mathematics (magnitudes and measurement) incorporating this knowledge to the idea of proportion with specific rules of operations that are parameters for actions of the discourses.

However, in relation to the answer provided to the problem, we realize that there may have been a misinterpretation of the rules of this field of mathematics, because in the problem formulated the student presents the result of the proportion between the amount of juice powder needed to dilute in 200 ml of water. Therefore, the answer should be 20g instead of

DOI: 10.20396/zet.v31i00.8672196

0.01. This misunderstanding in the result may be linked, according to Sfard (2008), to how the routine of the discourse may have been memorized.

Some previously endorsed narratives may be immediately available and some others may have to be reconstructed. Such mediated recollections involve special routines that probably depend on the way in which the remembered narratives were memorized at the beginning. That is, such a factor may be evidenced in the failure to successfully solve the problem presented, indicating that the student may have mistakenly memorized the rules of discourse.

P5A2T8 (Figure 5) stands out for the quality of the writing in the ideas' organization, since it makes clear the existing proportion relationship.

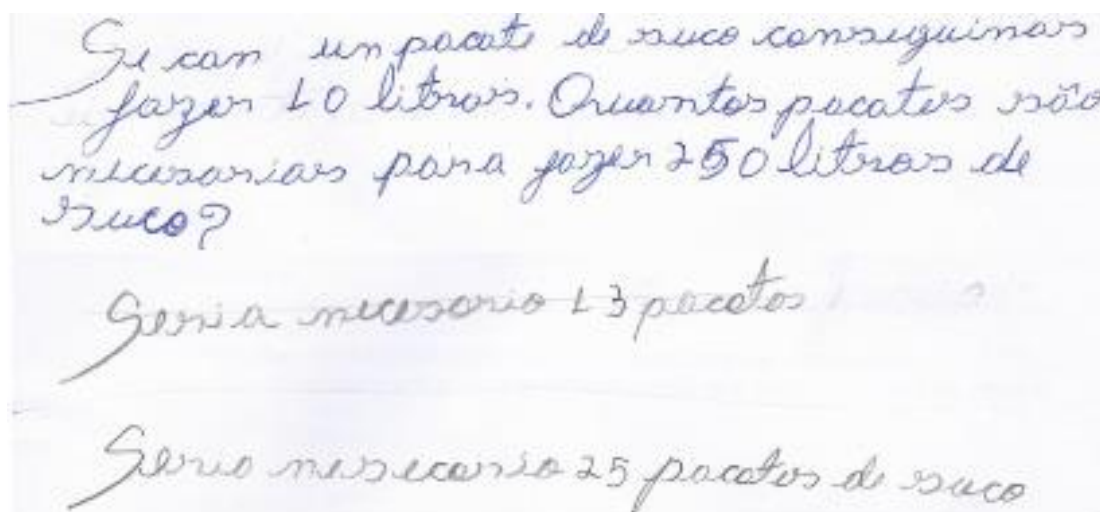


Figure 5 - Problem elaborated by the student - P5A2T8

Source: Research data

If with one pack of juice we can make 10 liters. How many packages are needed to make 250 liters of juice?

A: It would take 13 bottles

A: It would take 25 cartons of juice

Although the problem was well written, it was not possible to identify flexibility and/or originality. The student did not get rid of the information contained in the image, resulting in a common problem, without much innovation.

As the first value, which represents the wrong answer, seems to have been the first answer found, and the 25, which is the correct answer, a second one, leads us to infer that this student was aware that he had reached the wrong solution in the first moment, redoing the resolution process and finding the correct answer. In other words, this action of revising the answer is a result of the individual (author of the action) being supported by a domain (set of rules) according to Csikszentmihalyi (1998). Under the theoretical lens of Sfard (2008), we can classify this action of narrative recall. Even if he did not present the calculations justifying the answer, we infer that the student has resorted to previously acquired mathematical knowledge, because by resuming the resolution changing from 13 to 25, we

identify the success in the resolution, which may be the indication of the student having resorted to the simple rule of three algorithm, already endorsed and widely used by the mathematics community in solving proportionality problems.

Acts and Creativity

P9A1T7 is a clear and well-crafted narrative from the standpoint of its clarity and the present mathematical relationship (Figure 6).

ALINE COMPROU 20 COPOS POR R\$ 1,50 CADA UM
 PARA PODE LEVAR PRA CASA DE SUA VÓ POIS
 IRIA FAZER SUCO PARA SEUS NETOS. QUANTO
 ALINE GASTO?

$$\begin{array}{r} 20 \\ \times 1,50 \\ \hline 30 \end{array}$$

$$\begin{array}{r} 15 \\ + 15 \\ \hline 30 \end{array}$$

$$\begin{array}{r} 1,50 > 3 > 6 \\ 1,50 > 3 \\ 1,50 > 3 \\ 1,50 > 3 > 6 \\ 1,50 > 3 > 6 \\ 1,50 > 3 \\ 1,50 > 3 \\ 1,50 > 3 > 3 \\ 1,50 > 3 > 3 \\ 1,50 > 3 > 3 \end{array}$$

ALINE GASTOU 30 REAIS PELOS COPO QUE ELA COMPROU

Figure 6 - Problem elaborated by the student - P9A1T7

Source: Research data

Aline bought 20 glasses for 1.50 each so she could take them to her grandmother's house to make juice for her grandchildren. How much will Aline spend?

$$20 + 1,50 = 30 \quad 15 + 15 = 30$$

Aline spent 30 reais for the glasses he bought

The narrative presents a clear writing, being easily identifiable the context, the data, and the question: to find out how much Aline spent to buy the glasses. However, we notice little relation with the elements of the thematic image A1 (Figure 1), since the problem does not deal with the preparation of juice directly, but with the expense of buying glasses. However, what draws attention is the solution of the problem, for its innovative and original character.

According to Sfard (2008), in this procedure the student evokes a meta-rule linked to the when of a routine, since he evaluates whether the actions taken are appropriate to solve the problem. In the first answer, he uses the grouping method, adding ten times the value of the juice glass, which is R\$1.50. Then he adds these values two by two, finding 3 as the result. Next, he sums these new parcels two by two with the one not yet added up, finding 15. Finally, it doubles this value and finds 30.

From Csikszentmihalyi's (1998) perspective, we consider this action as a creative performance, since the student resorts to other mathematical information, considering

DOI: 10.20396/zet.v31i00.8672196

multiplication as addition of equal parcels. For this problem we also identified creativity elements pointed out by Gontijo (2007), such as flexibility, since the narrative presents different procedures for finding the answer, represented here by the product between the number of cups by the unit value and the sum of each cup values. We also classify a production with originality, since the process used by the student is differentiated if compared to the answers presented by other students.

P12A2T7 has little relation to the thematic image (Figure 7). But the process of construction and solution draws attention.

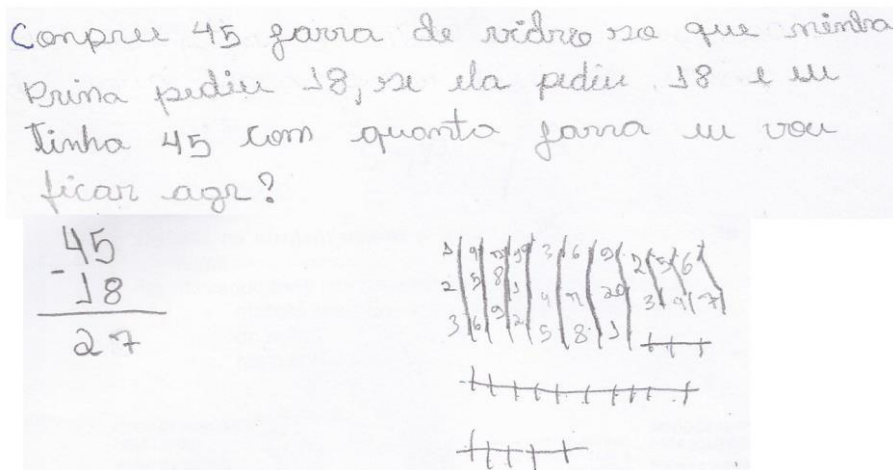


Figure 7 - Problem-solving elaborated by the student - P12A2T7

Source: Research data

I bought 45 glass jars but my cousin asked for 18. If she asked for 18 and I had 45, how many jars will I get?

$$45 - 18 = 27$$

We identified in this production a very superficial treatment in relation to the elements of A2's thematic image. The student limited himself to use only one element, the "jar". We understand that this may have occurred due to the way this student may have retained abstract information such as 5 spoons, 200 ml, 10 L etc., demonstrating greater ability to manipulate those that he considers more tangible or physical objects when indicating a relationship between the amount of glass jugs bought with the ones his cousin had. In other words, this student also seems to be more concerned about what to do with the objects listed in the figure instead of establishing some kind of connection between them, configuring it as an act.

However, it draws attention the two solution forms. We understand that this answer presents characteristics of originality, since none of the other students produced something similar, and therefore it is an unusual answer. We also identify flexibility, because the student presented different paths of solution for the problem he created.

In the answer presented, we also noticed more evidence of an act implementation, because the symbols manipulation (straws cut in half) indicates numbers that will be subtracted from the operation $45 - 27$. This is an application of meta-rule, which according to Sfard (2008) has to do with the actions of the discourses and what they do to substantiate or

DOI: 10.20396/zet.v3i1i00.8672196

validate a performance. We notice the student's concern to justify the subtraction algorithm through a less formal procedure. Such action is recognized by Csikszentmihalyi (1998) as a creativity act, since he agrees that the individual's action is based on a domain (group of rules and procedures), identified here as the subtraction algorithm, applied informally in counting toothpicks and number sequence. With the successful problem solution, we can say that the school field (experts in the area), which in the school is represented by the mathematics teacher, could evaluate the method as positive and, therefore, a creative performance.

Rituals and Creativity

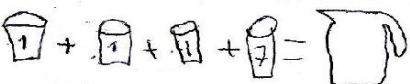
The selected answers for discussion in this type of routine draw attention by their specificities in the theme approach of the proposed activity, for example P15A1T7 (Figure 8).

① Entendo aqui se como no questionário e responde:

A) Qual se como no questionário?
R: Se como o preparo da suca

B) Como você chegou ^{nessa} ~~nessa~~ ^{resposta} ~~resposta~~
R: na imagem diz o modo de preparar como a suca

C) Com base na imagem acima crie uma equação por você.
R: _____



D) Qual o formato de preparo que você utilizou o Cimo
R: Eu coloquei 3 partes de suca concentrada de morango e misturei com 7 partes de água, deixo fermentar ~~um pouco de morango~~ ^{um pouco de suco de morango}

Figure 8 - Problem elaborated by the student - P15A1T7

Source: Survey data

Understand what is going on in the question and answer:

What is going on in the question?

A: The preparation of the juice

How did you get the answer

A: In the image it says "how to prepare the juice"

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Based on the above image create a similar situation.

A: $1 + 1 + 1 + 7 =$ a whole pitcher

What preparation method did you use above

A: I put 3 parts strawberry juice concentrate and mixed it with 7 parts waters inside the pitcher I prepared a pitcher of strawberry juice

This production stands out for the way in which the student elaborates questions from the theme of A1. We understand that the student seems to reproduce the very common model in textbooks in Brazil, those of the type 'interpret the text', 'understand what is going on in the question and answer it', as he proposes questions to be answered, like a script. For example, the first question is "What is going on in the question?", and is immediately answered how "The preparation of juice passes? We glimpse in the situation the result of a ritualistic performance, since the specificity of this routine is the reproduction or imitation of an action or a specific performance of a community, which in this case represents the format of the exercises from the books and/or the teacher.

However, despite the fact that the problem suggests an imitation of the practice of a certain community, it was still possible to perceive originality aspects in this production, because comparing to the other problems produced by the other students, it was the only one who followed this path in the problem situation elaboration.

From A2, we highlight P24A2T9 (Figura 9).

Pedrinho foi ao supermercado e comprou um pacote de suco. É com esse pó de suco dar para fazer 10 Litros de suco e como 5 colheres de pó dar para fazer 1 litro de suco. Suponha que ele conseguiu fazer 5,7 litros de suco. Quantas colheres de suco em pó ele usou?

Pacote = 10 Litros
 5 colheres = 1 litro

colheres	suco/Litros
5	1
x	5,7

$27 \cdot 1 = 27$
 $27 \cdot 5 = 135$

$x : 1 = 57$
 $5 \cdot 5,7 = 27$

Ele usou
 27 colheres
 de pó de suco.

DOI: 10.20396/zet.v31i00.8672196

Figure 9 - Problem elaborated by the student - P24A2T9

Source: Research data

*Pedrinho went to the supermarket and bought a juice package. With this juice powder he can make 10 liters of juice and as 5 spoons of powder he can make 1 liter of juice. Suppose he can make 5.7 liter of juice. How many spoonsful of juice powder did he use?
He will use 27 tablespoons of juice powder.*

In this production, the student had the concern to formulate a narrative bringing elements that evidence the juice preparation, as proposed by A2 (Figure 2). The student proposes a proportion problem involving the powder and the quantity of liters in which the juice would result. However, despite the quality in the writing, it was not possible to perceive creativity aspects in this formulation, since it not exceeded the limits of the information contained in the activity.

As for the solution presented, we noticed the implementation of a ritual, used by the student when replicating a meta-rule, an action identified in the application of the product of the means by the extremes or simple rule of three. In other words, rules to solve a proportionality like the one presented by the student. Although the problem represents an imitation of the performative action of teachers in the classroom, we noticed originality signs in the production, even if the student did not succeed in the answer. If compared to the other students' narratives for this type of routine, he was the only one to use the rule-of-three algorithm in the solving process.

Final Considerations

This research sought to comprehend the strategies and what they reveal about the creative potential of students in activities of formulation and solution of problems of school mathematical discourse. As an analysis process, we initially tried to classify the students' productions based on the routines of mathematical discourse based on the theoretical assumptions of Sfard (2008). This option comes from the understanding that this theory can contribute to the development of creativity, since the author considers routine as ordered actions used by the discourses in the narrative's construction. She also states that in mathematical discourse routines are linked to the activities of defining, conjecturing, estimating, among others. That is, in this process, of selection or construction of arguments, the student can manifest an idea with traces of creativity.

In this sense, among the routines of exploration, acts and rituals implemented by the students, it does not seem to us that there are significant differences between the creative productions, although the first two, in contrast to the last one, are more conducive to creativity due to the creation of a repertoire of arguments used to substantiate a discourse. Nevertheless, it was possible to identify elements of creativity in problem formulation and solution activities with regard to the aspects of flexibility and originality. Thus, the research corroborates discussions about strategies involved in Mathematics Education, since our defense is around stimulating the improvement of mathematical reasoning, or even, awakening the taste for discovery. Furthermore, research suggests that the student involved in

the process of problem formulation can expand his knowledge to the extent that he practices and predicts the solution process of the problem he is presenting. In this process the students tried to remember knowledge they had experienced in other moments of their lives, either in school or in their daily lives, thus showing the importance of developing this model of activity in mathematics classes.

We hope to expand discussions on this theme, since this research identified few productions in the area of problem formulation. In addition, we hope to contribute to awakening teachers to implement this type of activity in their teaching practices.

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