Zetetiké

DOI: 10.20396/zet.v32i00.8676348

CC BY NC ND

Lesson Study and Didactic Suitability Criteria in the Reflective Practice of Ecuadorian Teachers

Lesson Study y Criterios de Idoneidad Didáctica en la Práctica Reflexiva de Profesores Ecuatorianos

Eulalia Calle[1]

Víctor Sumba[2]

Abstract

The reflective competence in mathematics instruction processes has gained strength in several research programs, making necessary the use of didactic tools that contribute to the assessment of the practice, regardless of the methodology used. In this line, the objective of this study is to determine the Didactic Suitability Criteria that emerge in the Lesson Study framework to strengthen instructional processes; Thus, a group of teachers of the second year of General Basic Education, who are participating in a professional training program in Ecuador, discuss and reflect on a process of instruction on numeration in an Lesson Study context, and through a content analysis, these discussions are reinterpreted, obtaining as a result that in the reflection of teachers, in the different stages of the Lesson Study, the six dimensions of the Didactic Suitability Criteria emerge.

Keywords: Lesson Study; Didactic Suitability Criteria; Reflection on teaching practice.

Resumo

A competência reflexiva nos processos de instrução matemática tem ganhado força em vários programas de pesquisa, tornando necessária o uso de ferramentas didáticas que contribuam para a avaliação da prática, independentemente da metodologia utilizada. Nesta linha, o objetivo deste estudo é determinar os Critérios de Adequação Didática que emergem no âmbito do Estudo da Lição com o objetivo de fortalecer os processos de instrução; assim, um grupo de professores do segundo ano do Ensino Básico Geral, que participam de um programa de formação profissional no Equador, discutem e refletem sobre um processo de instrução para ensinar adição sem reagrupamento em um contexto de Estudo da Lição e, por meio de uma análise de conteúdo, reinterpretam essas discussões obtendo como resultado, que na reflexão realizada pelos professores emergem as seis dimensões dos Critérios de Adequação Didática.

Palavras-chave: Estudo da Lição; Critérios de adequação didática; Reflexão sobre a prática docente.

Resumen

La competencia reflexiva en los procesos de instrucción matemática ha tomado fuerza en diversos programas de investigación, haciendo necesario el uso de herramientas didácticas que aporten a la valoración de la práctica,

Submetido em: 18/04/2024 – **Aceito em**: 26/006/2024 – **Publicado em**: 15-08-2024

[1]Doctor en Didáctica de las Ciencias, las Lenguas, las Artes y las Humanidades, por la Universitat de Barcelona. Profesora de la Universidad de Cuenca, Ecuador. Email: eulalia.calle@ucuenca.edu.ec. https://orcid.org/0000-0001-9526-8832.

[2] Doctor en Educación por la Universidad Nacional de Rosario. Profesor de la Universidad Nacional de Educación UNAE, Ecuador. Email: victor.sumba@unae.edu.ec. https://orcid.org/0000-0002-8208-5233.

DOI: 10.20396/zet.v32i00.8676348

independiente de la metodología utilizada. En esta línea, el objetivo de este estudio es determinar los Criterios de Idoneidad Didáctica que emergen en el marco del Lesson Study con la finalidad de fortalecer procesos de instrucción; es así que un grupo de profesores de segundo año de Educación General Básica, quienes se encuentran participando de un programa de formación profesional en el Ecuador, discuten y reflexionan sobre un proceso de instrucción para enseñar sumas sin reagrupación en un contexto de Lesson Study y, mediante un análisis de contenido, se reinterpretan estas discusiones obteniendo como resultado que en la reflexión de los docentes, en las diferentes etapas del Lesson Study, emergen las seis dimensiones de los Criterios de Idoneidad Didáctica.

Palabras-clave: Lesson Study; Criterios de Idoneidad Didáctica; reflexión de la práctica docente.

Introduction

In the realm of investigating the reflection on teaching practice as a key competence for professional development and teaching improvement, several studies stand out. These include Lesson Study (LS), developed in Japan (Murata, 2011) and recognized internationally, as well as action research (Elliott, 2000) and reflective practice (Schön, 1998), among others. Of particular interest, for enhancing teachers' reflection on their own practice, are the construct of didactic suitability criteria (and its breakdown into components and indicators) proposed within the framework of the Onto-Semiotic Approach to Mathematical Knowledge and Instruction (EOS) (Godino, Batanero & Font, 2007). This construct can be used as a tool to organize teachers' reflection, as demonstrated in various training processes in Spain, Panama, Ecuador, Chile, and Argentina (Alcaraz, Breda & Sala, 2022; Esqué de los Ojos & Breda, 2021; Pochulu & Rodríguez, 2016; Seckel, 2016).

A particularly significant case among researchers in Mathematics Education is the way in which the Lesson Study methodology and the Didactic Suitability Criteria develop reflective practice in training processes and the potential for their joint use (Hummes, Breda, Font & Seckel, 2023; Font, Calle & Breda, 2023; Sol, Sánchez, Breda & Font, 2023). In Ecuador, the reflection has gained momentum with the LS methodology, yielding interesting results in teacher training processes (Sumba, Pérez & Sgreccia, 2022; Sumba, 2022), sparking interest in whether there are common characteristics that enable the joint application of these two constructs. This study aims to determine the Didactic Suitability Criteria (CID) that emerge within the framework of an LS cycle conducted with Ecuadorian teachers.

The Criteria of Didactic Suitability

In the Onto-Semiotic Approach (OSA), the didactic suitability of a teaching-learning process is understood as the degree to which this process (or a part of it) possesses certain characteristics that qualify it as optimal or adequate for achieving the alignment between the personal meanings acquired by students (learning) and the intended or implemented institutional meanings (teaching), considering the available circumstances and resources (environment) (Godino, Batanero & Font, 2007).

This is a multidimensional construct broken down into the following partial criteria of didactic suitability (Font, Planas & Godino, 2010):

DOI: 10.20396/zet.v32i00.8676348

Epistemic Suitability: Refers to the degree of representativeness and interconnection of the implemented (or intended) institutional meanings in relation to a reference meaning. Tasks or problem situations are a fundamental component of this dimension and must involve various mathematical objects and processes.

Ecological Suitability: Degree to which the study process aligns with the educational project of the center, the school, and society, and adapts to the conditions of the environment in which it is developed.

Cognitive Suitability: Degree to which the intended and implemented meanings are within the students' potential development zone, as well as the proximity of the achieved personal meanings to the intended/implemented meanings.

Affective Suitability: Degree of students' involvement (interests, emotions, attitudes, and beliefs) in the study process.

Interactional Suitability: Degree to which the didactic configurations and classroom discourse allow for the identification of potential semiotic conflicts (detectable a priori) and the resolution of conflicts that arise during the instructional process.

Mediational Suitability: Degree of availability and adequacy of the material and temporal resources necessary for the development of the teaching-learning process.

This tool can be very useful not only for organizing and analyzing teachers' discursive practices regarding how the instructional process should be but also for evaluating the practices involved in determining the intended, implemented, and assessed meanings. The use of the Criteria of Didactic Suitability has played a significant role in various teacher training processes, as it is a content to be taught with the objective of being used as a guideline to organize one's practice (Font, Calle & Breda, 2023).

To operationalize the Criteria of Didactic Suitability, it is necessary to break them down into components and indicators (Breda & Rosário Lima, 2016; Breda, Pino-Fan & Font, 2017) according to Table 1, which can serve as a guide for the analysis and evaluation of the didactic suitability of any instructional process.

Table 1 - Criteria of Didactic Suitability (CID) and their Components

CID	Components
Epistemic	Errors, Ambiguities, Richness of Processes, Representativity
Cognitive	Prior Knowledge, Curricular Adaptation to Individual Differences, Learning, High Cognitive Demand
Interactional	Teacher-Student Interaction, Student-Student Interaction, Autonomy, Formative Assessment
Mediational	Material Resources, Number of Students, Classroom Schedule and

DOI: 10.20396/zet.v32i00.8676348

	Conditions, Time
Affective	Interests and Needs, Attitudes, Emotions
Ecological	Curricular Adaptation, Intra- and Interdisciplinary Connections, Socio-labor Utility, Didactic Innovation

Source: Based on Morales-López and Font (2019).

Lesson Study and Teacher Training

Lesson Study (LS) constitutes a strategy involving the development of action-research processes with characteristics inherent to this methodology, such as being participatory, collaborative, reflective, and transformative of practice. LS is related to action research due to its cyclical, spiral, open, and continuous process, which opens possibilities for constant inquiry into situations that concern teachers. Its interactive phases are: Phase 1) Define the problem, Phase 2) Cooperatively design an "experimental lesson," Phase 3) Teach and observe the lesson, Phase 4) Collect evidence and discuss, Phase 5) Analyze and revise the lesson, Phase 6) Develop the revised lesson in another class and observe again, and Phase 7) Discuss, evaluate, and reflect on new evidence and disseminate the experience (Soto-Gómez & Pérez-Gómez, 2015; Vásquez-Suárez, 2017).

Due to its contributions to strengthening teaching practice, LS has occupied an important place in both initial (Del Río et al..., 2019; Portilla & Leyva, 2017) and ongoing teacher training (Del Río, 2021; Luengo, Jaramillo, Bonito & Arias, 2021; Sumba, 2022) in various contexts such as Brazil (Veiga, Schaetzle & Baldin, 2018), Spain (Pérez-Gómez, Soto-Gómez & Serván-Núñez, 2015), and Chile (Mena, 2009), among others. The processes and characteristics associated with LS, which enhance the teacher's role, involve collaborative work. This requires a group of teachers, brought together by similar concerns, to embark on the inquiry process. As Sumba, Pérez and Sgreccia (2022) assert, "Thus, it is they who, based on the identification of knowledge, didactic, and pedagogical problems, jointly propose and develop alternative solutions, demonstrating an interest in strengthening student learning" (p. 232).

Collaborative work requires deliberate and active participation to propose alternative solutions (Phase 1 LS), leveraging the experiences and knowledge each teacher possesses to strengthen the action proposal. It also includes support during lesson development (Phase 2 LS) to observe, record data, and later analyze it (Phases 5 and 7 LS). Promoting collaborative work, especially in the Ecuadorian context, involves dispelling the individualistic culture and the idea of the teacher as the "owner of the classroom" to open it to colleagues for observation, learning, and re-constructing knowledge that underpins practice.

Additionally, reflection is an inherent activity in LS, as teachers identify their strengths, limitations, successes, failures, etc., from the experience generated in lesson development. According to Schön (1992; 1998), two moments stand out: in and on action. The first moment (reflection in action) generally goes unnoticed and occurs during the very act of

DOI: 10.20396/zet.v32i00.8676348

teaching, in other words, teachers “act spontaneously without thinking about the action to be taken” (Elliott, 2000, p. 90). The second moment (reflection on action) happens when teachers revisit what happened, based on recorded observations, photographs, videos, etc. According to Ponte, Quaresma, Mata, and Baptista (2016), this collaborative reflection allows teachers to observe their colleagues’ practice, reflect critically on their own practice.

These and other characteristics lead to considering LS as an alternative in teacher training by promoting the reconstruction of practical knowledge into practical thinking. Practical knowledge encompasses a set of knowledge, skills, attitudes, values, and emotions that operate unconsciously, automatically, and implicitly in the teacher’s daily life. The processes, characteristics, and active participation in LS bring such dimensions to a conscious, reflective, and explicit state (Soto-Gómez & Pérez-Gómez, 2015). In this sense, LS is linked with training and research processes recursively (Sumba, 2023); these contribute to constructing the figure of the teacher-researcher (Elliott, 2000; Stenhouse, 2003), particularly in the Ecuadorian context.

In this vein, within the Ecuadorian context, particularly at the National University of Education (UNAE - a flagship university focused primarily on teacher training), processes of reflection, research, collaborative work, reconstruction of practical knowledge, and practicum, among others, are valued to train teachers capable of responding to current societal demands. Hence, LS is emphasized in the teacher training processes in its pedagogical model, considering that practical thinking and teacher competencies “require ongoing processes of research and reflection on action. [Thus] they must be trained as researchers of their own practice to identify and regulate the implicit and explicit resources that comprise their competencies and professional human qualities” (UNAE, 2013, p. 24).

Accordingly, LS is integrated into the curriculum plan of the programs, and this strategy is addressed from both theoretical and practical perspectives. Theoretically, it involves an introduction to its characteristics, phases, contributions, etc., while practically, it is linked to the practicum, where future teachers, in pairs or pedagogical triads, experience the LS phases. Each phase is accompanied by the UNAE teacher with the support of the professional tutor (classroom teacher). The experience generated in the development of LS is systematized in the integrative knowledge project (called PIENSA). This strategy thus favors the experimentation of theory and the theorization of practice through situations that stimulate the development and strengthening of professional competencies.

Methodology

This study was exploratory and analytical-interpretative in nature, aiming to determine the criteria, components, and indicators of Didactic Suitability that emerged from a Lesson Study (LS) process conducted virtually during the pandemic. The focus was on teaching addition without regrouping in the second year of primary education (EGB). The analysis was carried out systematically and objectively (Campos & Turato, 2009; Noguero, 2002), examining the content of information collected through interviews and recordings of

DOI: 10.20396/zet.v32i00.8676348

virtual sessions, covering the design, execution, and reflection of the lesson. The group (identified as G1LS) consisted of three teachers (D1, D2, and D3), who were participating in a distance teacher professionalization program (Sumba, Pérez & Sgreccia, 2022), organized by UNAE in collaboration with the Ministry of Education and subsequently with the Technical Secretariat of the Special Amazonian Territorial Circumscription.

The LS process, which allowed the identification of the Didactic Suitability Criteria (CID) that emerged in this context, was based on a publication by the second author of this article in 2022: Lesson Study in Teacher Training: Experience in a Remote Education Context for Teaching Addition without Regrouping (Sumba, Pérez & Sgreccia, 2022). This author also acted as the observer of the practice, applying the techniques and data sources listed in Table 2 (consistent with the respective phases of LS): semi-structured interviews (Interview2), document analysis (Act. 2.2 and 3.4), and discussion groups (Group-Disc1 and 3), which were revisited in this article for analysis and the connection between LS and CID.

Table 2 - LS Phases and Techniques Employed

LS Phases	Technique	Code
Phase 1	Micro-story	Microst
Phase 2	Document analysis	Act. 2.2
	Interview	Interv2
Phase 3	Observation	Observ1
Phase 4	Document analysis	Group-Disc1
Phase 5	Discussion group	Activ3.4
Phase 6	Observation	Observ2
Phase 7	Discussion group	Group-Disc3

Source: The authors, based on Sumba et al. (2022)

The identification of the Didactic Suitability Criteria (CID) that emerged in the LS discussion was carried out by analyzing what the teachers mentioned during the process. For example, in the discussion, "we changed the song Los pollitos suman to La gallina turuleca because it is more interesting and motivating" (Group-Disc1), the affective suitability, which values the interests and emotions of the students, could be easily identified. Thus, working with content analysis using a priori categories (the components and indicators of the Didactic Suitability Criteria) allowed the interpretation of what the group of teachers would consider in reflecting on their practice.

The cyclical process of the LS led the teachers to actively engage in investigating

DOI: 10.20396/zet.v32i00.8676348

their own practice through phases that required constant discussions, reflections, analysis, and collaborative work, among others. The phases developed by the group were: Phase 1. Definition of the problem; Phase 2. Cooperative design of an "experimental lesson"; Phase 3. Teaching and observation of the lesson; Phase 4. Collection of evidence and discussion; Phase 5. Analysis and review of the lesson; Phase 6. Development of the revised lesson in another class and further observation; and Phase 7. Discussion, evaluation, and reflection on the new evidence and dissemination of the experience.

Results

In Phase 1, when defining the problem for the development of LS, some of the DSIs emerged during the participants' discussion. Teacher D1 highlighted the need to address mathematical problems in the second grade of EGB, due to difficulties in developing skills related to addition without regrouping, as "they do not identify the place value of units and tens, some do not have a clear concept of addition, difficulty in placing quantities on the positional board, lack of resources" (Sumba, Pérez & Sgreccia, 2022). This reflection relates to epistemic and cognitive suitability, which focus on prior knowledge and concept-based learning that students must achieve. Ecological suitability is also present due to the teacher's concern to achieve skill development at this level, as well as considering the suitability of means due to the lack of didactic resources for teaching additions.

The development of LS reveals certain questions related to the subjective dimension of practical knowledge, emotions: "I am happy because I continue working with my students, but not 100%, because it is not the same to be behind a computer as to be directly with my students in the classroom" (D2 - Entrev1). On the other hand, affective relationships were present between students and teachers, which according to these teachers were more favorable during face-to-face interactions. This is evidenced in some reflections: "For me, the most important thing is to work with the child in person, to be attentive to them, to motivate them, to accompany them, to feel what affection is" (D3 - Entrev1). Also, at the group level, for example, "The negative thing right now is not being able to connect with all the students and therefore there is no visual, auditory, kinesthetic contact" (G1LS - Activ2.1). Reflections that demonstrate the presence of the affective suitability criterion, which concerns the interests and needs of students, as well as their attitudes and emotions.

Both in the design and execution of the lesson (Phase 2 and Phase 3 LS), teachers consider relating new learning to students' prior experiences or knowledge (epistemic suitability indicator). For example, the proposed problem incorporates elements and characteristics accessible to students, contextualized in the rural environment where they live, as seen in Figure 1 (Observ1). This situation is affirmed by the group, stating that "the content of the problem must be in accordance with the context, according to the environment to make it easier for them" (Act. 2.2.).

DOI: 10.20396/zet.v32i00.8676348

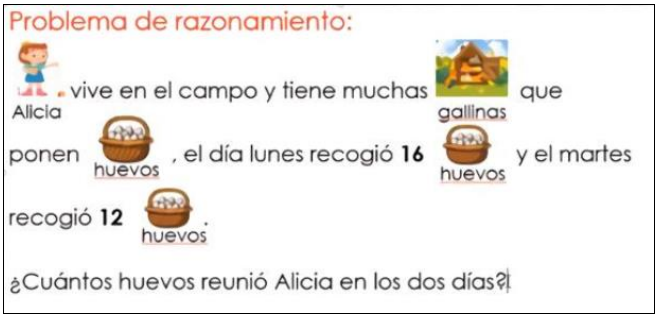

Figure 1. Problem statement: prior knowledge and its relationship with students' learning level.
Source: Sumba, Pérez and Sgreccia (2022)

Following this line of contextualization, the problem is easily tailored to the skills that children have regarding reading, for which teachers consider it necessary to use pictograms. This aspect refers to mediational suitability.

Teachers' reflections also reveal certain concerns regarding the method used for skill development, as evidenced by their previous experience in similar face-to-face processes, which now poses a challenge in adapting to the new virtual context, related to interactional suitability. In the words of one teacher:

> (...) we used to implement it in person, but now it's a bit challenging in this virtual way. That's what I mean, we need to delve deeper to see what utilities it has, what benefits, mainly how to develop this methodology virtually (D1-Entrev2).

Consequently, the impact of the new modality in which teachers had to conduct practice, virtuality, has been evidenced; reflecting on feasibility and the limitations they faced, including limited internet connectivity (mediational suitability indicators). In this context, teachers propose questions for students and prepare resources in advance, such as Cuisenaire rods, based on the graphical and symbolic phases of Mathematics (Sumba, Pérez, & Sgreccia, 2022), aiming to provide direct experiences to facilitate knowledge construction.

In a subsequent phase, in the Discussion Phase (Phase 4 LS, G1LS, Grup-Disc1), the group of teachers validates and emphasizes the importance of allowing students to first observe and manipulate concrete materials, before being able to graphically and symbolically represent mathematical processes necessary to solve problems. This perspective remains prominent in later LS phases, as revisited in the new analysis and reflection (Phase 7 LS) by teacher D2: "Math doesn't become boring for them, because Math is often tedious, they are even afraid of it. But if you give them the material they are happy, they even like the material from the middle". (Grup-Disc3).

At this LS stage, mediational suitability is easily identified when teachers ensure that students manipulate materials for their learning; additionally, epistemic suitability is evident in the teachers' interest in students' understanding of graphical and symbolic representation of mathematical processes, and affective suitability highlights the "liking" for learning mathematics.

Sumba, Pérez and Sgreccia (2022) point out that in Phase 5 (Activ3.4), related to

DOI: 10.20396/zet.v32i00.8676348

lesson redesign, a space was created to restructure the proposal based on initial observations, considering adjustments through didactic worksheets for three students who could not attend synchronous sessions due to connectivity issues. Teachers ensured that the worksheet served as a guide for solving mathematical problems, emphasizing the use of the place value table for correct number placement; this activity was directed at those who missed sessions, ensuring they also developed skills and knowledge acquired by their peers. Another aspect in Phase 5 was the importance placed on problem statement; for this, teachers decided to present students with the data on which they were to base their problem statement. Thus, during lesson development (Phase 6 LS), teacher D3 asked four students to read the problem presented, noting that three presented data different from what was requested, while one presented it as proposed (Observ2). Reflection on this arose in Phase 7: "yes, it works out in person [...] I talked with D3 and said: let's do it this way. And I don't know now, it's a bit difficult to do it virtually" (D1, Grup-Disc3).

The development of LS in Phase 5, Phase 6, and Phase 7 allows visibility of the DID, specifically cognitive suitability, when teachers, through reflection, ensure that all students achieve knowledge (learning); interactional suitability is identified when they indicate that the process is better in face-to-face settings, and finally, media suitability emerges when reflecting on the resources used in this study modality.

The results of the coding performed in the LS phases with the described process demonstrate the feasibility of making connections by identifying and analyzing the DID emerging from reflection in each phase of this process and reaching consensus (Calle, Breda, & Font, 2022), as depicted in Figure 2; where the six criteria of EOS are also visible.

Table 3 - Relationship between LS phases and emerging CID

LS Phase	CID
Phase 1	ecological, cognitive, emotional, and mediational
Phase 2	epistemic, mediational, and interactional
Phase 3	epistemic, mediational, and interactional
Phase 4	mediational, epistemic, and affective
Phase 5	cognitive, interactional, and mediational
Phase 6	cognitive, interactional, and mediational
Phase 7	cognitive, interactional, and mediational

Source: Authors of this paper

Conclusions

The discussion led by participating teachers in LS easily promotes a reinterpretation in terms of criteria, components, and indicators of the CID construct. In the first phase, for

DOI: 10.20396/zet.v32i00.8676348

instance, the Ecological Suitability Criterion is identified, with its component "adaptation to the curriculum" and its descriptor: contents, their implementation, and evaluation align with curriculum guidelines. Additionally, the Cognitive Suitability Criterion, with the component "prior knowledge" and its descriptor: Students either possess or lack the necessary prior knowledge for studying the topic; the Emotional Suitability Criterion, with the component "emotions" and its descriptor: promoting self-esteem while avoiding rejection, phobia, or fear of mathematics; and finally, the Mediational Suitability Criterion, with the component "material resources" and its descriptor: using manipulative and computer resources to introduce situations, languages, procedures, and arguments adapted to the intended meaning.

This exercise of identifying and relating discussion moments in the first phase of LS with CID has been extended to each phase of the process, identifying moments of teacher practice reflection and the criteria, components, and indicators of CID emerging in the LS process. It demonstrates there is an implicit or explicit consensus between the class developer and participating-observing teachers in positively evaluating aspects considered in the instructional process (Breda, Font & Pino-Fan, 2018), as mentioned in Hummes, Breda, and Font (2022a; 2022b).

Reinterpreting the contributions of participating LS teachers in terms of CID criteria, components, and indicators leads to the inference that the trend toward reflecting on their own practice occurs in all LS phases, starting with the foundations they work on in their teaching practice: level, prior knowledge, student interests, and resources to be used. This situation has allowed for the identification of CID evidenced in a stronger relation with suitability of means and interactional suitability, in addition to cognitive, ecological, and affective suitability, albeit to a lesser extent, epistemic aspects, coinciding with Calle and Breda (2019) on the difficulty of incorporating this last suitability in instructional processes.

The six dimensions of CID emerging in LS phases determine the possibility of combining these two constructs since, as mentioned in Hummes, Font, and Breda (2019), certain Didactic Suitability Criteria are present in the process of designing, developing, and reflecting on the lesson. Therefore, working with this tool helps enhance teacher practice reflection, generating skills on what and why to observe, a weakness identified in teaching practice during lesson development (Sumba, 2023), seeking to ensure a thorough observation of the aspects required for the cyclical process of teacher competence research and formation. This situation coincides with the study conducted by Breda, Hummes, Silva, and Sánchez (2021), recommending a more significant role for the observation phase in implementing the LS methodology, reaching consensus on what has been observed before moving to the reflection phase. CID, as a tool that defines guidelines for teacher practice reflection, can better guide the LS methodology, promoting group debates or discussions in each phase and facilitating autonomous analysis and reflection by the group itself. In other words, CID serves as scripts for observing and reflecting, without necessarily requiring a third person (the researcher) to intervene with provocative questions (Sumba, 2023) related to topics identified in the initial phase to ensure the consolidation of the proposed objectives.

DOI: 10.20396/zet.v32i00.8676348

The aforementioned aligns with Font, Calle and Breda (2023), in the sense that the LS methodology can be considered a somewhat loosely structured and broad reflection phase aimed at improving the mathematics teaching and learning process. Therefore, it is expected that in the planning phase, observation phase, reflection phase, and redesign phase aimed at improvement, teachers implicitly or explicitly use many CID indicators and components to evaluate certain aspects of the experience conducted. Thus, the significant contribution of this tool to teacher practice reflection is defined.

Acknowledgments

Work carried out within the framework of the project PID2021-127104NB-I00 (MICIU/AEI/10.13039/501100011033) and by "FEDER A way of making Europe" and co-financed by the Vice-Rectorate of Research of the University of Cuenca, Cuenca-Ecuador.

References

Alcaraz, D., Breda, A., & Sala, G. (2022). Valoração de uma unidade didática sobre sistemas de equações lineares: um trabalho de reflexão sistematizada sobre a própria prática. In E. S. Vasconcelos, J. F. Staffa da Costa, V. M. do Rosário Lima (Org.). *O entrelaçar das pesquisas com as práticas pedagógicas:* notas sobre o ensino de Ciências da Natureza e da Matemática na Educação Básica (pp. 261-281). Porto Alegre: EDIPUCRS.

Breda, A., & do Rosário Lima, V. (2016). Estudio de caso sobre el análisis didáctico realizado en un trabajo final de un máster para profesores de matemáticas en servicio. *Journal of Research in Mathematics Education, 5*(1), 74–103. https://doi.org/10.17583/redimat.2016.1955

Breda, A., Font, V., & Pino-Fan, L. (2018). Criterios Valorativos y Normativos en La Didáctica de las Matemáticas: el Caso del Constructo Idoneidad Didáctica. *Bolema, 32*(60), 255-278. https://doi.org/10.1590/1980-4415v32n60a13

Breda, A., Hummes, V., da Silva, R. S., & Sánchez, A. (2021). El papel de la fase de observación de la implementación en la metodología Estudio de Clases. *BOLEMA: Boletim de Educação Matemática, 35*(69), 263–288. https://doi.org/10.1590/1980-4415v35n69a13

Breda, A., Pino-Fan, L. R., & Font, V. (2017). Meta Didactic-Mathematical Knowledge of Teachers: Criteria for The Reflection and Assessment on Teaching Practice. *Eurasia Journal of Mathematics, Science and Technology Education, 13*(6), 1893–1918. https://doi.org/10.12973/eurasia.2017.01207a

Calle, E., & Breda, A. (2019). Reflexión sobre la complejidad de los objetos matemáticos en la formación inicial de profesores. In D. Aguilar, M. Cobos, L. C. Cortés, & E. Campozano (Eds), *La Investigación Educativa en un Mundo en Constante Transformación* (pp. 29-50). Cuenca: ASEFIE. Available in: https://unae.edu.ec/noticias/libro-la-investigacion-educativa-en-un-mundo-en-constante-transformacion/.

Calle, E., Breda, A., & Font, V. (2022). The complexity of the notion to teach in the assessment of the pre-professional practice of future Ecuadorian mathematics teachers.

DOI: 10.20396/zet.v32i00.8676348

REDIMAT – Journal of Research in Mathematics Education, 11(3), 218-249. https://doi.org/10.17583/redimat.10986

Campos, C. J. G., & Turato, E. R. (2009). Análisis de contenido en investigaciones que utilizan la metodología clínico-cualitativa: aplicación y perspectivas. *Revista Latino-Americana de Enfermagem*, 17, 259-264.

Del Río, J. L. (2021). La 'Lesson Study' como estrategia formativa para aprender a enseñar en la universidad. Algunas reflexiones derivadas de un Proyecto de Innovación234 Educativa desarrollado en la Universidad Nacional de Educación (UNAE). *Revista Complutense de Educación*, 32(2), 171-180. https://doi.org/10.5209/rced.68205

Del Río, J., Rodríguez, M., Barberi, O., & Tur, G. (2019). Caracterización de los procesos de formación en investigación e innovación educativa desarrollados en la Universidad Nacional de Educación (UNAE). In J. Del Río (Ed), *Investigar para Conocer, Innovar para Mejorar. La Formación de Docentes Investigadores en Educación Inicial* (pp.11-136). Editorial UNAE. Available in: http://repositorio.unae.edu.ec/bitstream/56000/1299/1/Investigar%20conocer%20%2009.pdf

Elliott, J. (2000). *La Investigación Acción en Educación*. Morata. (Trabajo original publicado en 1990).

Esqué de los Ojos, D., & Breda, A. (2021). Valoración y rediseño de una unidad sobre proporcionalidad utilizando la herramienta Idoneidad Didáctica. *Uniciencia*, 35(1), 38–54. http://dx.doi.org/10.15359/ru.35-1.3

Font, V., Calle, E., & Breda, A. (2023). Uso de Los Criterios de idoneidad Didáctica y la metodología Lesson Study en la formación del profesorado de matemáticas en España y Ecuador. *PARADIGMA*, 44(2), 376-397. https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p376-397.id1424

Font, V., Planas, N., & Godino, J. D. (2010). Modelo para el análisis didáctico en educación matemática. *Infancia y aprendizaje*, 33(1), 89–105. https://doi.org/10.1174/021037010790317243

Godino, J., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. ZDM. *The International Journal on Mathematics Education*, 39(1-2), 127-135.

Hummes, V. B., Font, V., & Breda, A. (2019). Uso combinado del estudio de clases y la idoneidad didáctica para el desarrollo de la reflexión sobre la propia práctica en la formación de profesores de matemáticas. *Acta Scientiae*, 21(1), 64-82.

Hummes, V., Breda, A., & Font, V. (2022a). El desarrollo de la reflexión sobre la práctica en la formación de profesores de matemáticas: una mirada desde el Lesson Study y los criterios de idoneidad didáctica. In J. Luco, L. Pin, M. Pochulo & W. Castro (Eds.), *Enfoque onto-semiótico del conocimiento y la instrucción matemáticos: Investigaciones y desarrollos en América Latina* (pp. 221-241). Universidad de Los Lagos Editorial.

Hummes, V., Breda, A., & Font, V. (2022b). Critérios de adequação didática implícitos na reflexão de professores quando planejam, implementam e redesenham uma aula em uma experiência de Lesson Study. In A. Richit, J. P. da Ponte, E. S. Gómez (Eds), *Lesson*

DOI: 10.20396/zet.v32i00.8676348

Study na formação inicial e continuada de professores (53–88). São Paulo: Livraria da Física.

Hummes, V., Breda, A., Font, V., & Seckel, M. J. (2023). Improvement of Reflection on Teaching Practice in a Training Course That Integrates the Lesson Study and Criteria of Didactical Suitability. *Journal of Higher Education Theory & Practice*, 23(14), 208–224. https://doi.org/10.33423/jhetp.v23i14.6395

López-Noguero, F. (2002). El análisis de contenido como método de investigación. *XXI, Revista de Educación*, 4, 167-179.

Luengo, D., Jaramillo, N., Bonito, M., & Arias, M. (2021). La Lesson Study: una experiencia pedagógica exitosa en la Universidad Nacional de Educación. *Mamakuna*, 17, 57–66. https://revistas.unae.edu.ec/index.php/mamakuna/article/view/525

Mena, A. (2009). *El Estudio de Clases Japonés en Perspectiva*. Colección Digital Eudoxus.

Morales-López, Y., & Font, V. (2019). Valoración realizada por una profesora de la idoneidad de su clase de matemáticas. *Educação e Pesquisa*, 45, 1-19. https://doi.org/10.1590/S1678-4634201945189468

Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education* (pp. 1–12). Springer.

Pérez-Gómez, Á. I., Soto-Gómez, E., & Serván-Núñez, M. J. (2015). Lesson Studies: re-pensar y re-crear el conocimiento práctico en cooperación. *Revista Interuniversitaria de Formación del Profesorado,* 29(3), 81-101. https://www.redalyc.org/articulo.oa?id=27443871006

Pochulu, M., Font, V., & Rodríguez, M. (2016). Desarrollo de la competencia en análisis didáctico de formadores de futuros profesores de matemática a través del diseño de tareas. *Revista Latinoamericana de Investigación en Matemática Educativa, 19*(1), 71-98. https://doi.org/10.12802/relime.13.1913

Ponte, J., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2016). O estudo de aula como processo de desenvolvimento profissional de professores de matemática. *Bolema, 30*(56), 868-891. http://dx.doi.org/10.1590/1980-4415v30n56a01

Portilla, G., & Leyva, A. (2018). Innovación de la Práctica Docente en la UNAE, Ecuador recurriendo a la Web 2.0. *Revista Scientific*, 3(7), 140–154. https://doi.org/10.29394/Scientific.issn.2542-2987.2018.3.7.7.140-154

Schön, D. (1992). *La Formación de Profesionales Reflexivos.* Paidós. (Trabajo original publicado en 1987).

Schön, D. (1998). *El Profesional Reflexivo. Cómo Piensan los Profesionales cuando Actúan*. Paidós. (Trabajo original publicado en 1982).

Seckel, M. J. (2016). *Competencia en análisis didáctico en la formación inicial de profesores de educación general básica con mención en matemática* [Tesis doctoral]. University of Barcelona Digital Repository. Disponible en: http://hdl.handle.net/2445/99644

Sol, T., Sánchez, A., Breda, A., & Font, V. (2023). Didactic suitability criteria in teachers' practical argumentation in the phase of design of a Lesson Study cycle about functions. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.), *Proceedings of the*

DOI: 10.20396/zet.v32i00.8676348

46th Conference of the International Group for the Psychology of Mathematics Education (Vol. 4, pp. 227–234). PME 46

Soto-Gómez, E., & Pérez-Gómez, Á.I. (2015). Lessons Studies: Un viaje de ida y vuelta recreando el aprendizaje comprensivo. *Revista Interuniversitaria de Formación del Profesorado,* 29(3), 15-28. https://www.redalyc.org/pdf/274/27443871002.pdf

Stenhouse, L. (2003). *Investigación y Desarrollo del Currículum*. Morata. (Trabajo original publicado en 1975).

Sumba, V. (2023). *Formación docente: Aportes de la Lesson Study en la profesionalización docente. Estudio de caso en la Universidad Nacional de Educación*. Tesis de doctorado en Educación [not published]. Rosario: Universidad Nacional de Rosario.

Sumba, V. (2022). La formación de docentes en ejercicio a través de la Lesson Study: preocupaciones en una realidad compleja. *Praxis Educativa,* 26(1), 1-17. https://doi.org/10.19137/praxiseducativa-2022-260109

Sumba, V., Pérez, P., & Sgreccia, N. (2022). Lesson Study en la formación docente: experiencia en contexto de educación remota para la enseñanza de suma sin reagrupación. En A. Richit, J. P. da Ponte & E. Soto (Coords.), *Estudos de aula na formação inicial e continuada de professores* (pp. 231-264). São Paulo: Livraria da Física.

UNAE. (2013). *Modelo Pedagógico de la UNAE*. Available in: http://repositorio.unae.edu.ec/bitstream/56000/1180/1/M_B73-73-98.pdf

Vásquez-Suárez, K. (2017). *La reconstrucción del conocimiento práctico en docentes de infantil com relación a la metodología de enseñanza a través de la Lesson Study:* Dos estudios de caso. Tesis de Doctorado em Educación. Facultad de Ciencias de la Educación - Departamento de Didáctica y Organización Escolar. Málaga: Universidad de Málaga. Repositorio Institucional de la Universidad de Málaga: https://riuma.uma.es/xmlui/handle/10630/15968

Veiga, M., Schaetzle, J., & Baldin, Y. (2018). Lesson Study como meio para a formação inicial e continuada de professores de Matemática - Entrevista com Yuriko Yamamoto Baldin. *Boletim Gepem,* 73, 115-130. https://doi.org/10.4322/gepem.2018.020.